Active Filters

ECE 321: Electronics II

Lecture #7

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Background:

Filters: Circuits whose behaviour changes with frequncy

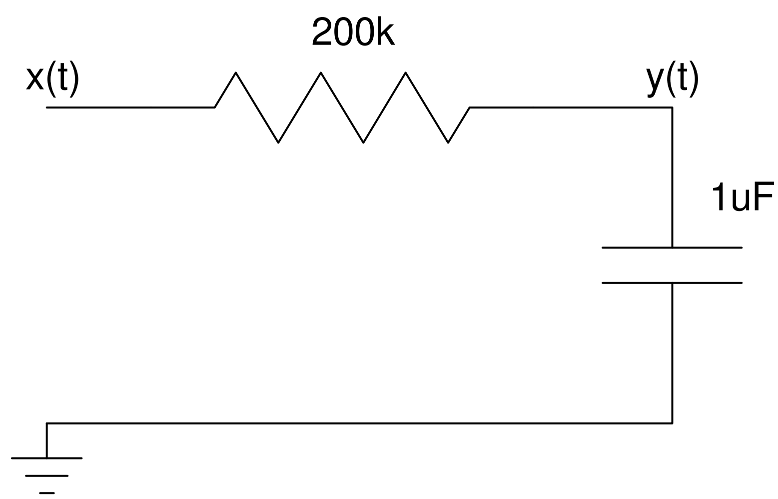
- Any circuit with capacitors and/or inductors

Sinusoids are used to analysis

- Allows you to use phasor analysis

Example: RC Filter

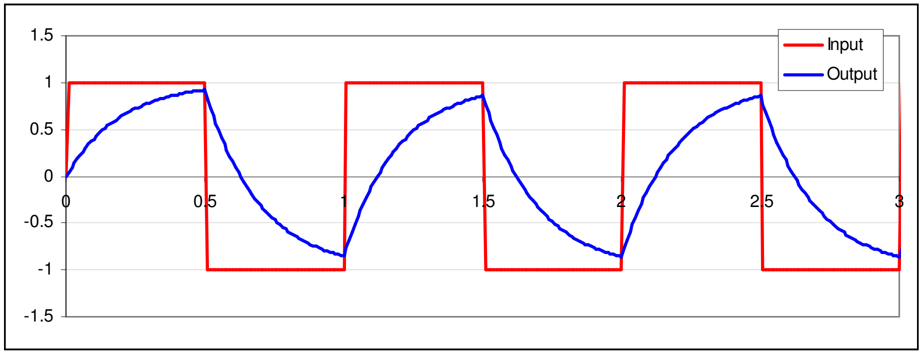
$$\frac{dy}{dt} + 5y = 5x$$



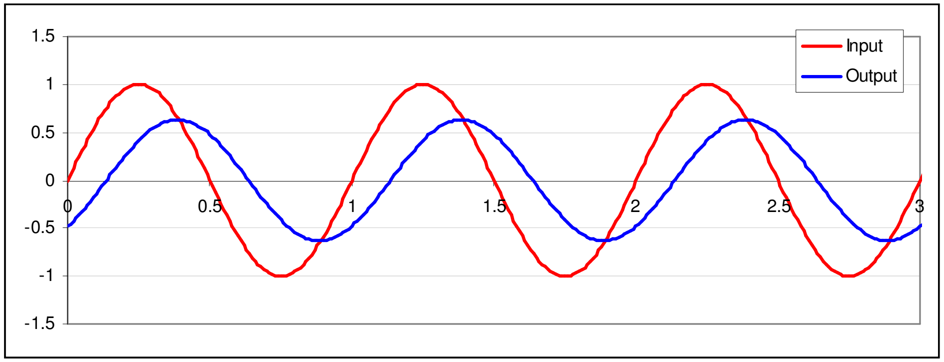
Sinusoids

Eigenfunctions: Output is the same as the input

- Not true for square waves



- Only true for sine waves



Phasor Anysis

Forced response with sinusoidal inputs

Example: find y(t)

$$\frac{dy}{dt} + 5y = 5x$$

$$x(t) = \sin(6t)$$

Solution:

