Filter Design Example

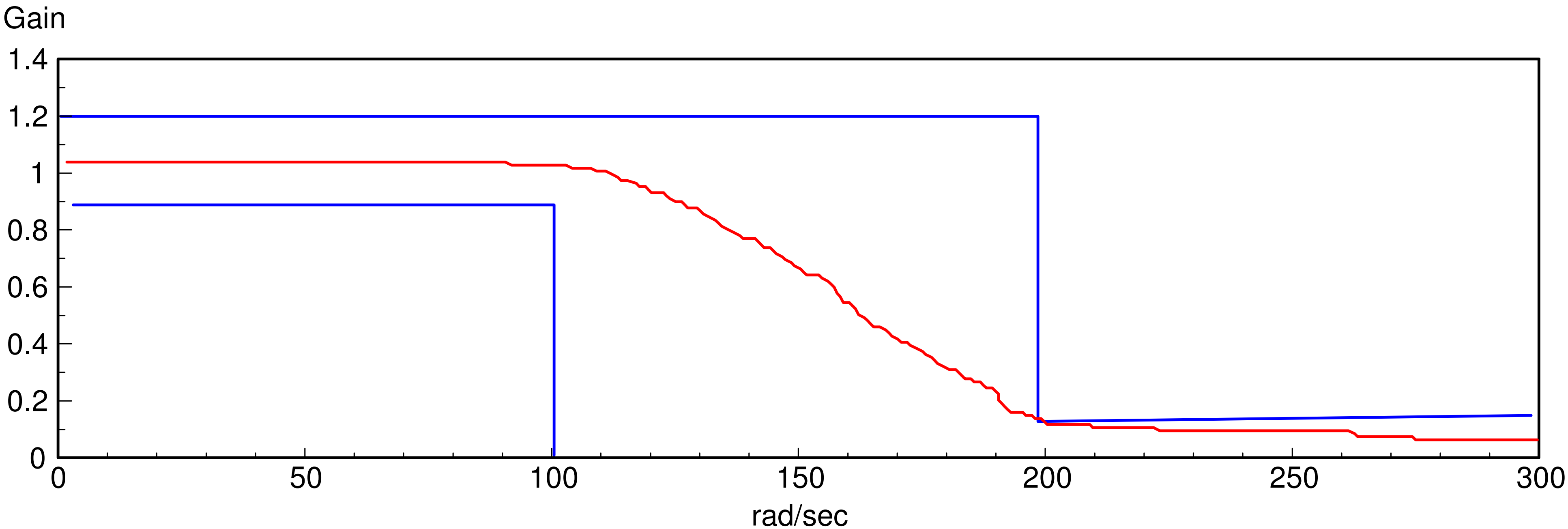
ECE 321: Electronics II

Lecture #10:

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Design a filter which

- Has a maximum gain < 1.2
- Has a gain > 0.9 for frequencies below 100 rad/sec and
- Has a gain < 0.1 for frequencies above 200 rad/sec



Step 1: Determine the order of the filter

The gain drops off as $\left(\frac{1}{\omega}\right)^n$ for an nth-order filter. Assuming the gain at 100 is one, the order required is

$$\left(\frac{100}{200}\right)^n < 0.1$$

$$n > 3.32$$

So, you need at least a 4th-order filter to meet these requirements.

Let n=5 just to be safe.

