Butterworth, Elliptic, & Chebychev Filters

ECE 321: Electronics II

Lecture #9:

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

Butterworth, Elliptic, Chebychev Filters

Objective:

- Know what each filter tries to optimize
- Know how these filters compare

Filter Types:

- RC
- Butterworth
- Chebychev (Type-1)
- Elliptic (Type-2 Chebychev)

RC Filter

Closest approximation to an ideal low pass filter subject to

- There are N poles
- All poles are real
- There are no zeros
- The maximum gain is 1.000

An n-pole RC filter has all n-poles on the real axis.

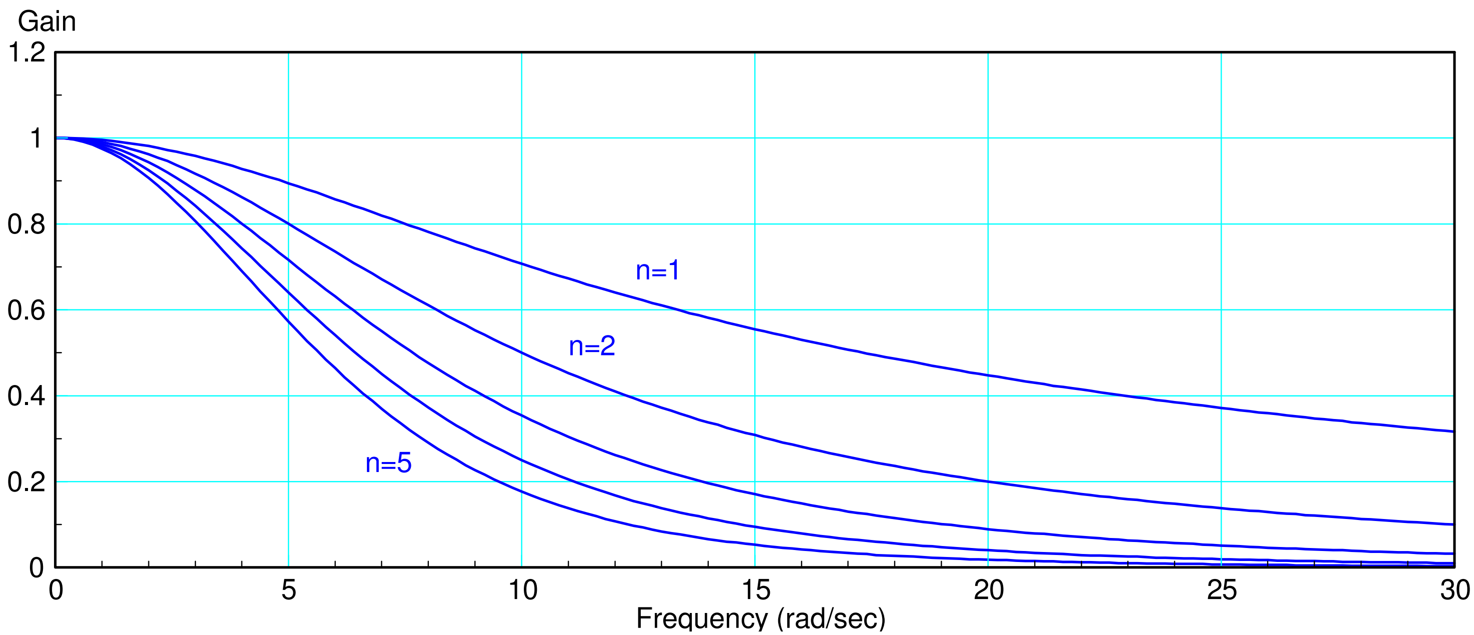
- It's advantage is you can build it with a passive RC filter (good)
- It's problem is it's a pretty poor filter.

RC Filter Example:

$$G(s) = \left(\frac{10}{s+10}\right)^n$$

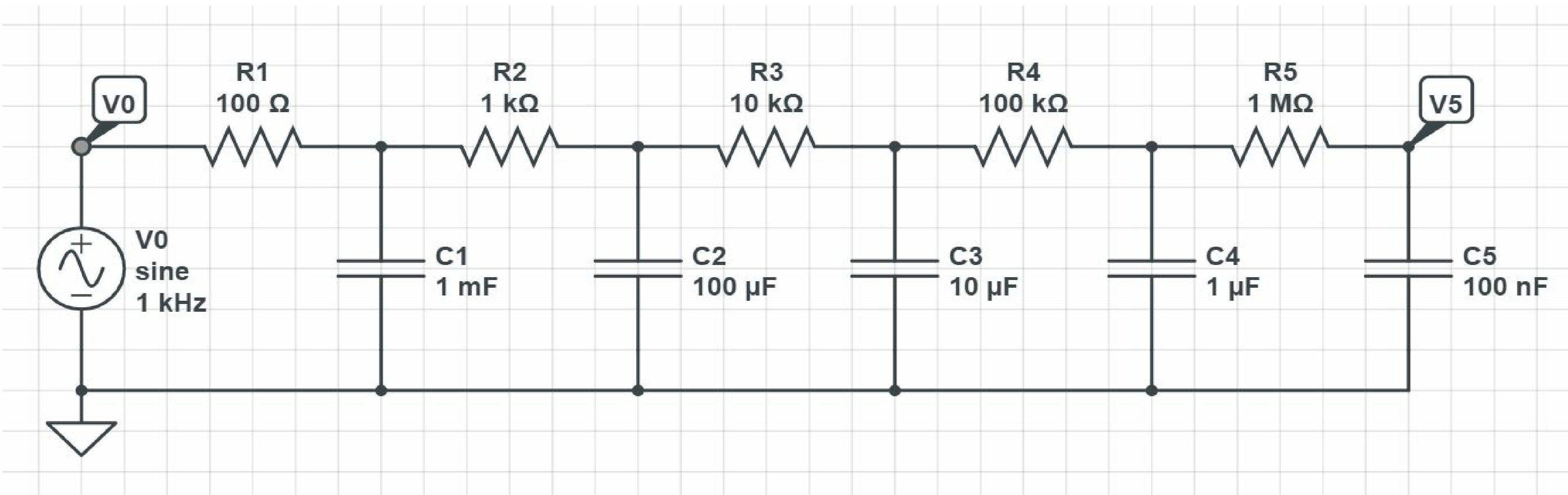
As n increases...

- The high-frequency gain gets smaller and smaller (good)
- The gain below 5 rad/sec starts to droop more an more (bad)