$$sY + 5Y = 5X$$

$$Y = \left(\frac{5}{s+5}\right) X$$

$$Y = \left(\frac{5}{s+5}\right)_{s=j6} \cdot (0 - j1)$$

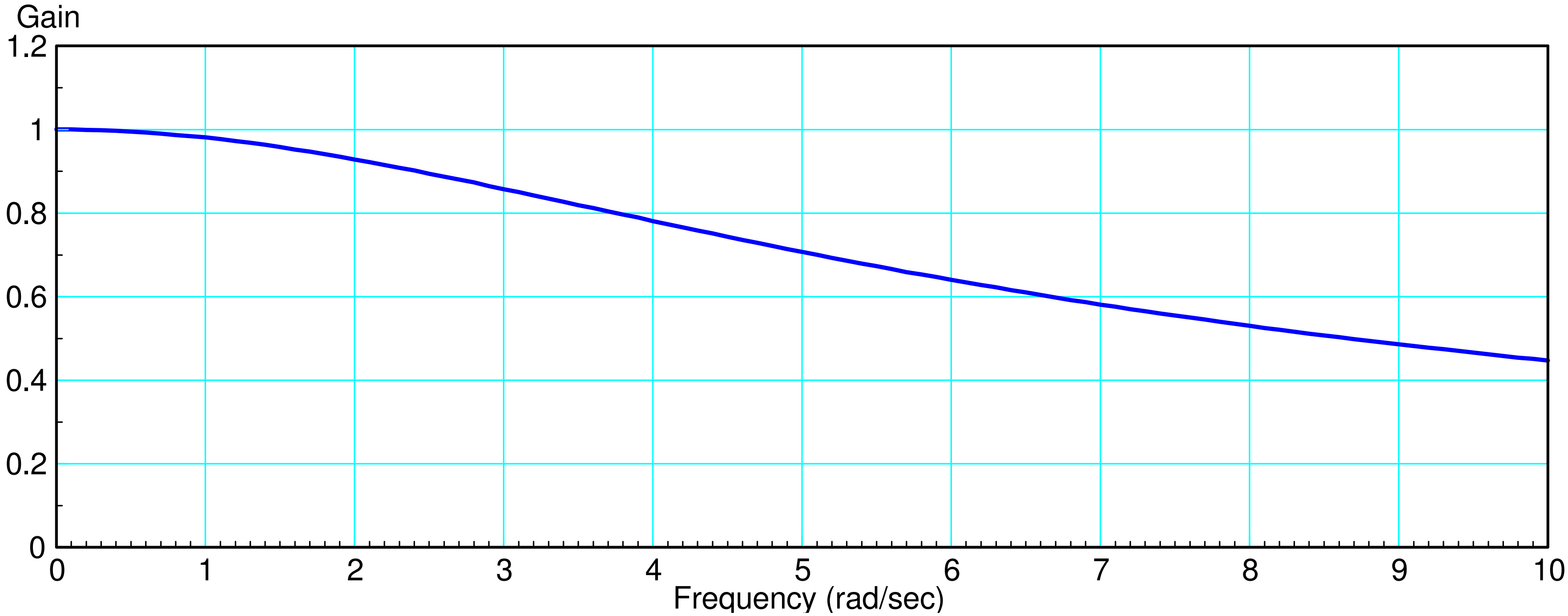
$$Y = -0.4918 - j0.4098$$

$$y(t) = -0.4918\cos(6t) + 0.4098\sin(6t)$$

Bode Plot

A Bode plot is graph showing the gain vs. frequency

```
w = [0:0.01:10]';
G = 5 ./ (j*w + 5);
plot(w, abs(G));
```



Active Filters

A filter with an op-amp

Op-Amps allow:

- Gains larger than one
- High input impedances
- Low output impedances
- Real poles, and
- Complex poles using only resistors and capacitors

Inductors tend to be large, lossy, prone to coupling, and expensive.

Circuits which only use capacitors and resistors tend to work much better.

Generalized Filter:

In general, a filter will be of the form

$$G(s) = k\left(\frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)(s+p_3)}\right)$$

where

zi are the zeros of the filter,

pi are the poles of the filter, and

k is a gain.

Today's lecture covers different circuits to implement a filter with

- Real poles, and
- Complex Poles

Real Poles: Passive RC Filters

Problem: Design a circuit to implement

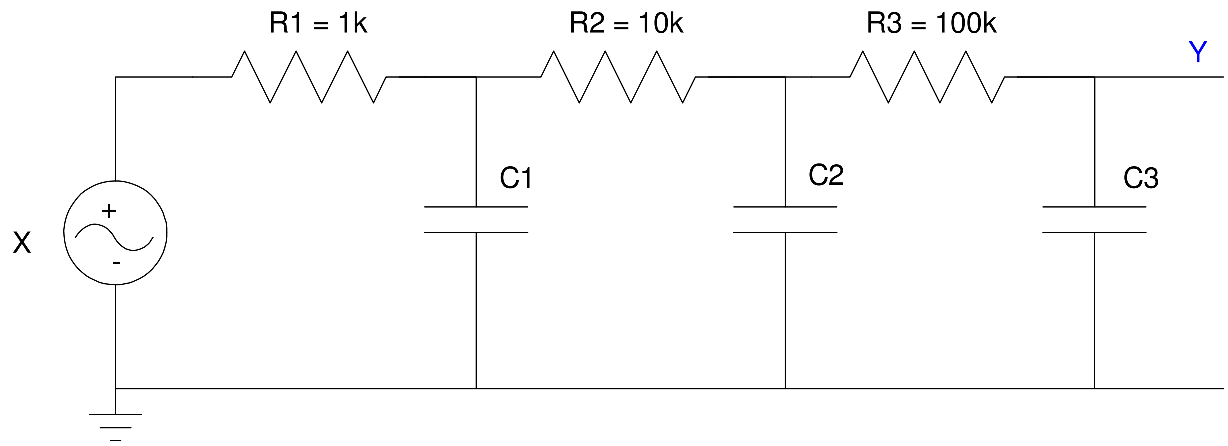
$$Y = \left(\frac{abc}{(s+a)(s+b)(s+c)}\right)X$$