Step 2: Determine the type of filter

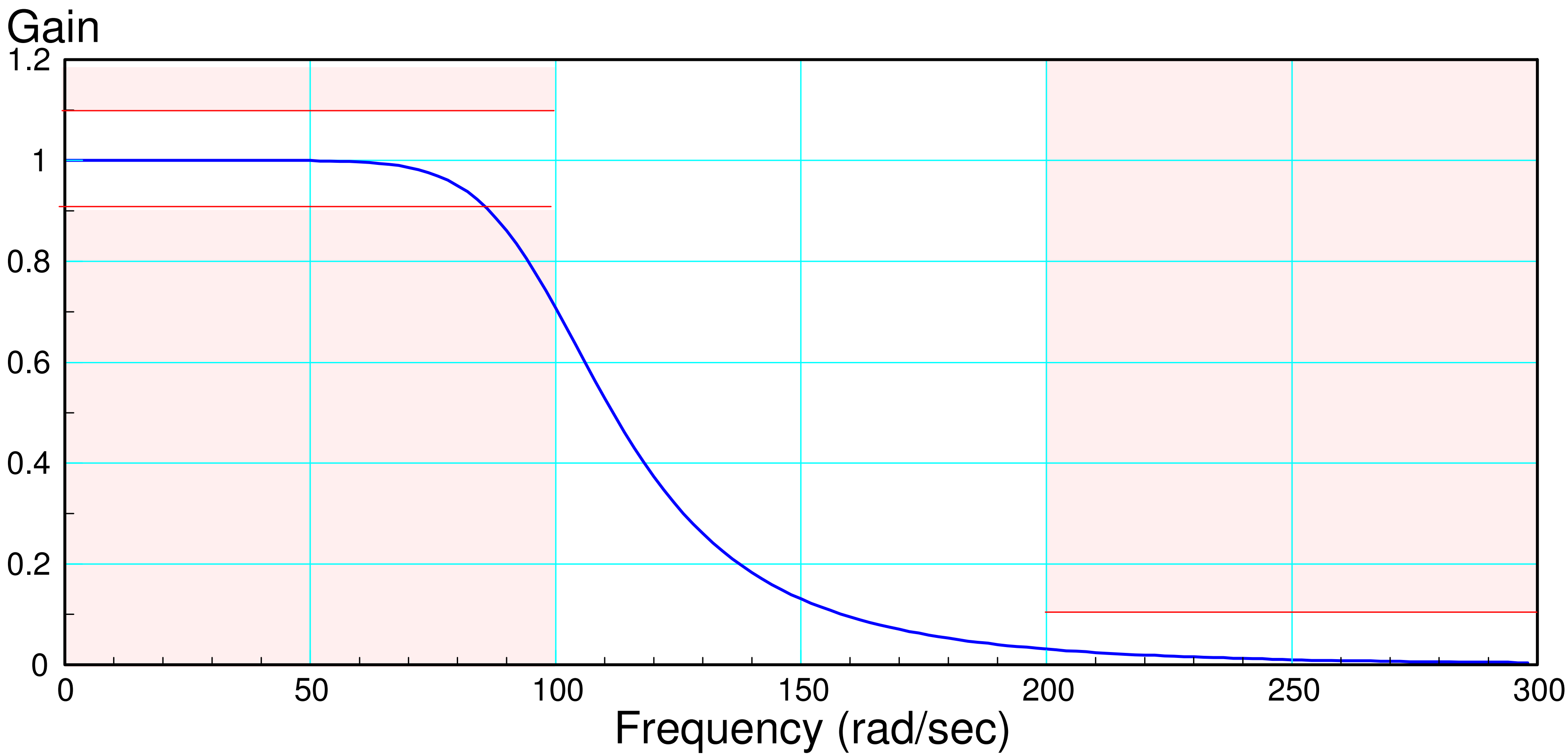
Assume a Butterworth low-pass filter because Butterworth filters are easy to design.

Step 3: Choose the filter's corners.