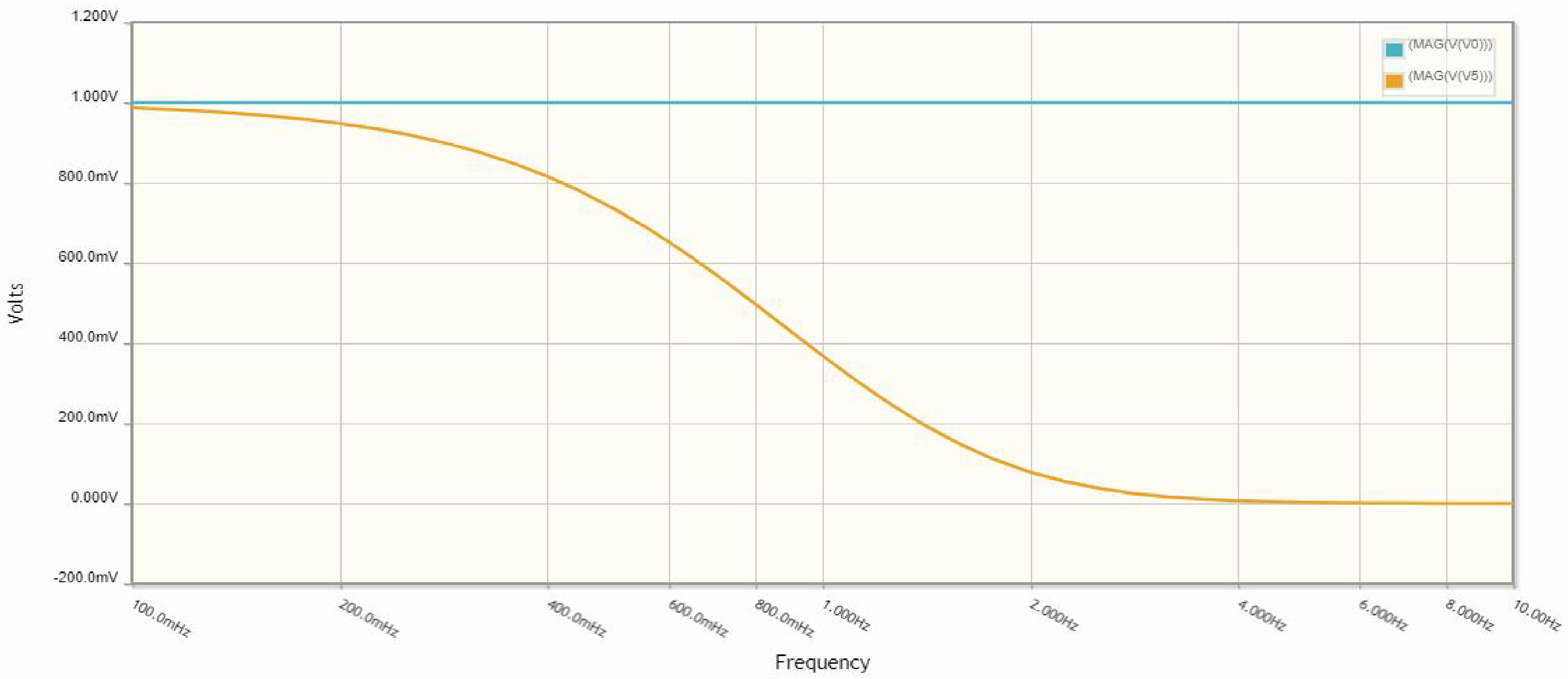
5th-Order RC Filter

- Hardware Design
- 1/RC = 10
- R increases 10x each stage to reduce loading



CircuitLab Simulation: 5th-Order RC Filter

- Frequency Sweep
- Not a very good filter



Butterworth Filter:

Closest approximation to an ideal low pass filter subject to

- There are N poles
- There are no zeros
- The maximum gain cannot exceed 1.0000

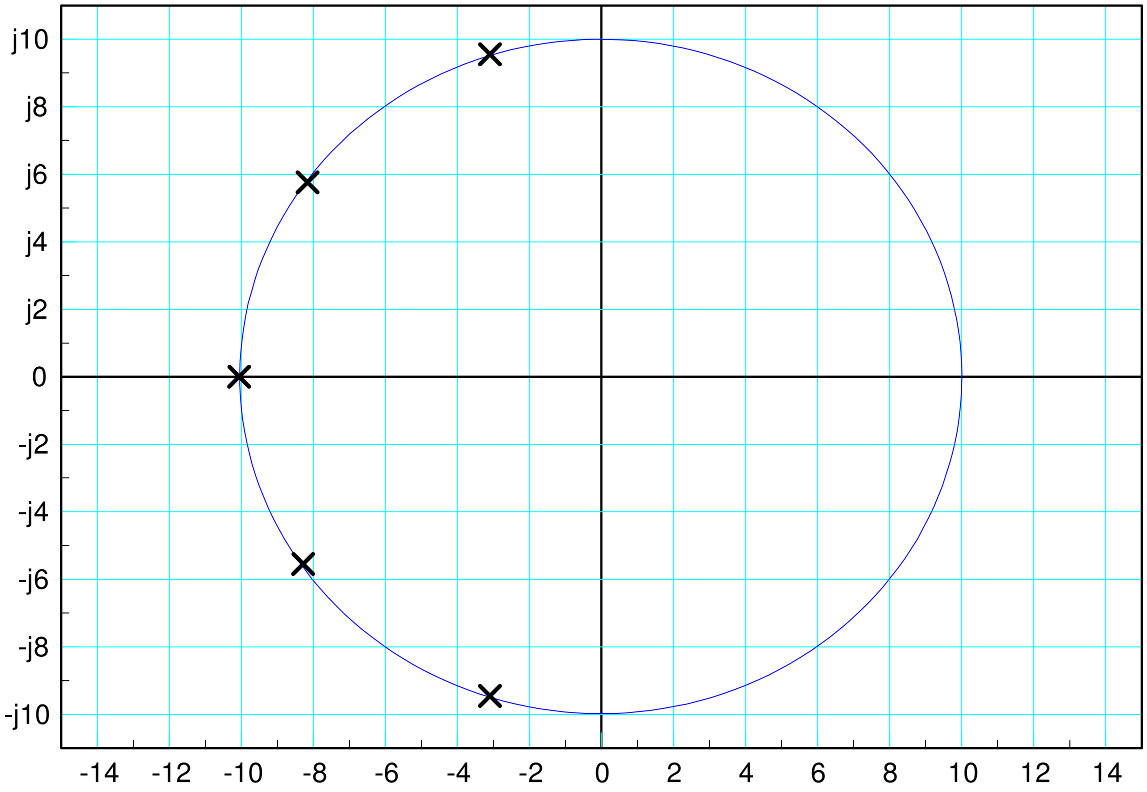
Solution:

- Magnitude of poles = corner frequency
- Angle between poles: $\phi = \frac{180^0}{N}$

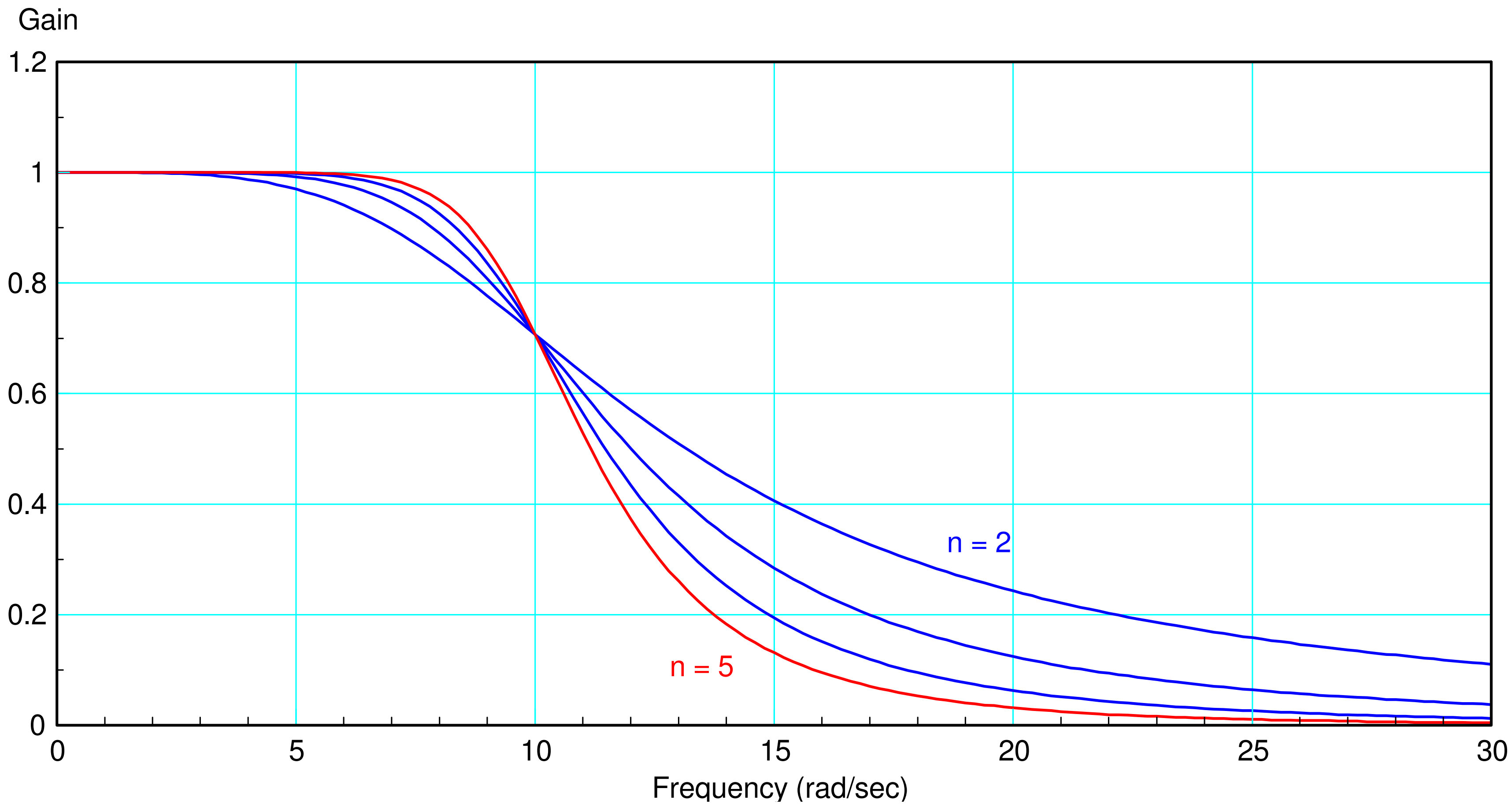
Butterworth Pole Locations: Corner = 1 rad/sec					
	N=2	N=3	N=4	N=5	N=6
zeros	none	none	none	none	none
poles	$-1.0\angle\pm 45^0$	-1.0 $-1.0\angle\pm 60^0$	$-1.0\angle\pm 22.5^0$ $-1.0\angle\pm 67.5^0$	-1.0 $-1.0\angle\pm 36^0$ $-1.0\angle\pm 72^0$	$-1.0\angle\pm 15^0$ $-1.0\angle\pm 45^0$ $-1.4\angle\pm 75^0$