Solution:

$$a = \left(\frac{1}{R_1 C_1}\right)$$

$$b = \left(\frac{1}{R_2 C_2}\right)$$

$$c = \left(\frac{1}{R_3 C_3}\right)$$



Notes:

- This filter is easy to build (good), but
- It's not a very good filter (gain drops off with frequency very fast)

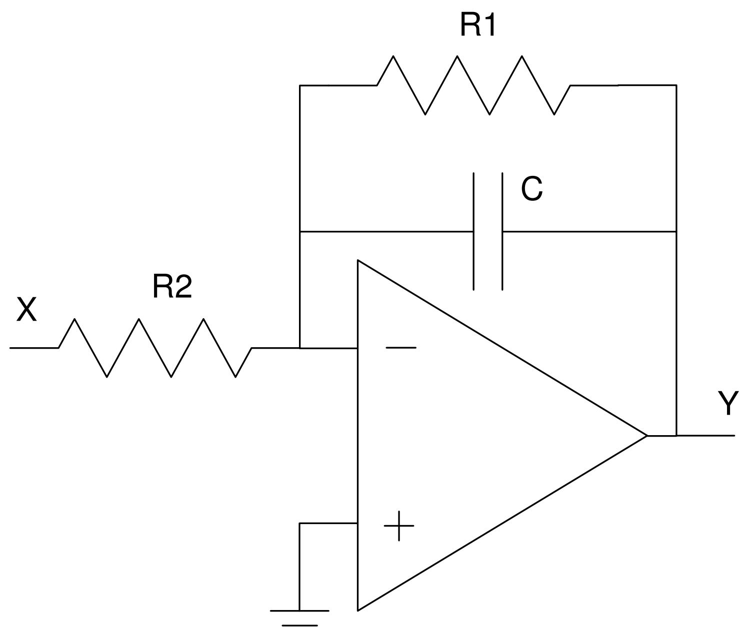
Real Poles, No Zeros (take 2)

$$Y = -\left(\frac{a}{s+b}\right)X$$

where

$$a = \frac{1}{R_2 C}$$

$$b = \frac{1}{R_1 C}$$



Example:

$$Y = -\left(\frac{50}{s+100}\right)X$$

Let

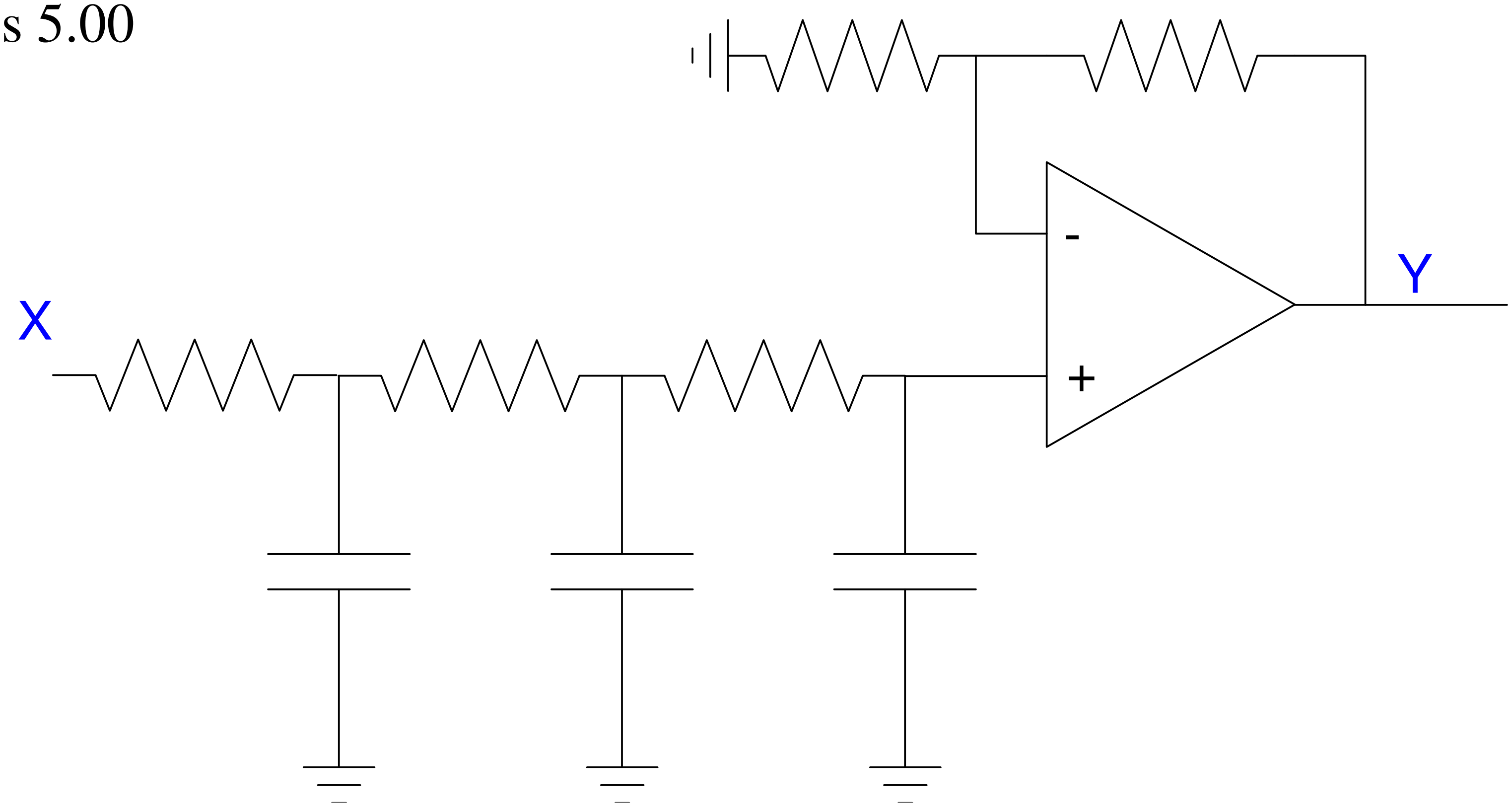
- C = 1uF (arbitrary)
- R1 = 10k
- R2 = 20k