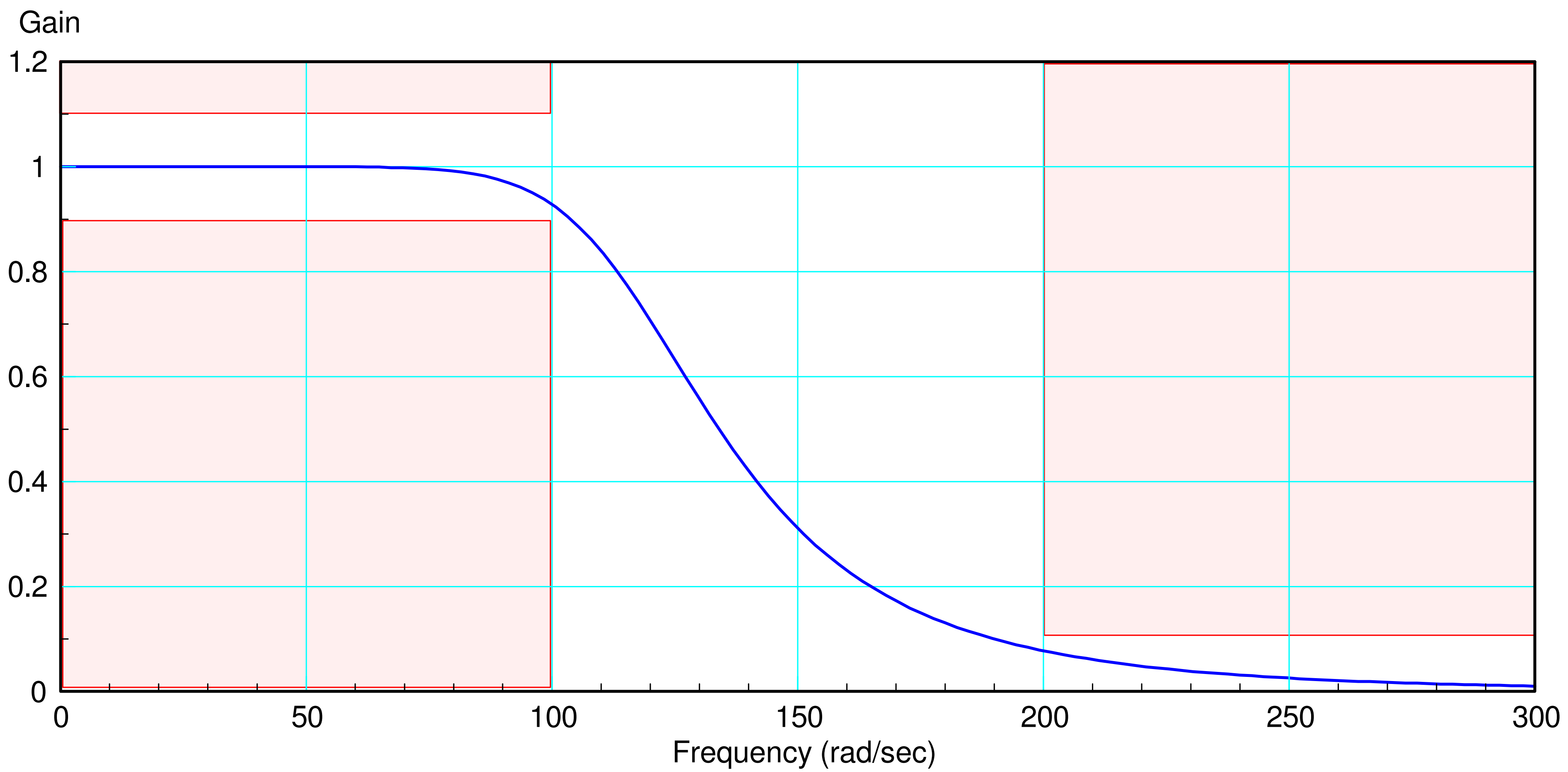
Assume a corner at 100 rad/sec as a start. This gives

$$G(s) = \left(\frac{100^5}{(s+100)\left(s+100\angle\pm 36^0\right)\left(s+100\angle\pm 72^0\right)} \right)$$

Plot the gain vs. frequency along with the requirements



Adjust the corner to 120 rad/sec



This works, so

$$G(s) = \left(\frac{120^5}{(s+120)(s+120\angle\pm 36^0)(s+120\angle\pm 72^0)} \right)$$

Circuit Implementation:

To build this circuit, build it in three sections:

$$G_1 = \left(\frac{120}{(s+120)} \right)$$

$$G_2 = \left(\frac{120^2}{\left(s+120\angle +36^0\right)\left(s+120\angle -36^0\right)} \right)$$

