Calibration and Noise

ECE 321: Electronics II

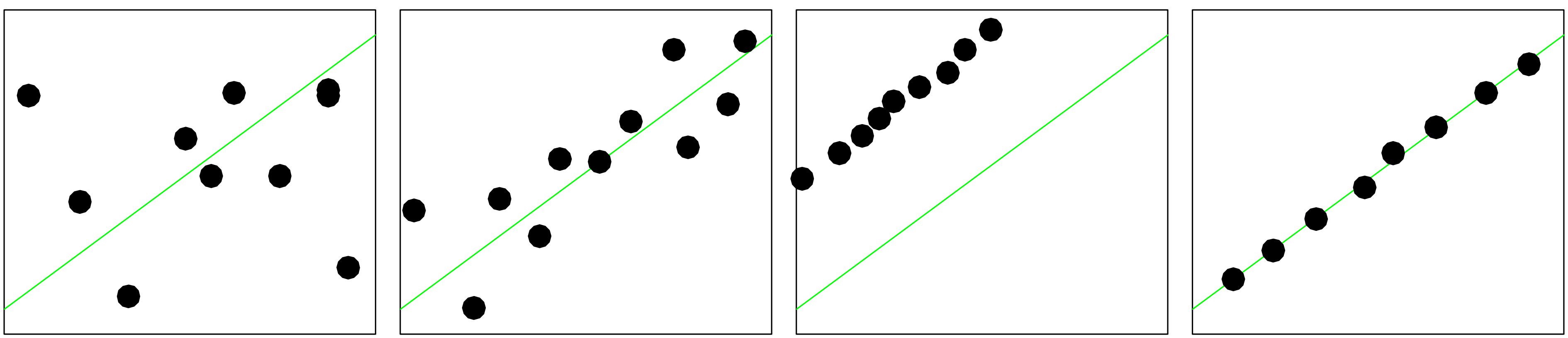
Lecture #6

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Calibration

Accuracy: (mean) The difference between the expected value of a sensor's output and the actual value of the quantity being measured. The mean of a set of data is related to the accuracy of the data.

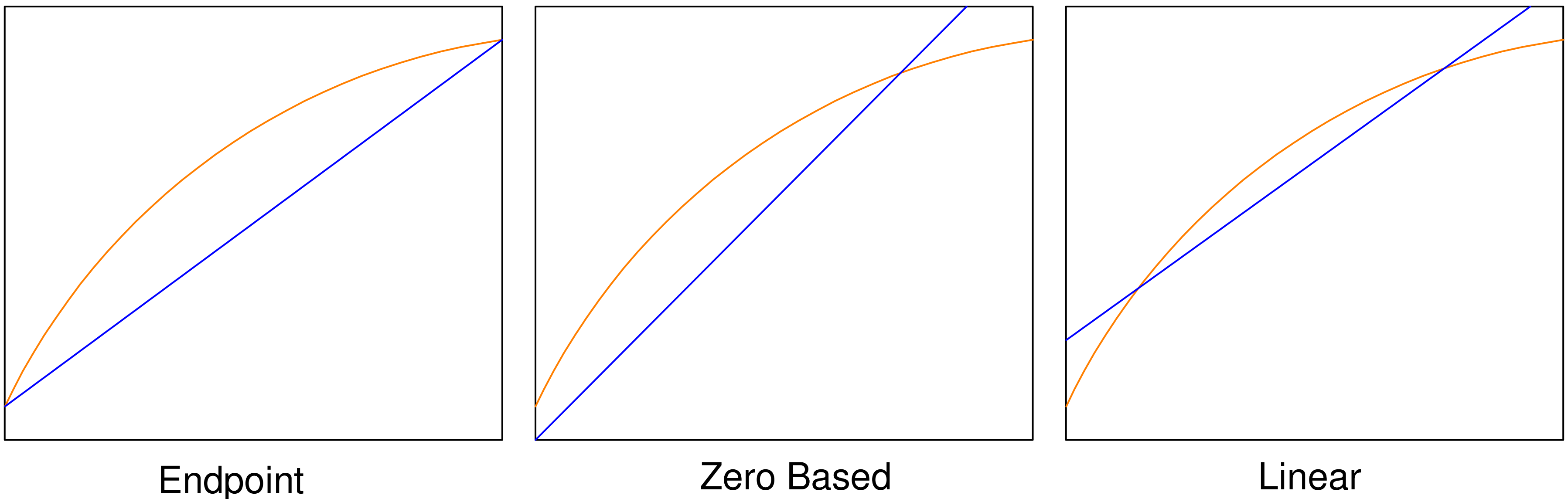
Precision: (standard deviation) The variations in a measured signal due to noise or other phenomena. The standard deviation of a set of data is a measure of the precision.