Example: Design a filter to implement

$$Y = \left(\frac{500}{(s+2)(s+5)(s+10)} \right) X$$

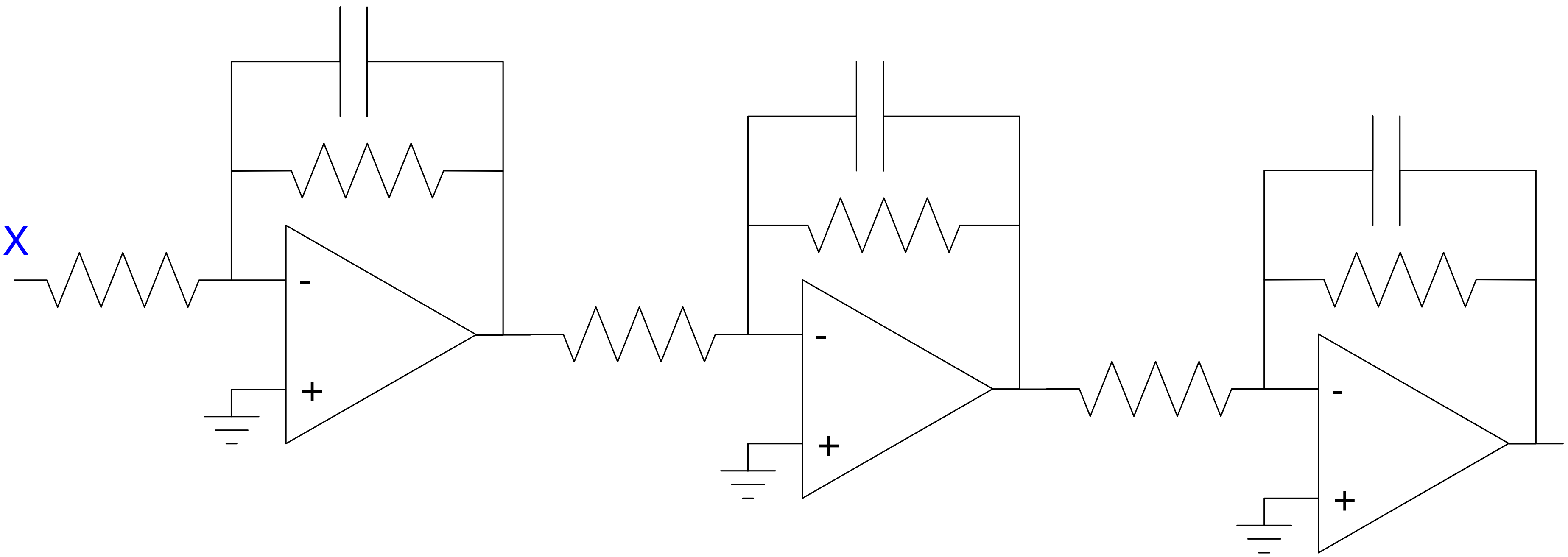
Option #1:

- 3-stage RC filter (poles at -3, -5, -10)
- DC gain is 5.00



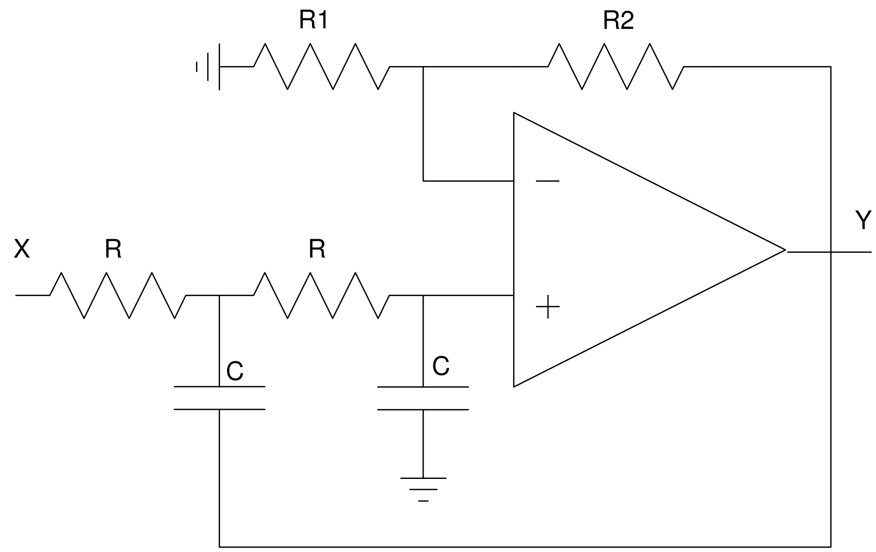
Option #2: Three 1st-order filters

$$-Y = \left(\frac{-500}{(s+2)(s+5)(s+10)}\right)X = \left(\frac{-5}{s+2}\right)\left(\frac{-10}{s+5}\right)\left(\frac{-10}{s+10}\right)X$$



