ECE 341 - Test #1

Combinations, Permitations, and Discrete Probability

Open-Book, Open Notes. Calculators, Matlab, Tarot cards, Internet allowed. Just not other people.

Please sign if possible (i.e. you did not get help from someone else).

No aid given, received, or observed: __________________________________

Due Sunday, May 31st, 8am

Please make the subject "ECE 341 HW#4" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

1. Combinationas and Permutations (dice)

In game of Farkle, you roll six dice to start the game. What is the probabilty of rolling 3-pair?

- note: 4 of a kind + pair counts as 3-pair
- dice = { xx yy zz, xxxx yy }

xx yy zz

(6 numbers, choose 3) (6 spots for x, choose 2)(4 spots left for y, choose 2)(2 spots for z, choose 2)

$$M_a = \binom{6}{3}\binom{6}{2}\binom{4}{2}\binom{2}{2} = 1800$$

xxxx yy

(6 numbers choose 2)(2 numbers, pick 1 for x)(6 spots for x, choose 4)(2 spots left for y, choose 2)

$$M_b = \binom{6}{2}\binom{2}{1}\binom{6}{4}\binom{2}{2} = 450$$

The total number of ways to get 3 pair is

$$M = M_a + M_b$$

$$M = 2025$$

The probability of 3 pair is then

$$p = \left(\frac{\text{\# successes}}{\text{\# combinations}}\right) = \left(\frac{2250}{6^6}\right) = \left(\frac{2250}{46{,}656}\right) = 0.048225$$

There is a 4.82% chance of rolling 3-pair in Farkle

2. Conditional Probability (cards)

Using conditional probability compute the odds of getting a full-house in 5-card draw.

- A: You are dealt a full house (xxx yy)
- B: You are dealt 3 of a kind (xxx yz), you discard the two cards that don't match, and get a full house
- C: You are dealt 2-pair (xx yy z), discard the off card, and get a full house,
- D: You are dealt a pair (xx abc), discard 3 then draw a full house, or
- E: You are dealt a high-card hand, discard 5, then draw a full house.

A: Dealt full house:

probability of being dealt a full house: $p = \left(\frac{3744}{2{,}598{,}960}\right) = 0.001441$

B: Dealt a 3 of a kind: xxx yz

To get a full house, you have to draw two matching cards

(12 cards with values other than x, choose 1)(4 cards in the deck, choose 2) over (47 cards choose 2)

$$p(X|B) = \frac{\binom{12}{1}\binom{4}{2}}{\binom{47}{2}} = \frac{72}{1081} = 0.066605$$

$$P(X|B)p(B) = 0.066605 \cdot \left(\frac{54{,}912}{2{,}598{,}960}\right) = 0.001407$$

C: Dealt 2-pair

(4 cards in the deck that match one of your cards) / (47 cards, choose 1)

$$p(X|C) = \left(\frac{4}{47}\right) = 0.085106$$

$$p(X|C)p(C) = 0.084106 \cdot \left(\frac{123{,}552}{2.598{,}960}\right) = 0.004046$$

D: Dealt a pair xx ---

Draw yyy (3 cards that match)

M =(12 cards that differ, choose 1)(4 cards in the deck, choose)

$$M_a = \binom{12}{1}\binom{4}{2} = 72$$

Draw xyy

M = (2 cards left, choose 1)(12 cards that differ, choose 1)(4 cards choose 2)

$$M_b = \binom{2}{1}\binom{12}{1}\binom{4}{2} = 144$$

$$p(X|D) = \left(\frac{72+144}{\binom{47}{3}} \right) = \left(\frac{216}{16{,}215} \right) = 0.013321$$

$$p(X|D)p(D) = 0.013321 \cdot \left(\frac{1{,}098{,}140}{2{,}598{,}960} \right) = 0.005629$$

F: High-card hand

same odds as drawing a full house with 5 cards (approx)

$$p(X|F) = 0.001441$$

$$p(X|F)p(F) = 0.001441 \cdot \left(\frac{1{,}302{,}540}{2.598{,}960} \right) = 0.000722$$

The total odds are then

p(full house) = 0.001441 + 0.001407 + 0.004046 + 0.005629 + 0.000722

p(full house) = 0.013245

From Lecture #3, the Monte Carlo simulation resulted in 1190 full houses in 100,000 hands, or p = 0.01190

3. Binomial Distribution

Assume you are rolling a 6-sided die.

- If you roll a 1 or 2, you get 1 point (p = 2/6).
- If you roll a 3-6, you get zero points (q = 4/6).