Example: 5th-Order Butterworth filter, Corner = 10 rad/sec

$$G(s) = \left(\frac{10^5}{(s+10)\left(s+10\angle 36^0\right)\left(s+10\angle -36^0\right)\left(s+10\angle 72^0\right)\left(s+10\angle -72^0\right)} \right)$$



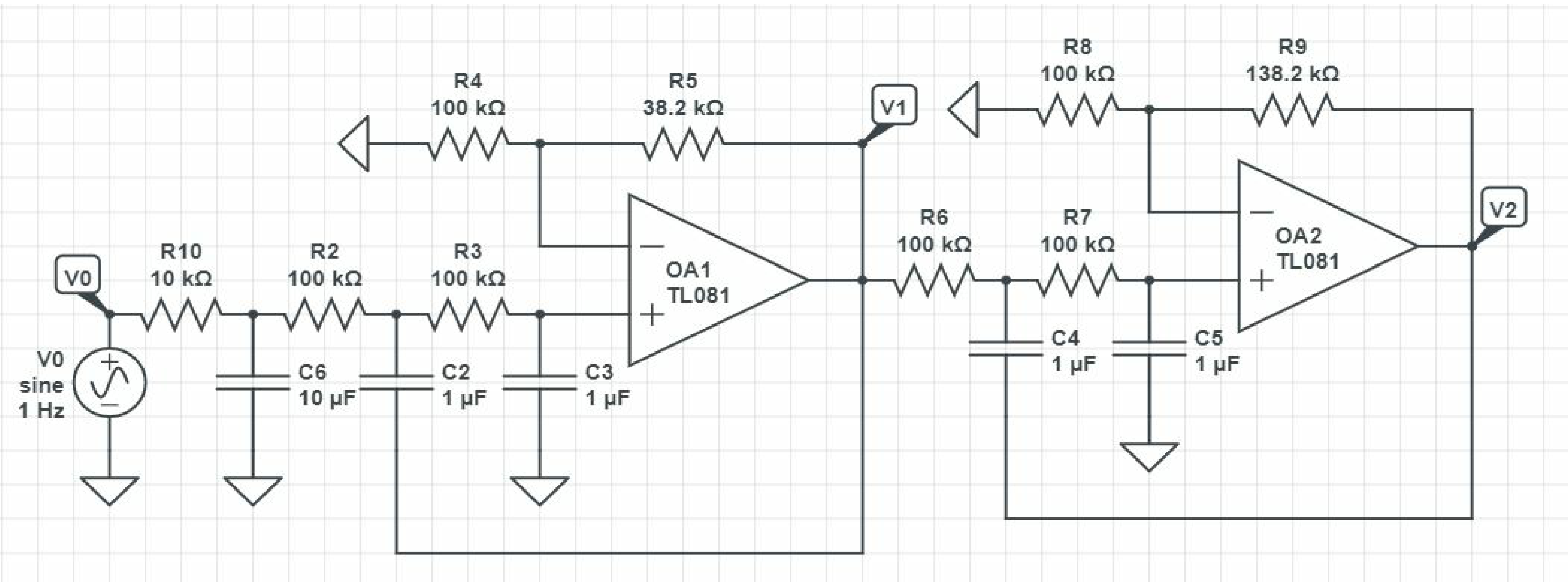
The resulting gain vs. frequency is then



Circuit Implementation

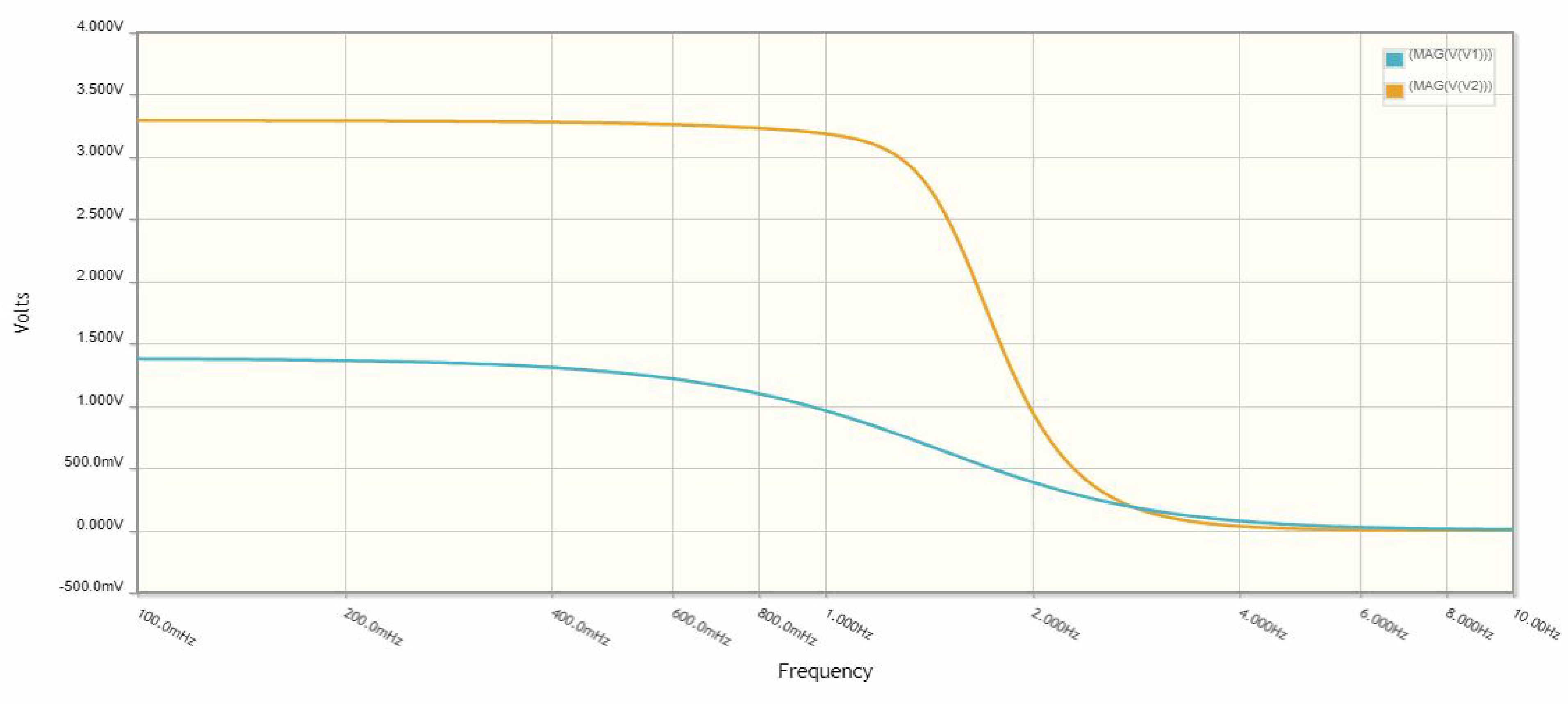
- $\left(\frac{1}{RC}\right) = 10$
- $3 - k = 2\cos(\theta)$

$$G(s) = \left(\frac{10^5}{(s+10)\left(s+10\angle 36^0\right)\left(s+10\angle -36^0\right)\left(s+10\angle 72^0\right)\left(s+10\angle -72^0\right)}\right)$$