Complex Poles, No Zeros

$$Y = \left(\frac{k \cdot \left(\frac{1}{RC}\right)^2}{s^2 + \left(\frac{3-k}{RC}\right) s + \left(\frac{1}{RC}\right)^2} \right) X$$

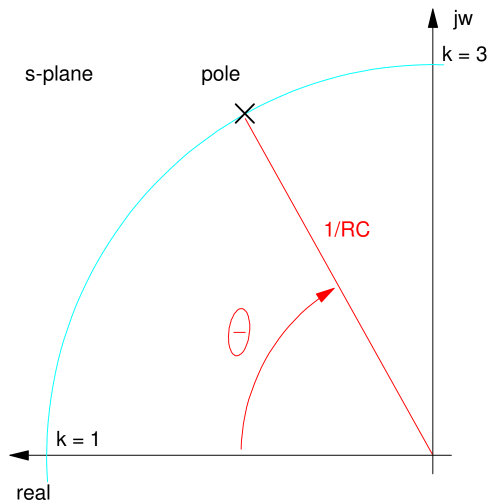


This filter has two complex poles with

- Amplitude = $\frac{1}{RC}$
- Angle: $3 - k = 2\cos\theta$
- DC gain $k = \left(1 + \frac{R_2}{R_1}\right)$

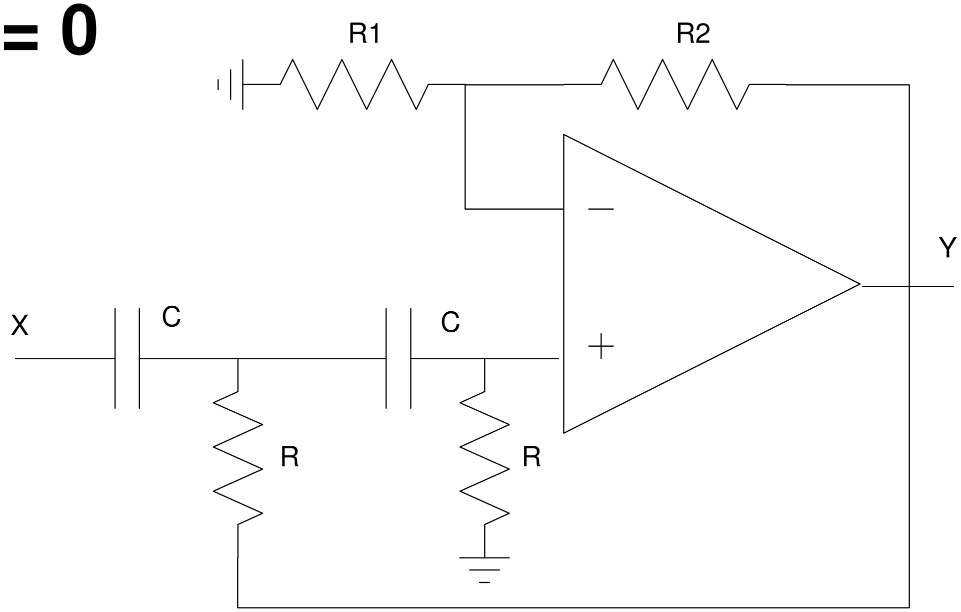
Note that the angle of the poles goes from

- 0 degrees when k =1
- 90 degrees when k = 3 (an oscillator)



Comples Polex, Two Zeros at s = 0

$$Y = \left(\frac{k \cdot s^2}{s^2 + \left(\frac{3-k}{RC}\right)s + \left(\frac{1}{RC}\right)^2} \right) X$$