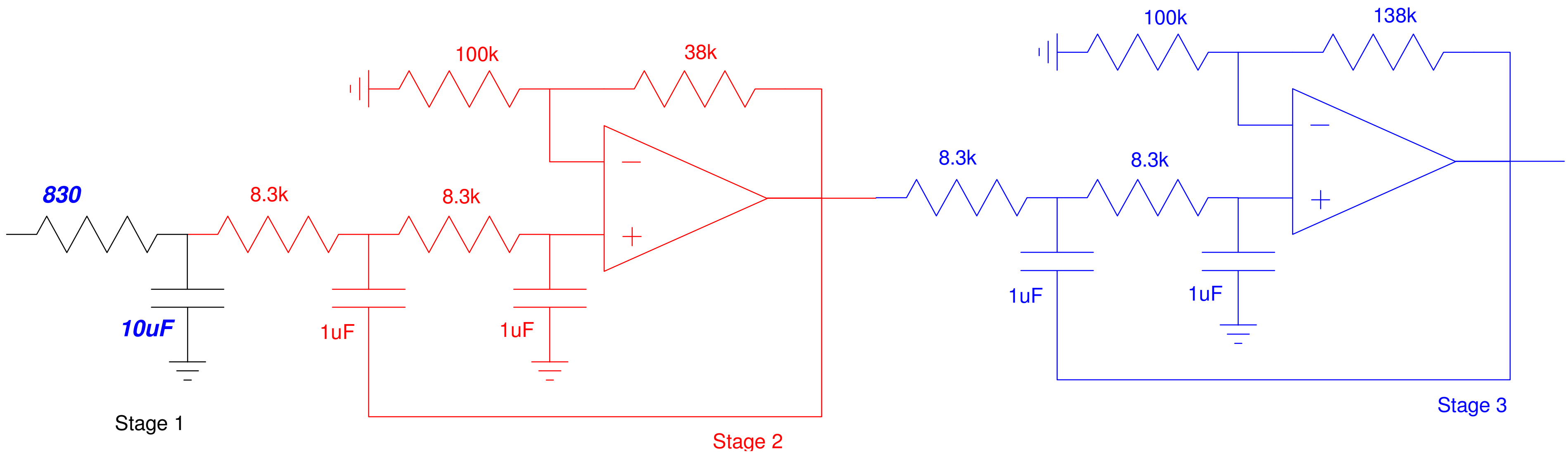
$$G_3 = \left(\frac{120^2}{\left(s+120\angle +72^0\right)\left(s+120\angle -72^0\right)} \right)$$

Section 1: (black)

$$\frac{1}{RC} = 120$$

C = 10uF,   R = 830

note: R is 10x smaller than stage 2 to avoid loading



Section 2: (red)

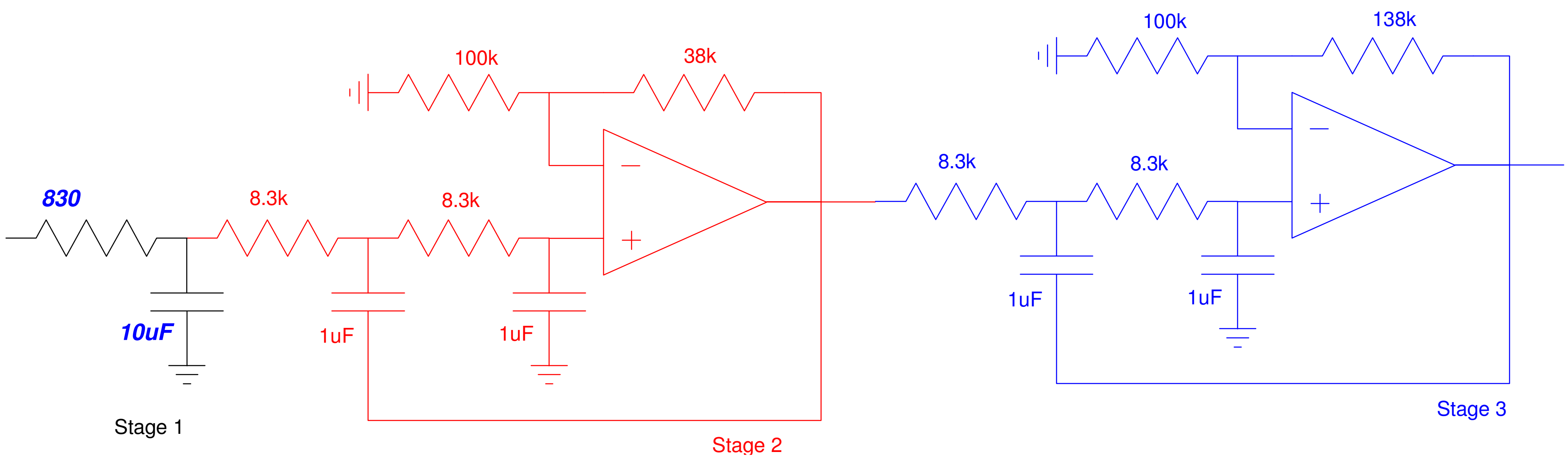
$$\frac{1}{RC} = 120$$

$$C = 1uF, \quad R = 8.3k$$

$$3 - k = 2\cos\left(36^0\right)$$

$$k = 1.3820 = 1 + \frac{R_1}{R_2}$$

R2 = 100k, R1 = 38.2k



Section 3: (blue)