CircuitLab Results

- Corner = 10 rad/sec (1.59Hz)



Type-1 Chebychev Filter

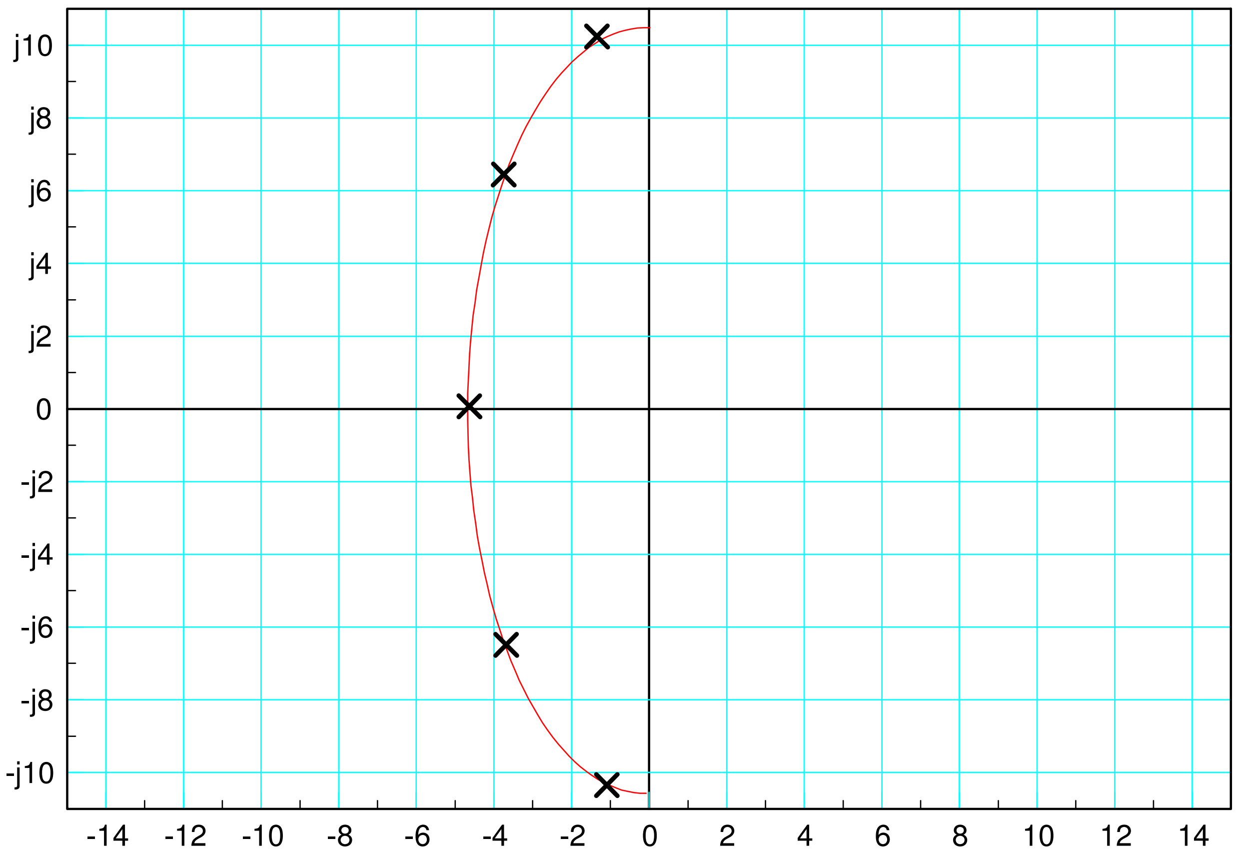
Closest approximation to an ideal low pass filter subject to

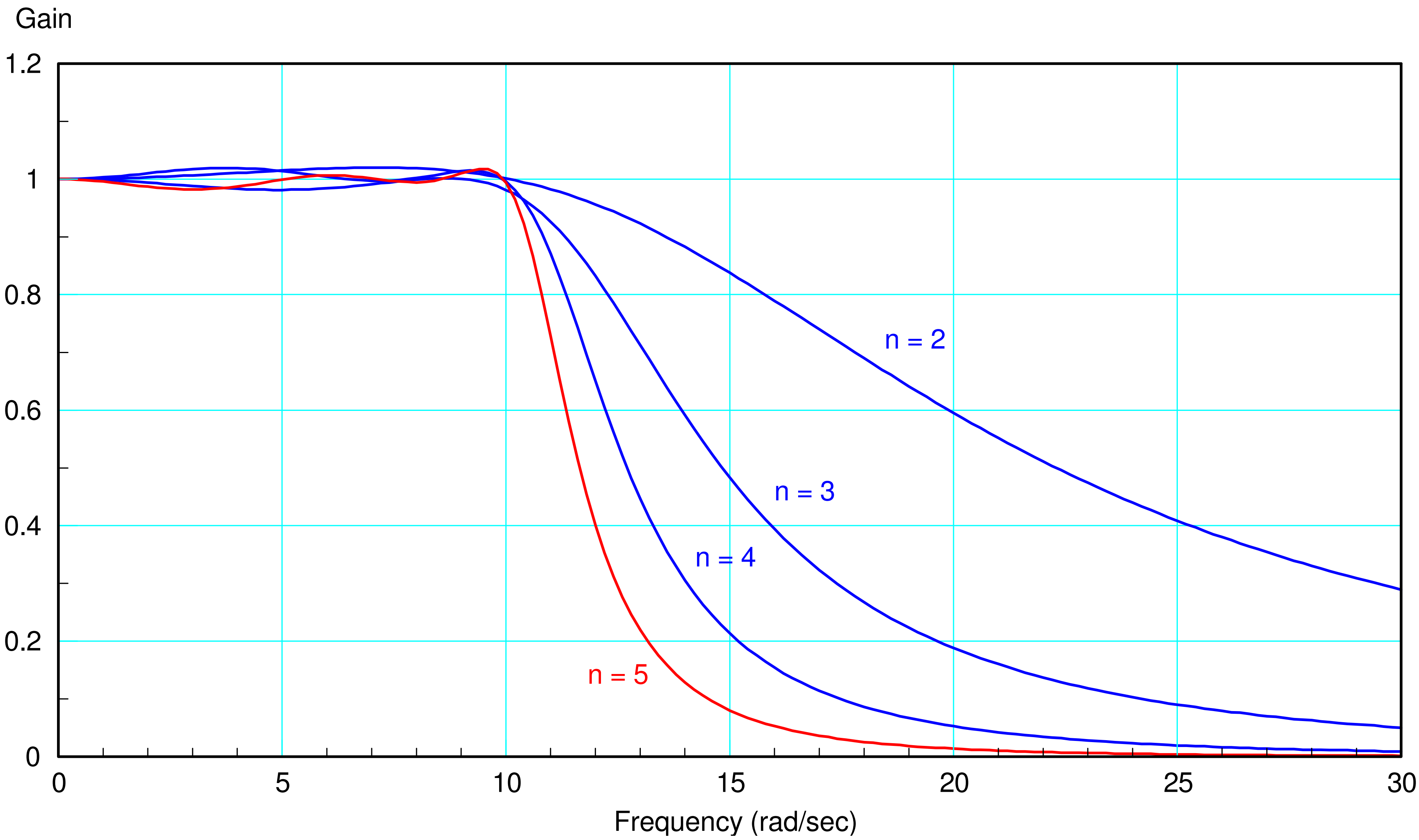
- There are N poles
- There are no zeros
- The maximum gain is $(1+\varepsilon)$. (Some ripple is permitted).