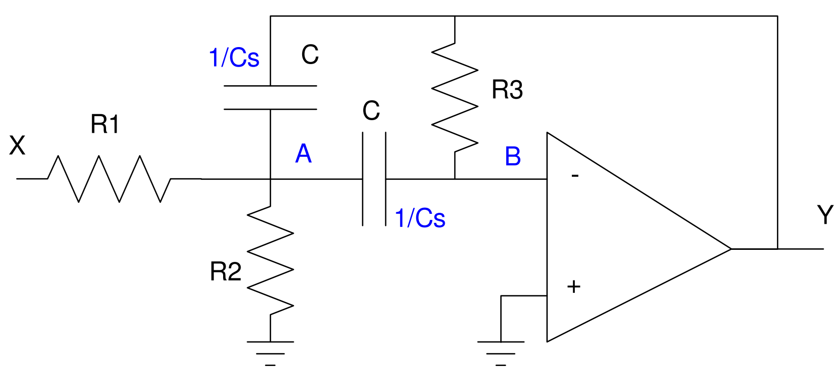
This filter has two complex poles with

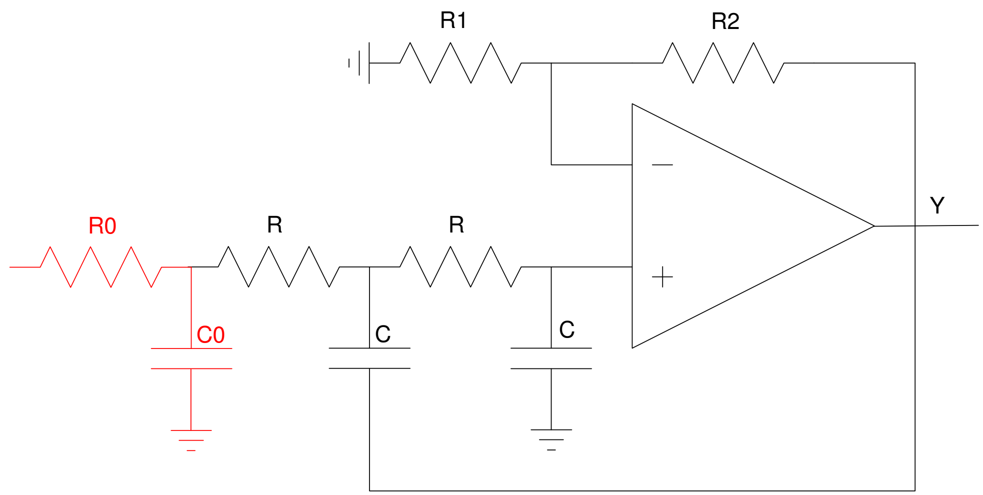
- Amplitude = $\frac{1}{RC}$
- Angle: $3 - k = 2\cos\theta$
- High Freq gain $k = \left(1 + \frac{R_2}{R_1}\right)$

Comples Polex, One Zeros at s = 0:

$$Y = \left(\frac{as}{s^2+bs+c}\right)X$$

$$Y = \left(\frac{-\left(\frac{1}{R_1 C}\right)s}{s^2+\left(\frac{2}{R_3 C}\right)s+\left(\frac{R_1+R_2}{R_1 R_2}\right)\left(\frac{1}{R_3 C^2}\right)}\right)X$$





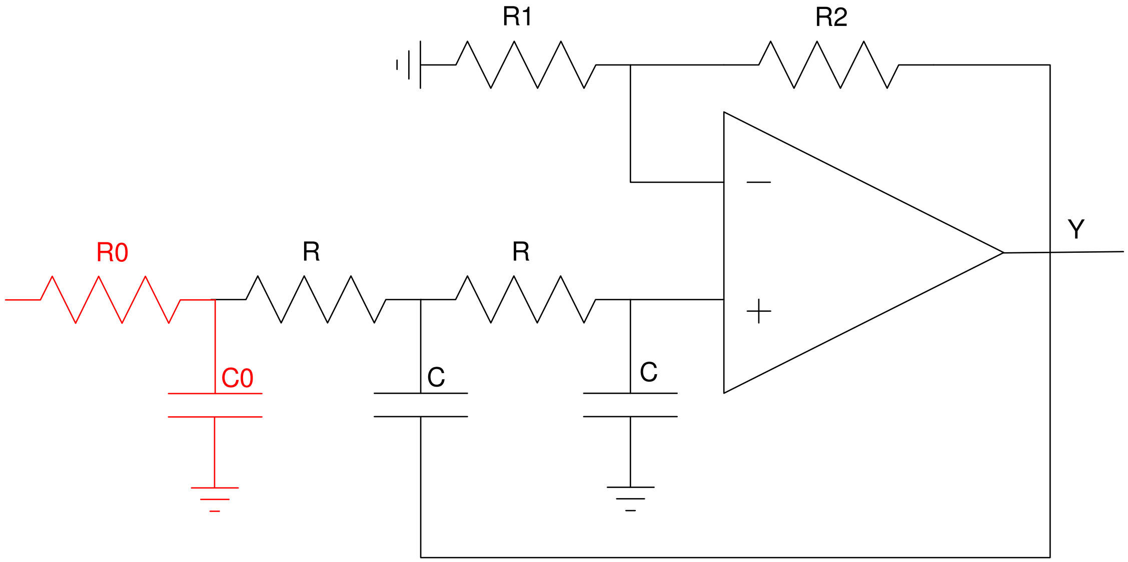
Example: Design a circuit to implement

$$Y = \left(\frac{1{,}244{,}485}{(s+85)(s+121\angle 69.5^0)(s+121\angle -69.5^0)} \right) X$$

Rewrite this as

$$Y = \left(\frac{85}{s+85} \right) \left(\frac{14{,}641}{(s+121\angle 69.5^0)(s+121\angle -69.5^0)} \right) X$$