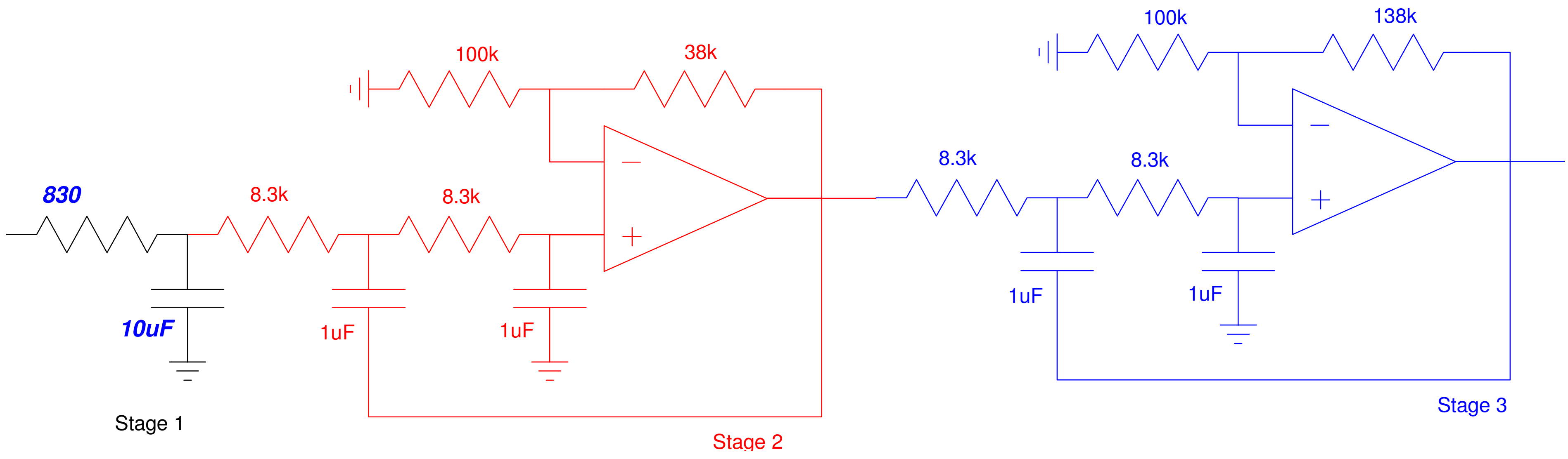
$$\frac{1}{RC} = 120$$

C = 1uF, R = 8.3k

$$3 - k = 2\cos(72^0)$$

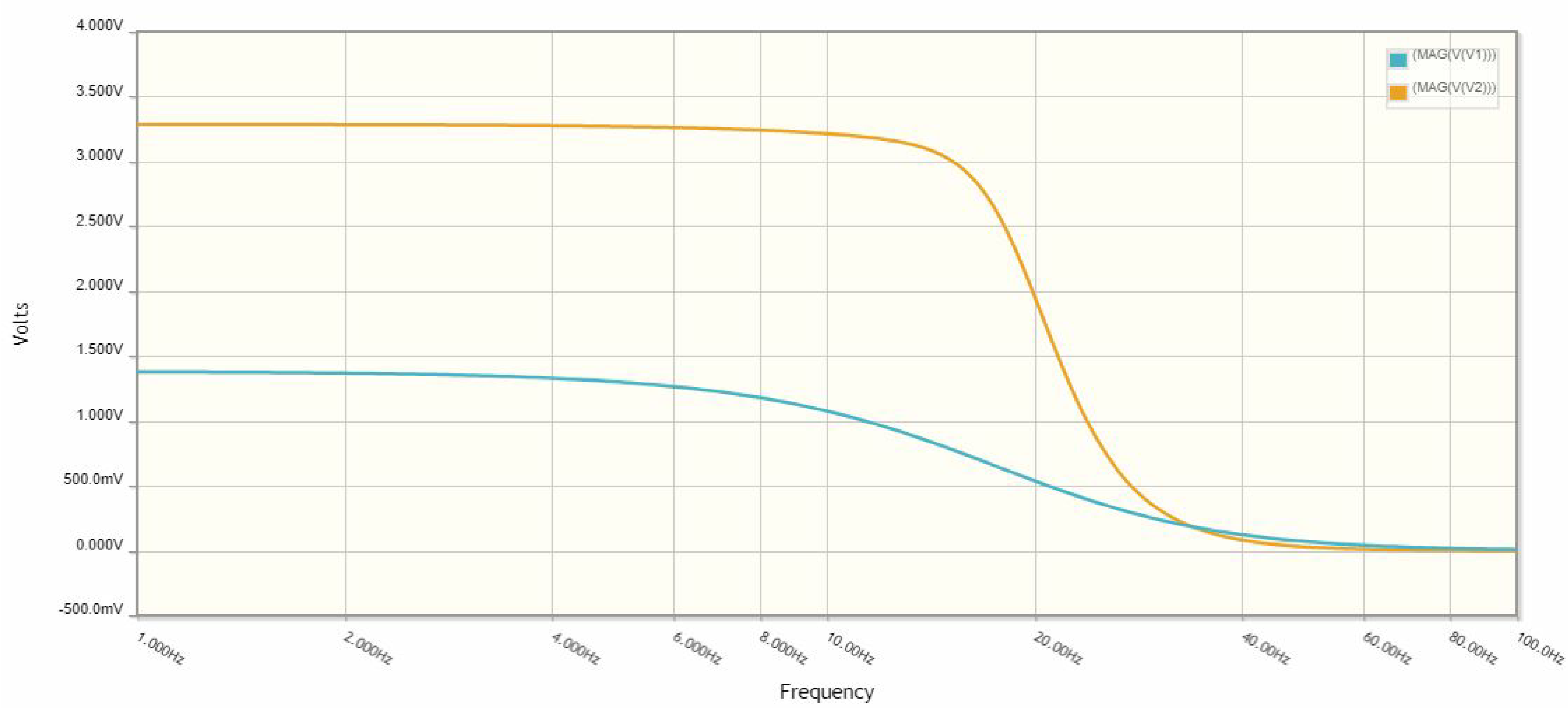
$$k = 2.3820 = 1 + \frac{R_1}{R_2}$$