Type-1 Chebychev Filter Pole Locations: Corner = 1 rad/sec					
	N=2	N=3	N=4	N=5	N=6
zeros	none	none	none	none	none
poles	$-1.60\angle\pm 50.7^0$	-0.85 $-1.21\angle\pm 69.5^0$	$-0.72\angle\pm 38.5^0$ $-1.11\angle\pm 77.8^0$	-0.48 $-0.76\angle\pm 59.3^0$ $-1.06\angle\pm 82.0^0$	$-0.47\angle\pm 36.1^0$ $-0.81\angle\pm 69.8^0$ $-1.04\angle\pm 84.4^0$

Example: Design a 5th-order Chebuchev filter, Corner = 10 rad/sec

$$G(s) = \left(\frac{4.8 \cdot 7.6^2 \cdot 10.6^2}{(s+4.8)\left(s+7.6\angle\pm 59.3^0\right)\left(s+10.6\angle\pm 82^0\right)} \right)$$

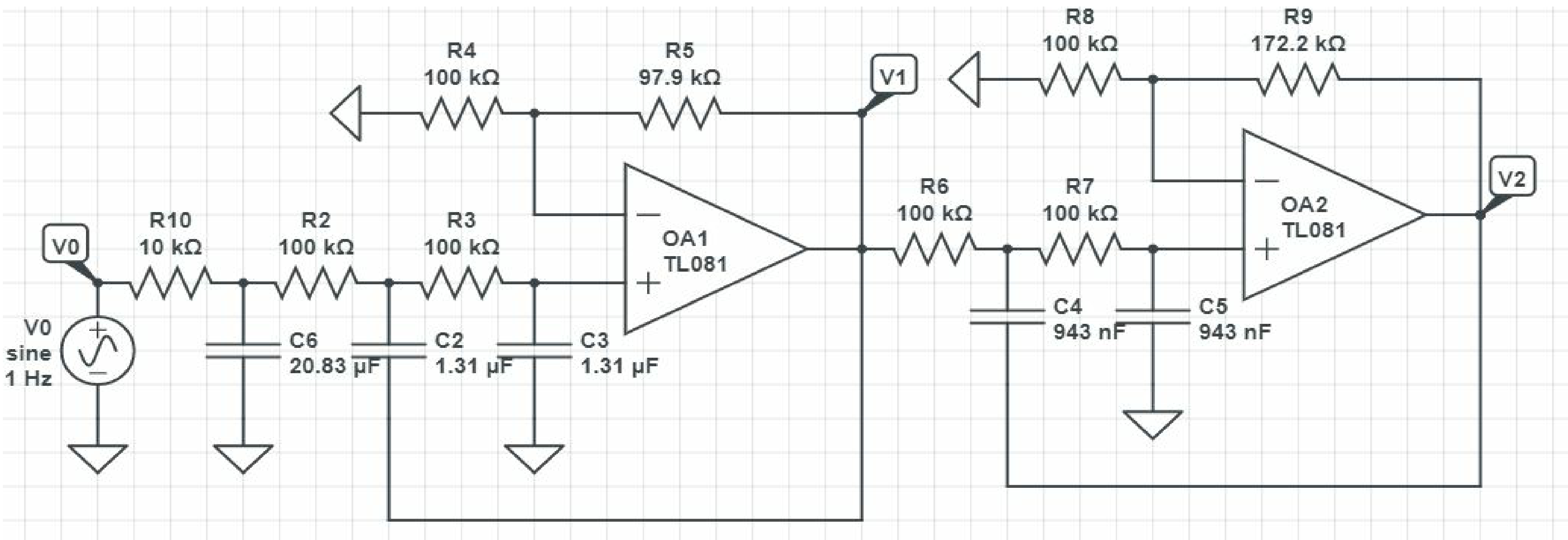



Gain
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
Frequency (rad/sec)
n = 2
n = 3
n = 4
n = 5

CircuitLab Implementation

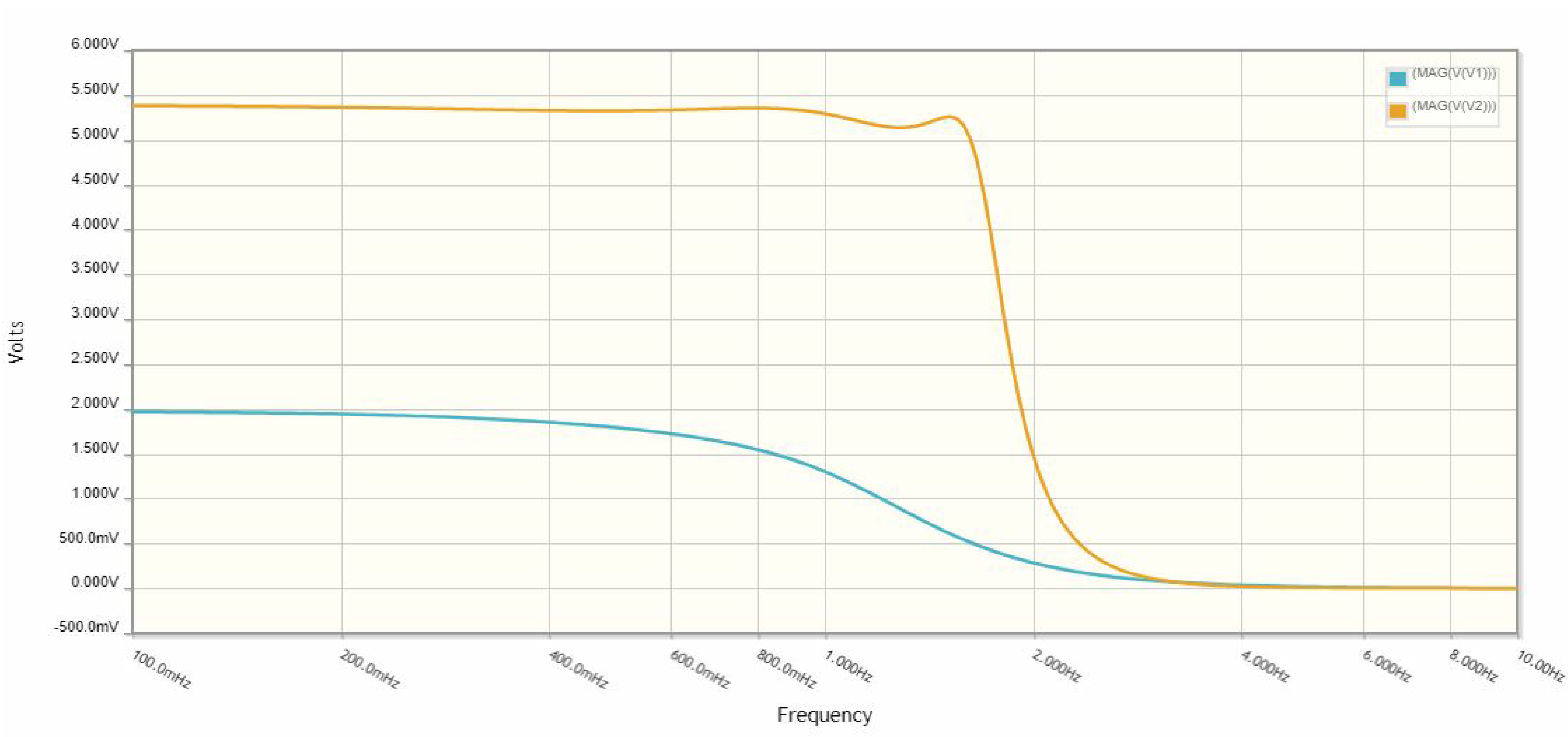
- $\left(\frac{1}{RC}\right) = pole$
- $3 - k = 2\cos\theta$

$$G(s) = \left(\frac{4.8 \cdot 7.6^2 \cdot 10.6^2}{(s+4.8)\left(s+7.6\angle\pm59.3^0\right)\left(s+10.6\angle\pm82^0\right)} \right)$$