Use the previous filters



To avoid loading, let

- R0 = 10k
- R = 100k

Matching terms in the denominator:

$$\left(\frac{1}{R_0 C_0}\right) = 85 \qquad C_0 = 1.17\mu F$$

$$\left(\frac{1}{RC}\right) = 121 \qquad C = 0.082\mu F$$

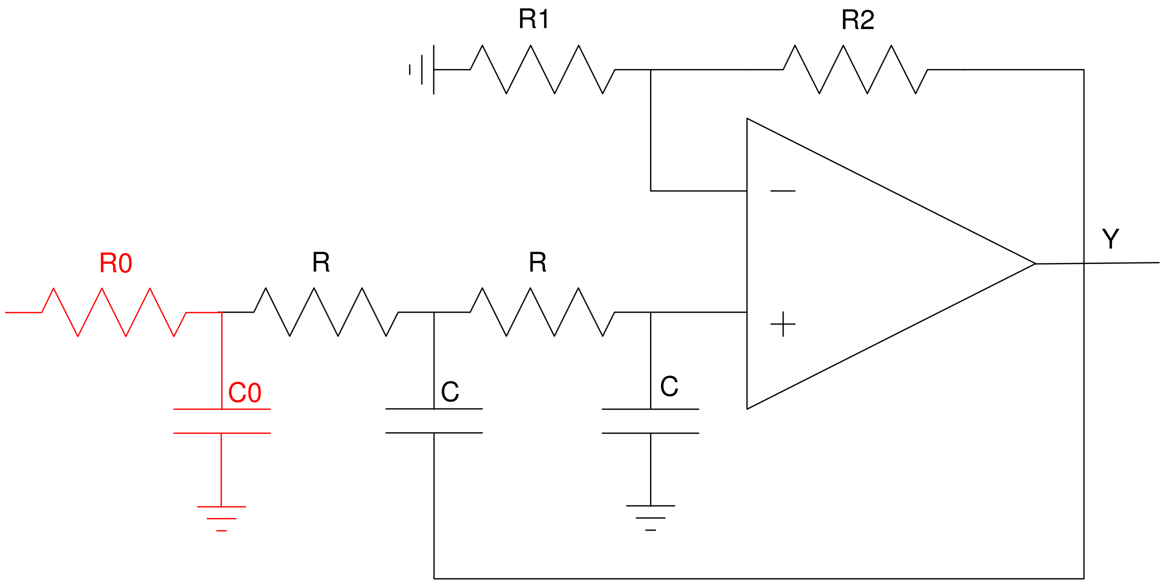
$$3 - k = 2\cos(69.5^0)$$

$$k = 2.3$$

$$1 + \frac{R_2}{R_1} = 2.3$$

R1 = 100k, R2 = 130k

Note: DC gain is 2.3.



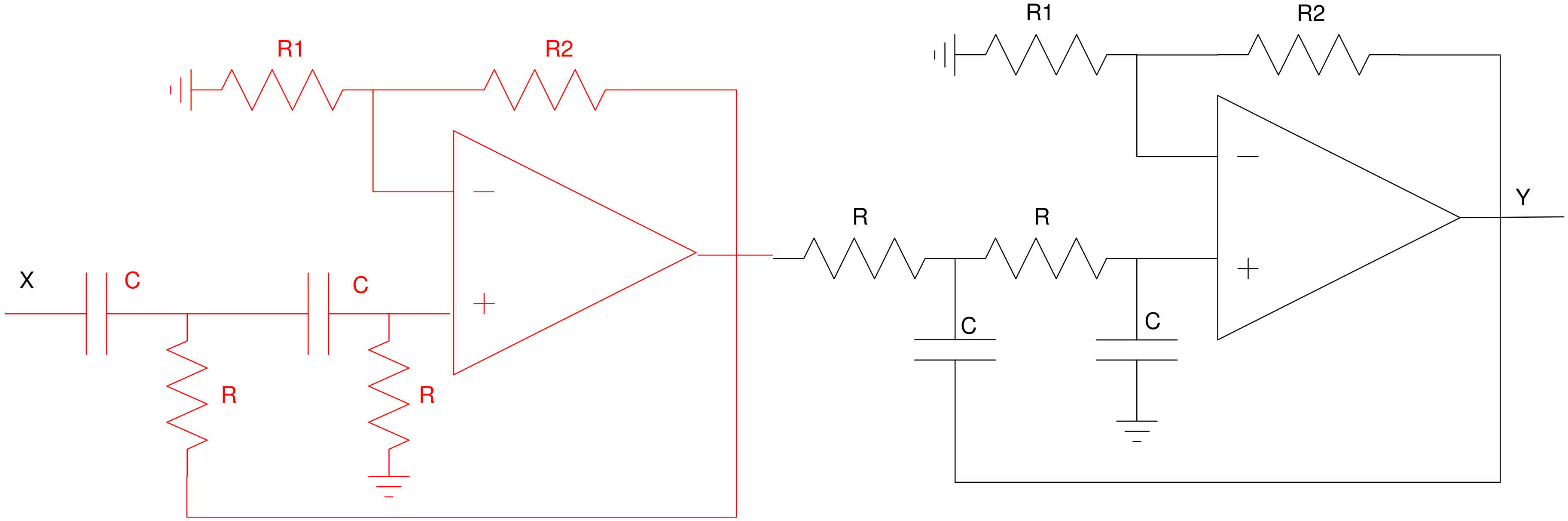
Example: Design a filter to implement

$$Y = \left(\frac{100{,}000 s^2}{\left(s^2+14s+100\right)\left(s^2+100s+10{,}000\right)} \right) X$$