R2 = 100k, R1 = 138.2k



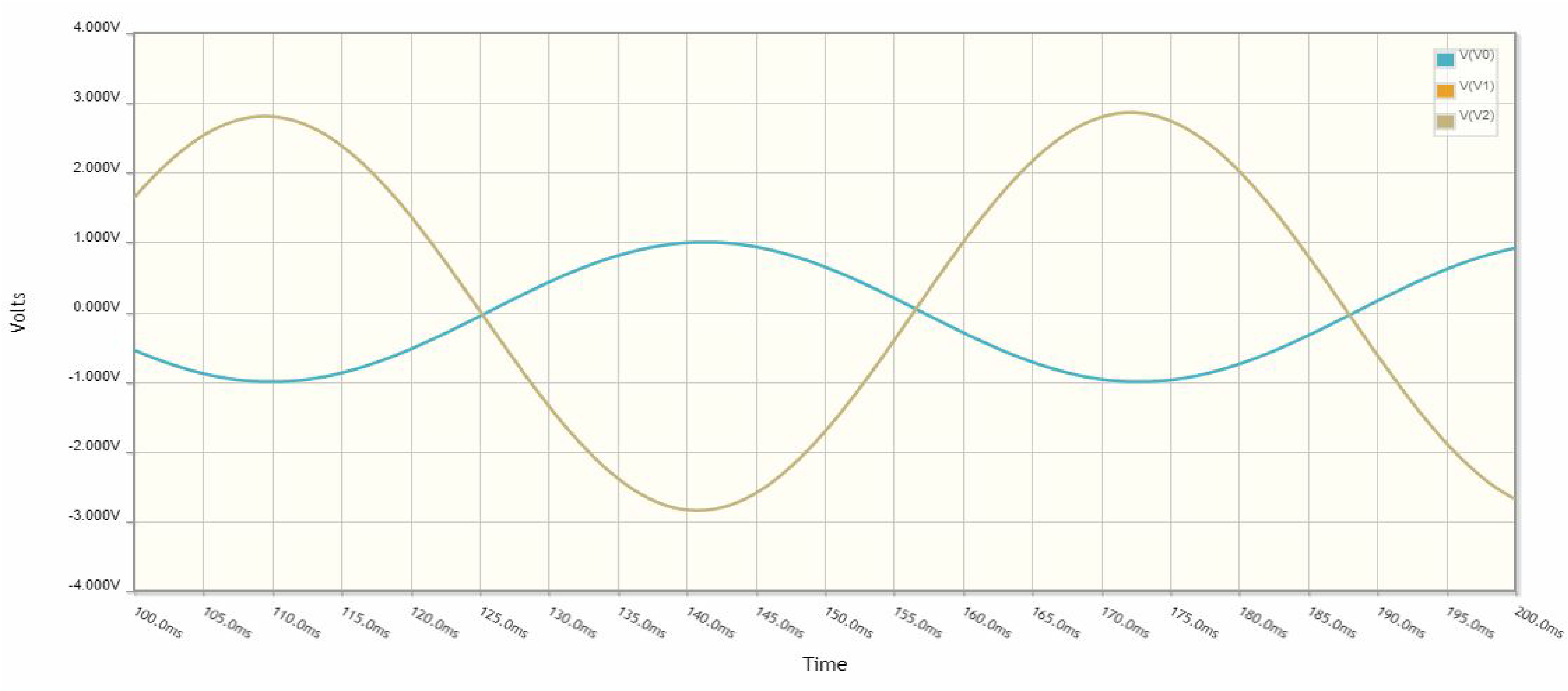
CircuitLab Results

- DC gain = 3.284
- Gain at 100 rad/sec (15.9 Hz) = 3.008 (91.6%)
- Gain at 200 rad/sec (31.8 Hz) = 0.256 (7.8%)



