ECE 341 - Solution to Homework #2

Card Games. Due Thursday, May 21st

Please make the subject "ECE 341 HW#2" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

The card game *bridge* uses a 52-card deck. Each person is dealt 13 cards for their hand.

1) How many different hands are possible? (order doesn't matter)

$$N = \binom{52}{13} = 635,013,559,600$$

different hands

2) What is the probability of having 7 cards of one suit in your hand?

(4 suits, choose 1) * (13 cards in any suit, choose 7) * (39 other cards in the deck, choose 6)

$$M = \binom{4}{1}\binom{13}{7}\binom{39}{6}$$

$$M = 22,394,644,272$$

The probability of having 7 cards of a suit are then

$$p = \left(\frac{M}{N}\right) = \left(\frac{22,394,644,272}{635,013,559,600}\right) = 0.035266$$

There is a 3.5266% chance of having 7 cards of one suit (28 : 1 odds against)

3) What is the probability of having all 4 Aces in your hand?

M = (4 aces, choose 4) (48 other cards, choose 9)

$$M = \binom{4}{4}\binom{48}{9} = 1,677,106,640$$

The probability of having 4 aces is

$$p = \left(\frac{M}{N}\right) = \left(\frac{1,677,106,640}{635,013,559,600}\right) = 0.002641$$

There is a 0.2641% chance of being dealt all four aces (378 : 1 odds against)

4) Compute the odds of a flush in 5-card stud.

N = number of hands = (52 cards, choose 5)

$$N = \binom{52}{5} = 2,598,960$$

The number of hands that are a flush are

M = (4 suits, choose 1) (13 cards of a suit, choose 5)

$$M = \binom{4}{1}\binom{13}{5} = 5148$$

The odds of being dealt a flush are thus

$$p = \left(\frac{M}{N}\right) = \left(\frac{5,148}{2,598,960}\right) = 0.001981$$

There is a 0.1981% chance of being dealt a flush. (504.8 : 1 odds against)

5) Compute the odds of a flush in 5-card draw. Assume you go for a flush if you have four cards of one suit in your opening hand (and draw one card).

This is a conditional probability. To get a flush (A), you could

- B: Be dealt a flush
- C: Be dealt 4 cards of a suit, discard the off card, then draw to a flush, or
- D: Be dealy no pairs, draw 5 cards, and the the 5 cards are all the same suit.

$$p(A) = p(A|B)\ p(B) + p(A|C)\ p(C) + p(A|D)\ p(D)$$

The first and last are easy

$$p(A|B)\ p(B) = (1.000)(0.001981)$$

$$p(A|D)\ p(D) = (0.001981)(0.5)$$

The middle one takes some work.

The number of hands which have 4 cards of the same suit are

M = (4 suits, choose 1) (13 cards of a suit, choose 4) (39 other cards, choose 1)

$$M = \binom{4}{1}\binom{13}{4}\binom{39}{1} = 111,540$$

$$p(C) = \left(\frac{111,540}{2,598,960}\right) = 0.042971$$

The probability of drawing to a flush is then

$$p(A|C) = \left(\frac{\text{9 cards of the same suit remaining in the deck, choose 1}}{\text{47 cards in the deck, choose 1}}\right) = \left(\frac{\binom{9}{1}}{\binom{47}{1}}\right) = 0.191489$$

so

$$p(A|C)p(C) = (0.191489)(0.042971) = 0.008218$$

Add them all up

$$p(A) = 0.011190$$

The probability of a flush in 5-card draw is 0.01190 (89.36 : 1 odds against)

6) Determine the odds of a flush using Matlab and a Monte-Carlo simulation

(kind of tricky - took three tries to get it to work)

5-Card Stud: 100,000 hands

Got 188 flush's

Calculated odds give 0.001981 * 100,000 = 198.1

Experimental is 5.1% lower than calculated

so that's pretty close

5-Card Draw: 100,000 hands

Got 976 flushes

Calculated odds give 1119 flushes

Difference is 12% low

Tried a second time: 100,000 hands

Got 1005 flushes

Tried a 3rd time: 100,000 hands

Got 1027 flushes