Solution: Rewrite this as the product of two filters:

$$Y = \left(\frac{s^2}{\left(s^2+14s+100\right)} \right) \left(\frac{10{,}000}{\left(s^2+100s+10{,}000\right)} \right) X$$

Using the previous circuits (building blocks),



$$\left(\frac{k\cdot s^2}{s^2+\left(\frac{3-k}{RC}\right)s+\left(\frac{1}{RC}\right)^2}\right)$$

$$\left(\frac{k\cdot\left(\frac{1}{RC}\right)^2}{s^2+\left(\frac{3-k}{RC}\right)s+\left(\frac{1}{RC}\right)^2}\right)$$

1st Stage:

$$\left(\frac{k\cdot s^2}{s^2+\left(\frac{3-k}{RC}\right)s+\left(\frac{1}{RC}\right)^2}\right)=\left(\frac{s^2}{\left(s^2+14s+100\right)}\right)=\left(\frac{s^2}{\left(s+10\angle 45^0\right)\left(s+10\angle -45^0\right)}\right)$$

Ignore the numerator gain. Match the denominator (the poles)

Matching the poles:

$$\left(\frac{1}{RC}\right)=10$$

C = 1uF,   R = 100k

$$3-k=2\cos\left(45^0\right)$$

k = 1.5858

R1 = 100k,   R2 = 58k

2nd Stage

$$\left(\frac{k\cdot\left(\frac{1}{RC}\right)^2}{s^2+\left(\frac{3-k}{RC}\right)s+\left(\frac{1}{RC}\right)^2}\right)=\left(\frac{10{,}000}{\left(s^2+100s+10{,}000\right)}\right)=\left(\frac{10{,}000}{\left(s+100\angle 60^0\right)\left(s+100\angle -60^0\right)}\right)$$

$$\left(\frac{1}{RC}\right)=100$$

C = 1uF,   R = 10k

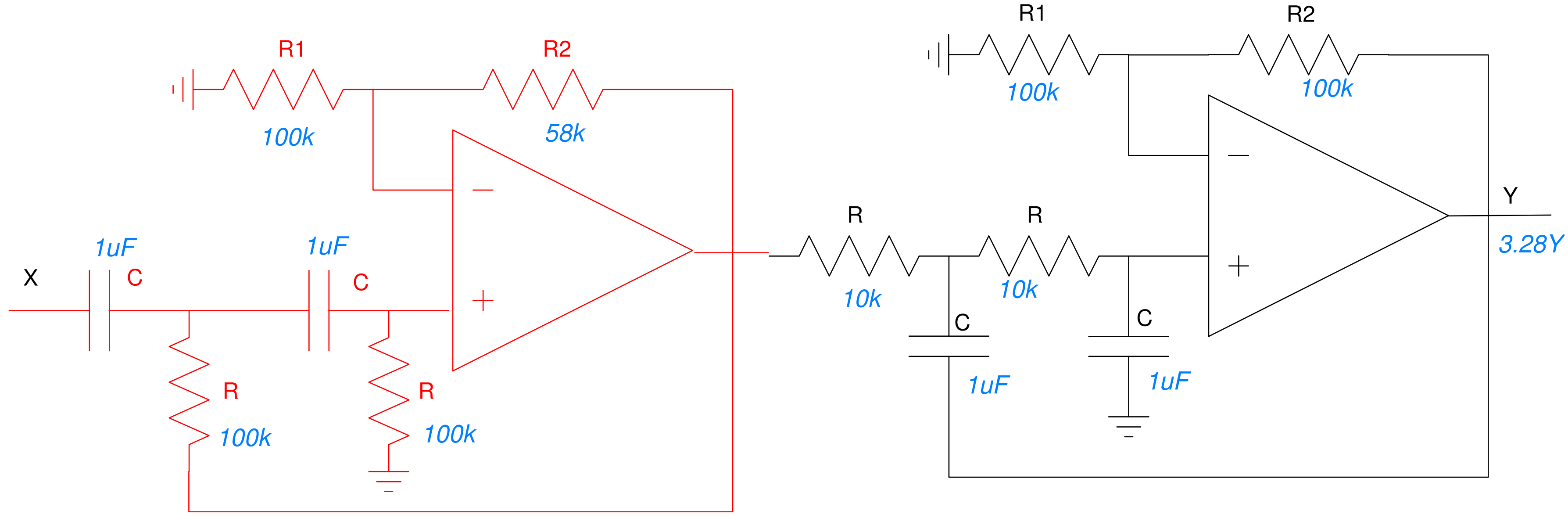
$$3-k=2\cos\left(60^0\right)$$

k = 2

R1 = R2 = 100k

Resulting Circuit

- midband gain is 3.28 (vs. 1.000)
- Call the output 3.28Y



Summary

Filter design is like using Legos: you cascade different building blocks

Step 1: Factor the filter into sections with real and complex poles

Step 2: Implement each section

- Single real pole: RC filter or RC active filter
- Complex poles with no zeros
- Complex poles with one zero at s=0
- Complex poles with two zeros at s=0

Step 3: Cascade sections