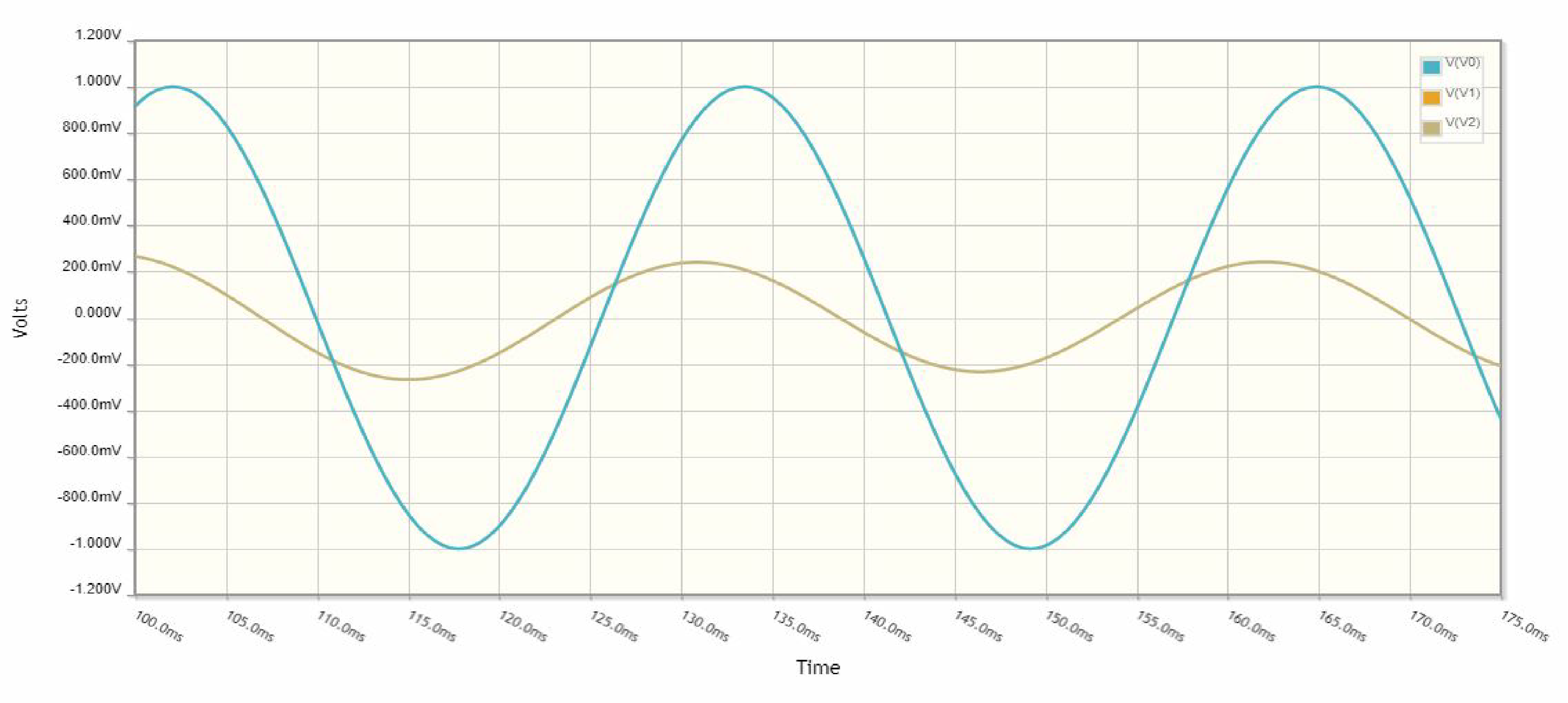
CircuitLab: 100 rad/sec

- gain = 2.860 (87% of DC gain)



CircuitLab: 200 rad/sec (31.8 Hz)

- gain = 0.240 (7.3% of DC gain)



Notes:

Don't go overboard with the requirements

- Pass frequencies < 100 rad/sec (0.9 < gain < 1.1)
- Reject frequencies > 120 rad/sec (gain < 0.1)
- Results in a 13th-order filter

$$n = -\left(\frac{\ln(0.1)}{\ln(1.2)}\right) = 12.63$$

- Do you really need a filter this selective?

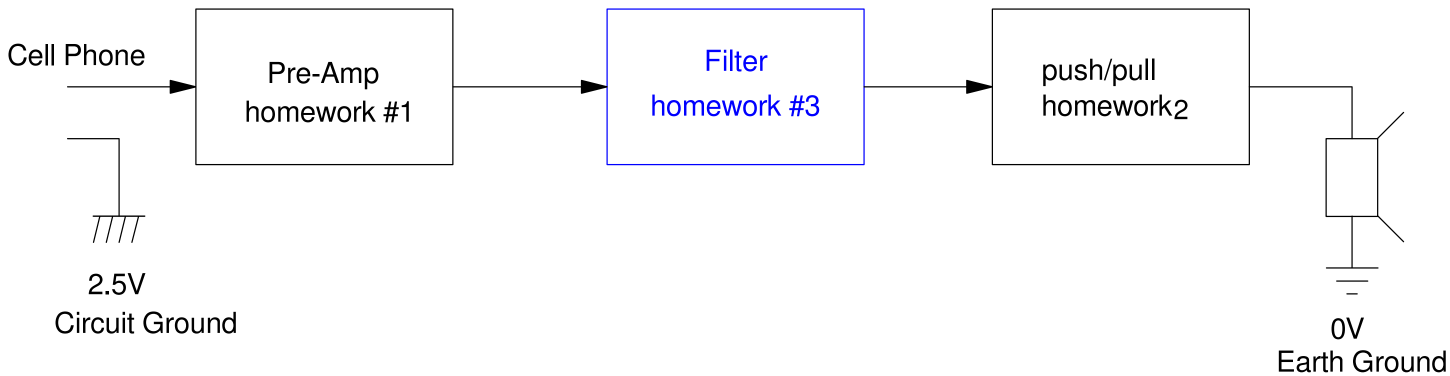
A Chebychev filter would work better

- Just a slight change in values for R's and C's

Notes: (cont'd)

The filters we designed have a DC gain

- You usually want to amplify the signal
- The filter provides some amplification (k) due to k setting the angle of the poles
- You usually need to add another amplifier to set the overall gain



Summary

- Filter design is fairly straight forward:

Requirements

- Specify the pass band (0.9 < gain < 1.1)
- Specify the reject band (gain < 0.1)

Calculate the Number of Poles Needed

- The tighter the reject / pass region are, the more poles you need

Pick your filter type

- Butterworth, Chebychev, Elliptic

Adjust the corner to meet the requirements