3a) What is the probability of getting 7 or more points if you roll 10 dice?

$$p(x) = \binom{10}{x}\left(\frac{1}{3}\right)^{x}\left(\frac{2}{3}\right)^{10-x}$$

Sum the total for x = { 7, 8, 9, 10 }

- 7: p(7) = 0.016258
- 8: p(8) = 0.003048
- 9: p(9) = 0.000339
- 10: p(10) = 0.000017

Total: 0.019662

3b) Assume the first three dice are all ones (3 points). Now what is the probabiityof getting 7 or more points when you roll 10 dice (total - including the 3 you already rolled).

Again, this is a binomial distribution for the remaining 7 rolls

$$p(x) = \binom{7}{x}\left(\frac{1}{3}\right)^{x}\left(\frac{2}{3}\right)^{7-x}$$

The sum for x = {4, 5, 6, 7}

- 4: p(4) = 0.128029
- 5: p(5) = 0.038409
- 6: p(6) = 0.006401
- 7: p(7) = 0.000457

Total = 0.173297

4. Uniform Distribution

Let

- A be the result of rolling a 4-sided die { 1, 2, 3, 4}
- B be the result of rolling a 6-sided die { 1, 2, 3, 4, 5, 6 }
- C be the result of rolling a 10-sided die { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }

4a) What is the pdf for the sum

$Y = A + B + C$

Y is the colvolution of A, B, and C

Y = {0, 1, 1, 1, 1} ** {0, 1, 1, 1, 1, 1, 1 } ** {0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1} / (4*6*10)

Easiest to do in matlab

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A = [0,1,1,1,1]' / 4;
B = [0,1,1,1,1,1,1]' / 6;
C = [0,1,1,1,1,1,1,1,1,1,1]' / 10;
AB = conv(A,B);
ABC = conv(AB,C)
```

x	p(x)
0	0
1	0
2	0
3	0.0042
4	0.0125
5	0.0250
6	0.0417
7	0.0583
8	0.0750
9	0.0875
10	0.0958
11	0.1000
12	0.1000
13	0.0958
14	0.0875
15	0.0750
16	**0.0583**
17	**0.0417**
18	**0.0250**
19	**0.0125**
20	**0.0042**

4b) What is the probability that the sum (Y) will be 16 or more?

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sum(ABC(17:21))
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p(x >= 16) = 0.1417

4c) What is the resulting mean and variance of Y?

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k = [0:20]';
m1 = sum(k .* ABC)
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mean = 11.5000

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var = sum( (k - m1).^2 .* ABC )
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variance = 12.4167

5. Geometric & Pascal Distribution

Let the moment generating function for F be

$$F(z) = \left(\frac{a(z-0.5)}{(z-0.9)(z-0.8)}\right)$$

5a) Determine 'a' so that this is a valid moment generating funciton, meaning (all equivalent)

- The total probabity is one
- The zeroth moment is 1.000
- $m_0 = F(z=1) = 1$

$$F(z=1) = \left(\frac{a(z-0.5)}{(z-0.9)(z-0.8)}\right)_{z=1} = 25a$$

a = 0.04

5b) Determine the pdf of F (the inverse z-transform of F(z))

$$F = \left(\frac{0.04(z-0.5)}{(z-0.9)(z-0.8)}\right)$$

$$F = \left(\frac{0.16}{z-0.9}\right) + \left(\frac{-0.12}{z-0.8}\right)$$

$$zF = \left(\frac{0.16z}{z-0.9}\right) + \left(\frac{-0.12z}{z-0.8}\right)$$

$$zf(x) = \left(0.16\,(0.9)^k - 0.12\,(0.8)^k\right)u(k)$$

$$\boxed{pdf(x) = \left(0.16\,(0.9)^{k-1} - 0.12\,(0.8)^{k-1}\right)u(k-1)}$$

5c) Determine the cdf of F (the inverse z-transform of $\left(\left(\frac{z}{z-1}\right)F(z)\right)$

$$C = \left(\frac{0.04(z-0.5)}{(z-0.9)(z-0.8)}\right)\left(\frac{z}{z-1}\right)$$

$$C = \left(\frac{0.04(z-0.5)}{(z-1)(z-0.9)(z-0.8)}\right)z$$

$$C = \left(\left(\frac{1}{z-1}\right) + \left(\frac{-1.6}{z-0.9}\right) + \left(\frac{1.6}{z-0.8}\right)\right)z$$

$$C = \left(\frac{z}{z-1}\right) + \left(\frac{-1.6z}{z-0.9}\right) + \left(\frac{0.6z}{z-0.8}\right)$$

$$\boxed{cdf(x) = \left(1 - 1.6\,(0.9)^k + 0.6\,(0.8)^k\right)\ u(k)}$$

$$\boxed{cdf(x) = \left(1 - 1.44\,(0.9)^{k-1} + 0.48\,(0.8)^{k-1}\right)\ u(k-1)}$$ same answer, different form