CircuitLab: Frequency Sweep

- Gain is closer to 1.00 out to 1.59 rad/sec

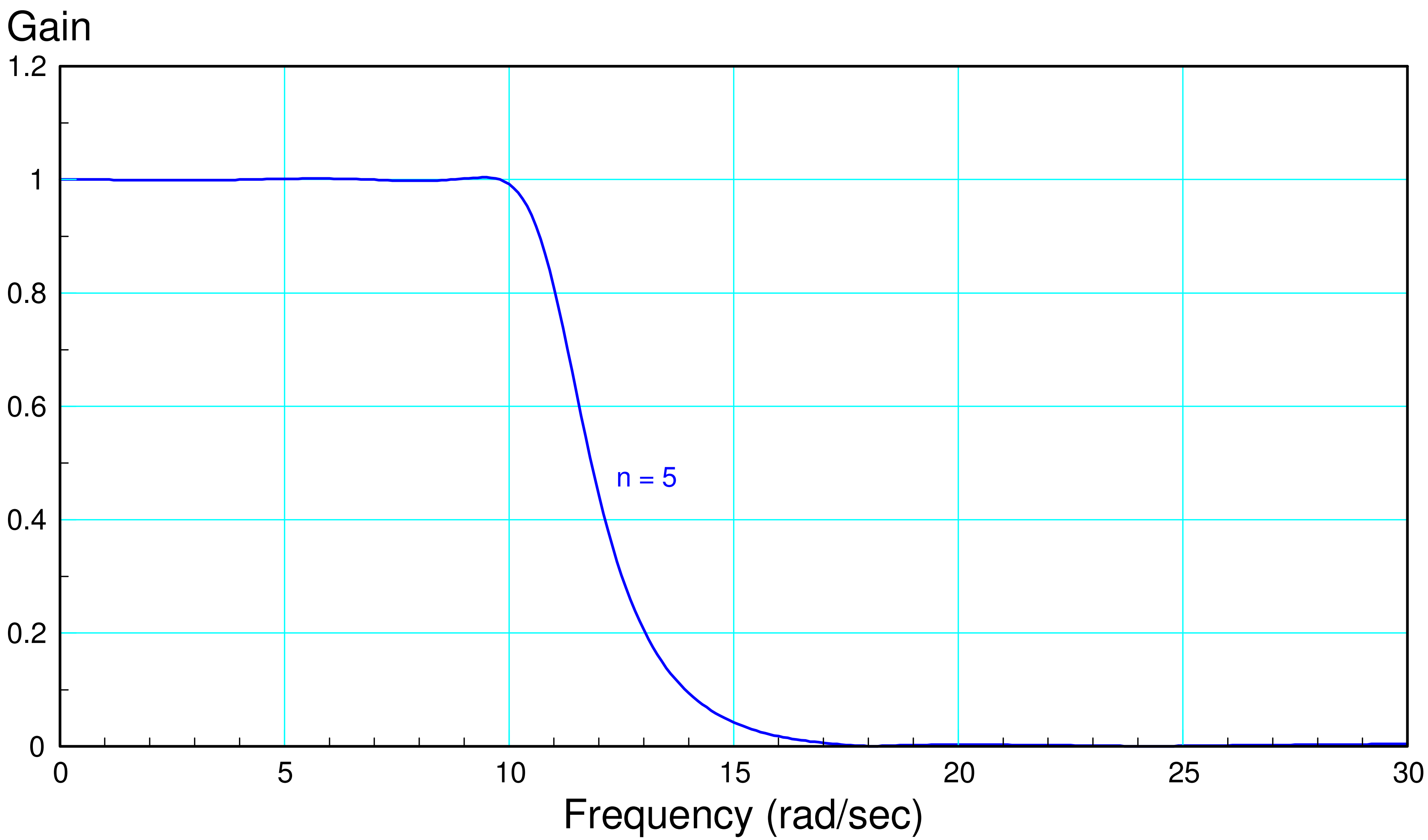


Elliptic Filters: (Type-2 Chebychev)

Closest approximation to an ideal low pass filter subject to

- There are N poles
- There are N zeros
- The maximum gain cannot exceed $(1+\varepsilon_1)$. (Some ripple is permitted).
- The maximum gain the band reject region cannot exceed ε_2

Pass	W1 = 0 to 1	W1 = 0 to 1	W1 = 0 to 1	W1 = 0 to 1
Reject	W2 = 9 to infinity	W2 = 3 to infinity	W2 = 1.7 to infinity	W2 = 1.3 to infinity
zeros	j 9.919	j3.246 j7.705	j1.805 j2.423	j1.316 j1.524 j2.491
poles	-0.391 + j1.242 -0.942	-0.572 + j0.467 -0.221 + j1.076	-0.807 -0.529 + j0.784 -0.151 + j1.095	-0.437 -0.331 + j0.583 -0.159 + j0.907 -0.040 + j1.024



Gain
1.2
1
0.8
0.6
0.4
0.2
0
n = 5
0
5
10
15
20
25
30
Frequency (rad/sec)

CircuitLab Implementation

- Can't implement complex zeros with circuits given

Frequency Scaling

The previous circuits had a corner at 10 rad/sec

- Redesign so that the corner is 100Hz

Change the capacitors

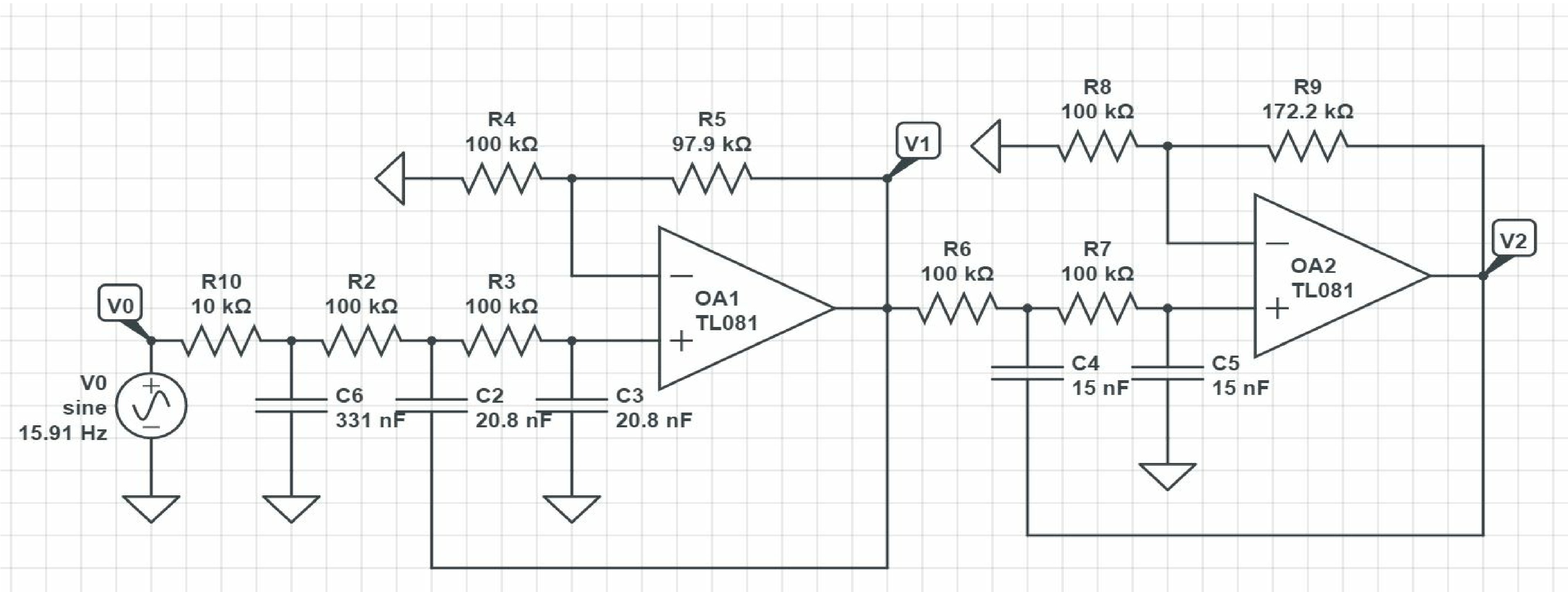
- Capacitors are integrators
- If you make the capacitor 10x smaller,
  - They integrate 10x faster
  - The corner frequency becomes 10x larger

To make the corner 100Hz

- Scale the capacitors by x 10/628

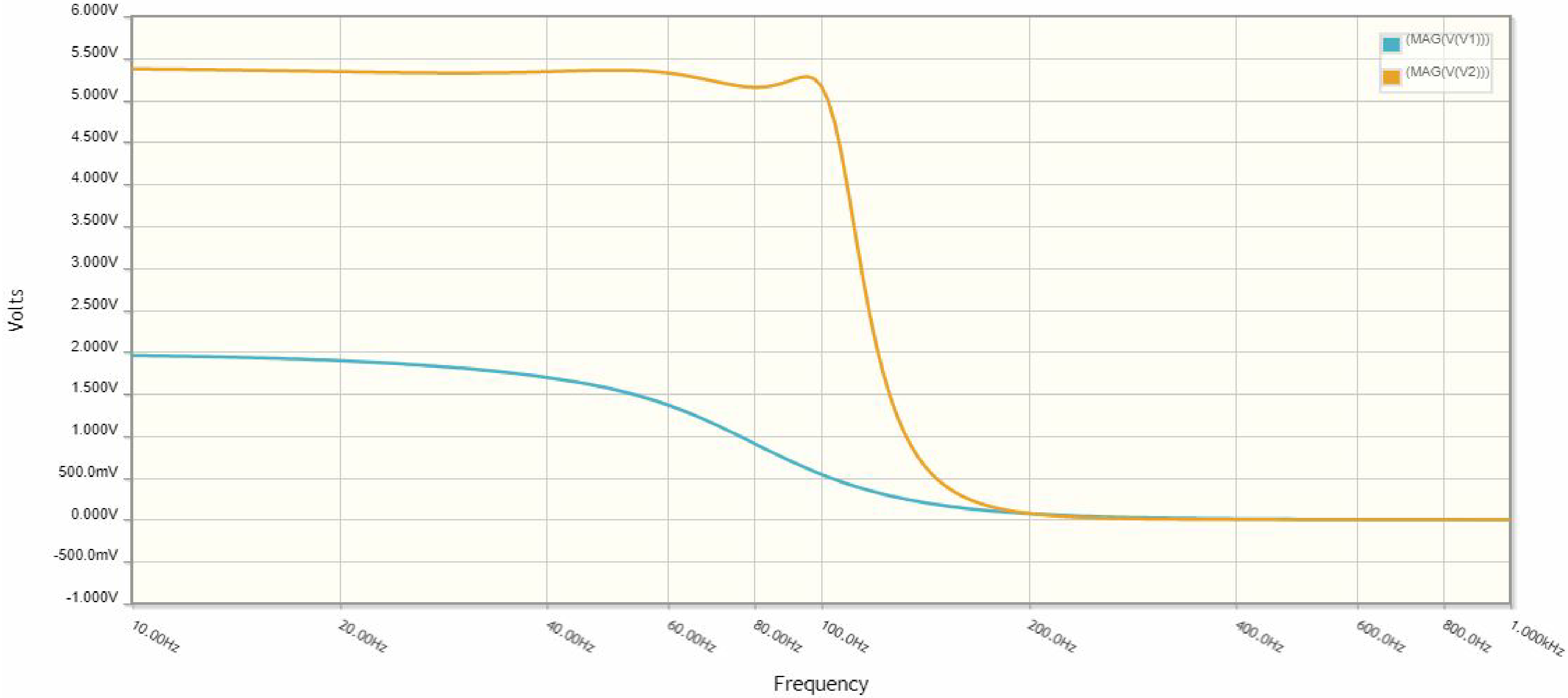
5th-Order Chebychev filter with a corner at 100Hz

- Capacitors scaled by 1 / 62.8



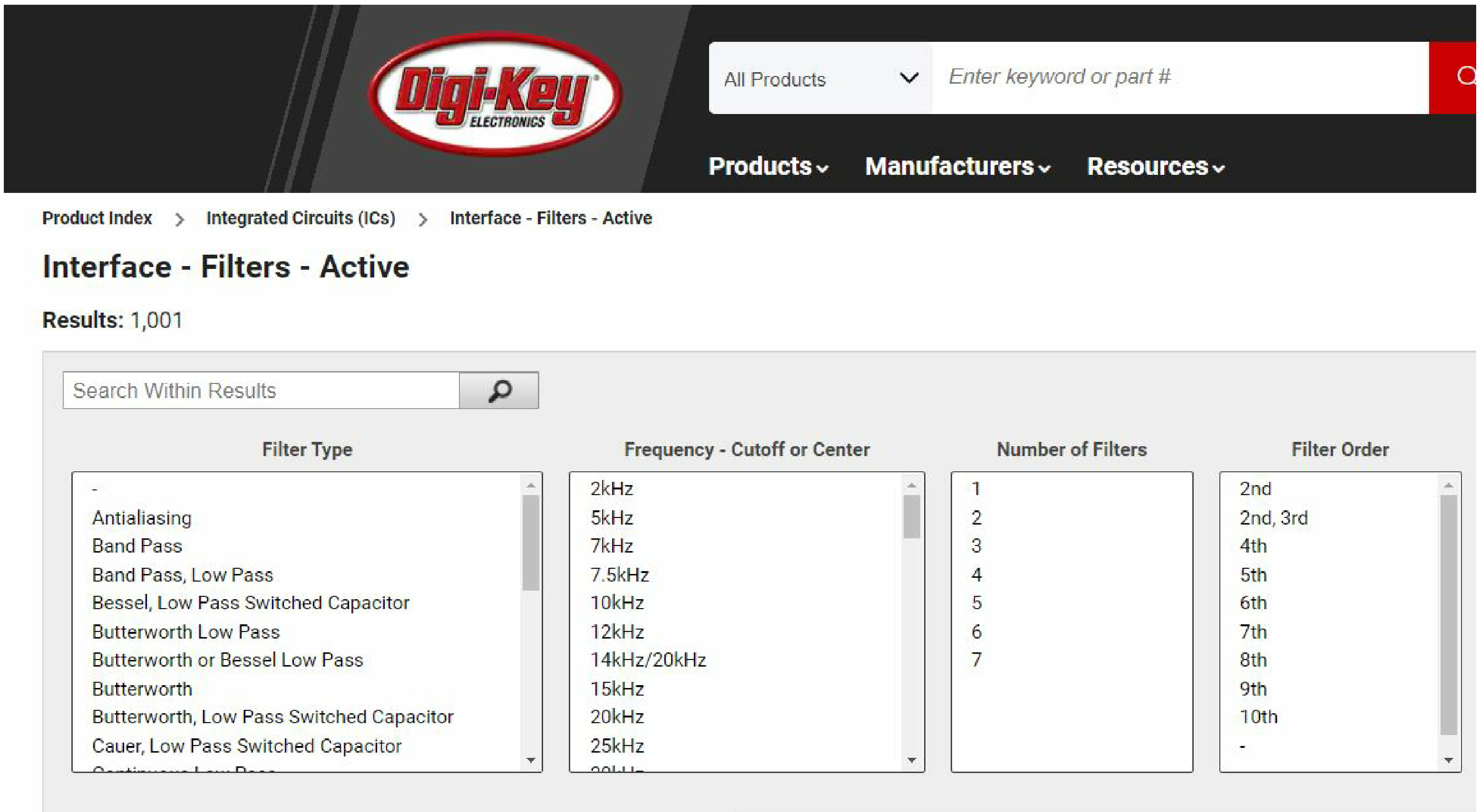
5th-Order Chebychev Low-Pass Filter

- CircuitLab simulation



Digikey

- Filters are really common and useful
- Digikey likewise sells 1001 filters on a chip

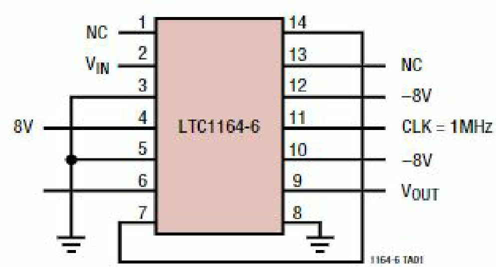


Example: LTC 1164

- 8th-order Elliptic filter
- Corner frequency is set by a clock input
- Good for senior design (easy to implement)
- Not so good for Electronics II (doesn't demonstrate knowledge of electronics)

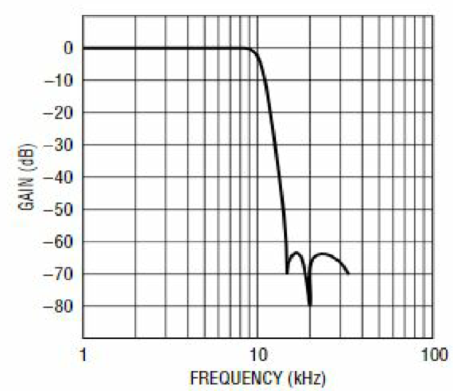
TYPICAL APPLICATION

10kHz Anti-Aliasing Elliptic Filter



WIDEBAND NOISE = 115μV_{RMS}
NOTE: THE CONNECTION FROM PIN 7 TO PIN 14 SHOULD BE MADE UNDER THE PACKAGE. THE POWER SUPPLIES SHOULD BE BYPASSED BY A 0.1μF CAPACITOR AS CLOSE TO THE PACKAGE AS POSSIBLE.

Frequency Response



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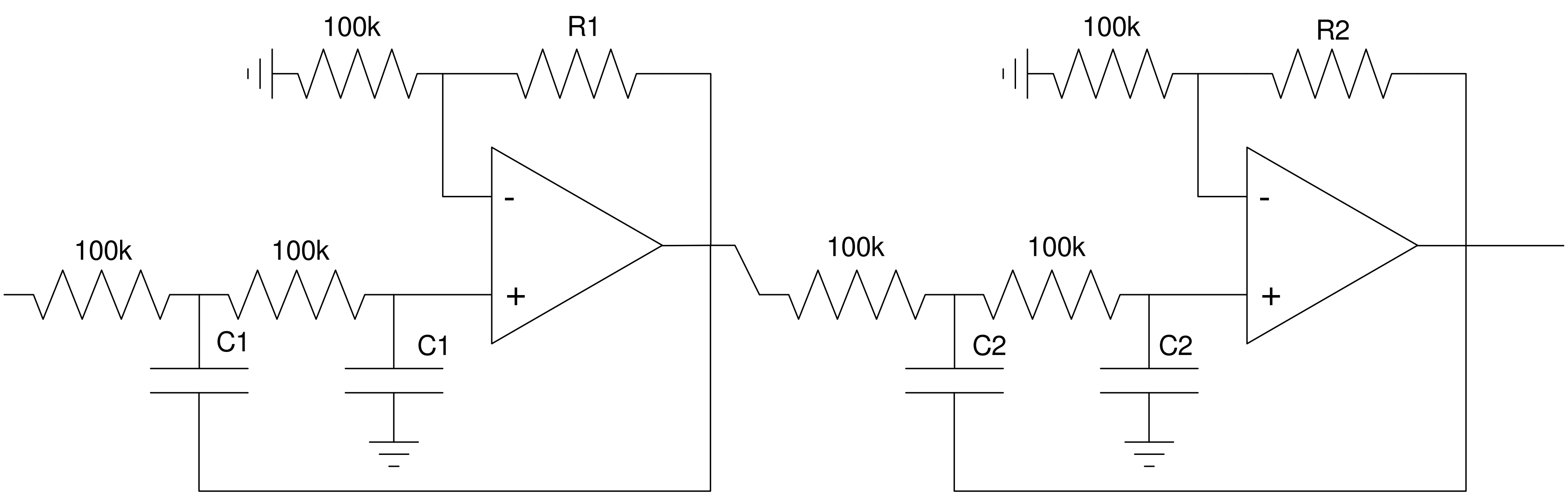
Handout:

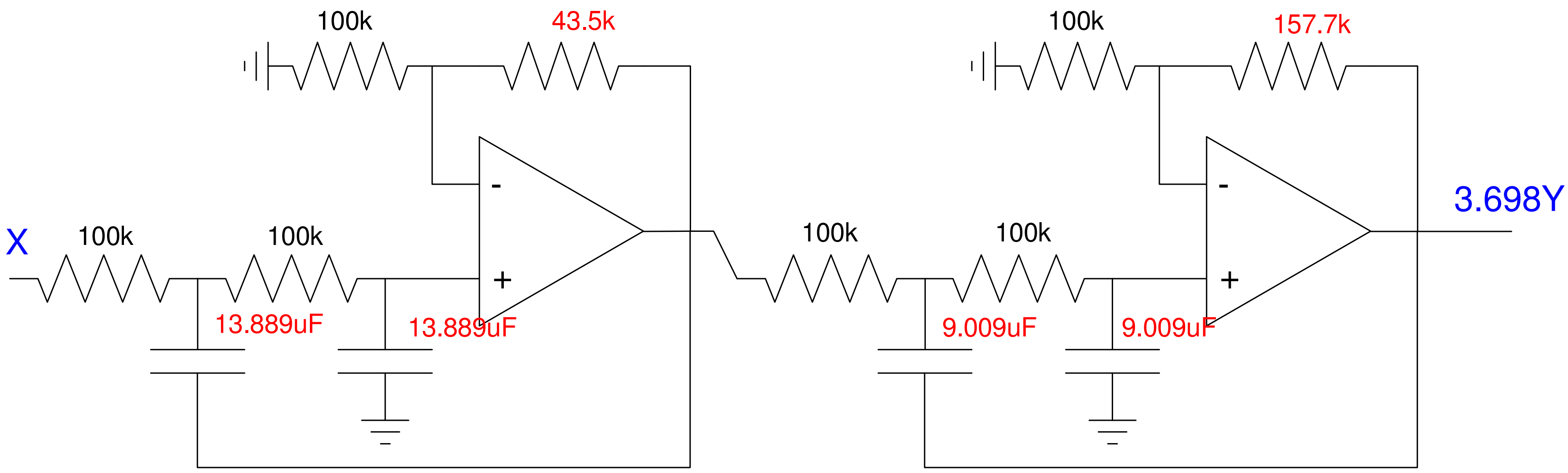
Find R and C to implement a Chebychev filter with a corner at 1 rad/sec

$$G(s) = \left(\frac{0.639}{\left(s+0.72\angle\pm 38.5^0\right)\left(s+1.11\angle\pm 77.8^0\right)} \right)$$



Solution

- To make the corner 628 rad/sec (100Hz), scale each C by 1/628



Summary

- If you are willing to use complex poles, you can do much better than an RC filter
- A Butterworth filter is one option:
  - Maximum gain at DC
  - Fairly straight forward design
- A Chebychev filter is another option
  - Slight resonance
  - Much better gain vs. frequency (closer to ideal)
  - Need to use a table (or Wikipedia) to design
- Changing the corner frequency is really easy
  - Scale the capacitors