Accurate | Precise | Accurate and Precise

Types of Calibration

- $y = f(x)$ theoretical
- $y = ax + b$ endpoint, passes through end points
- $y = ax$ zero based
- $y = ax + b$ linear, a, b free



Least Squares Curve Fitting:

Given a function (or data)

$$y = f(x)$$

determine (a, b, c) to approximate y() as

$$\hat{y} \approx ax^2 + bx + c$$

such that the sum-squared difference is minimized

$$J = \sum (y_i - \hat{y}_i)^2$$

Solution

Express in matrix form for n data points

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$Y_{nx1} = B_{nx3} A_{3x1}$$

Multiply by B^T:

$$B^T Y = B^T B A$$

Sove for A

$$A = (B^T B)^{-1} B^T Y$$

Weighted Least Squares:

$$J = \sum q_i (y_i - \hat{y}_i)^2$$

Define Q:

$$Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & \ddots \end{bmatrix}$$

Solve for A:

$$Y = BA$$

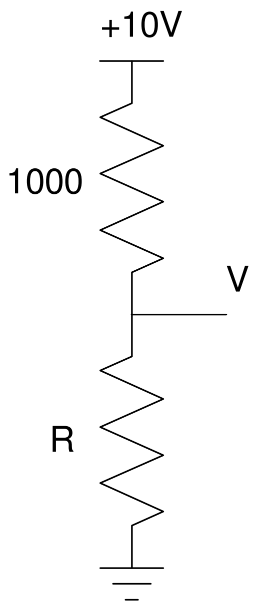
$$QY = QBA$$

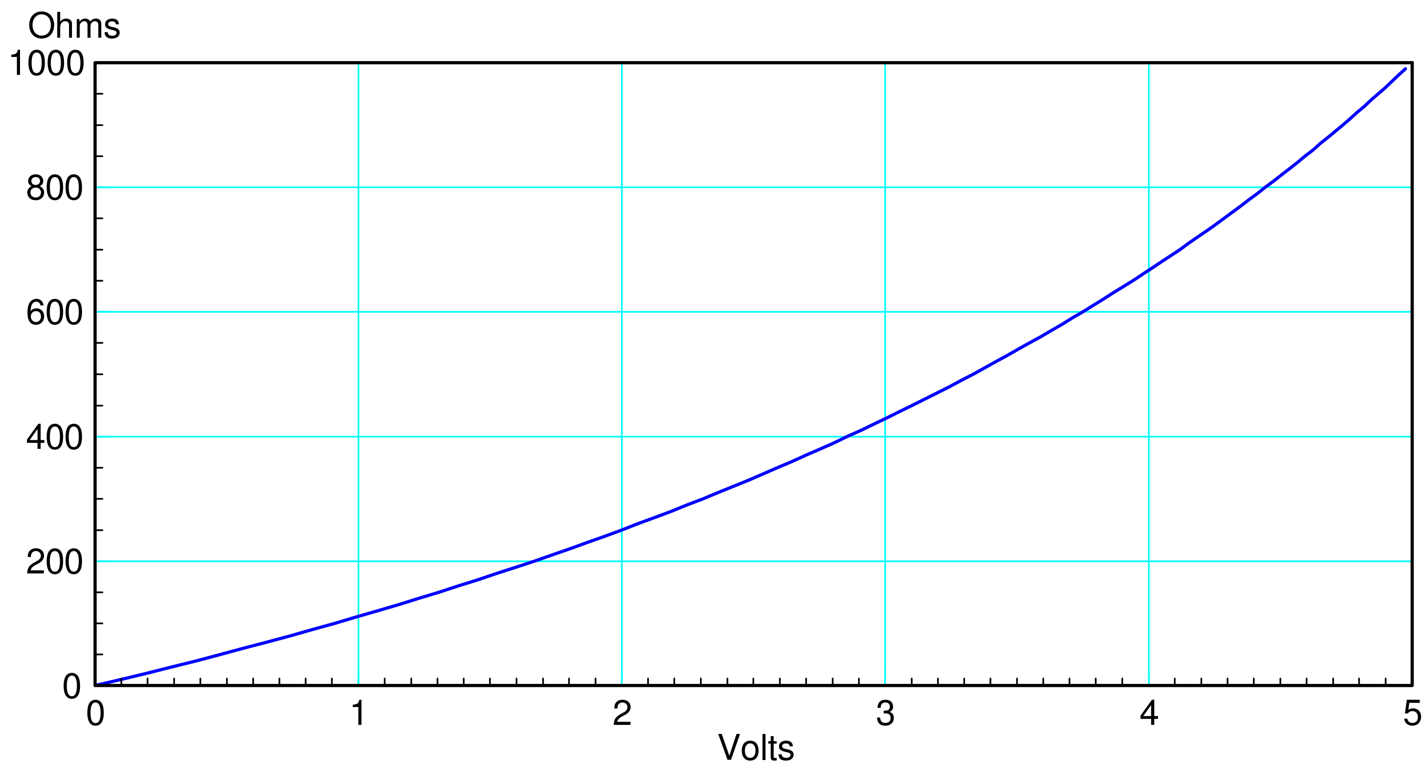
$$B^T QY = B^T QBA$$

$$A = (B^T QB)^{-1} B^T QY$$

Example: Determine R = f(V)

- Range: 0 < R < 1000 Ohms





Zero-Based Calibration: First, set this up as

$$R \approx aV = AX$$

Basis Function:

$$B = [V]$$

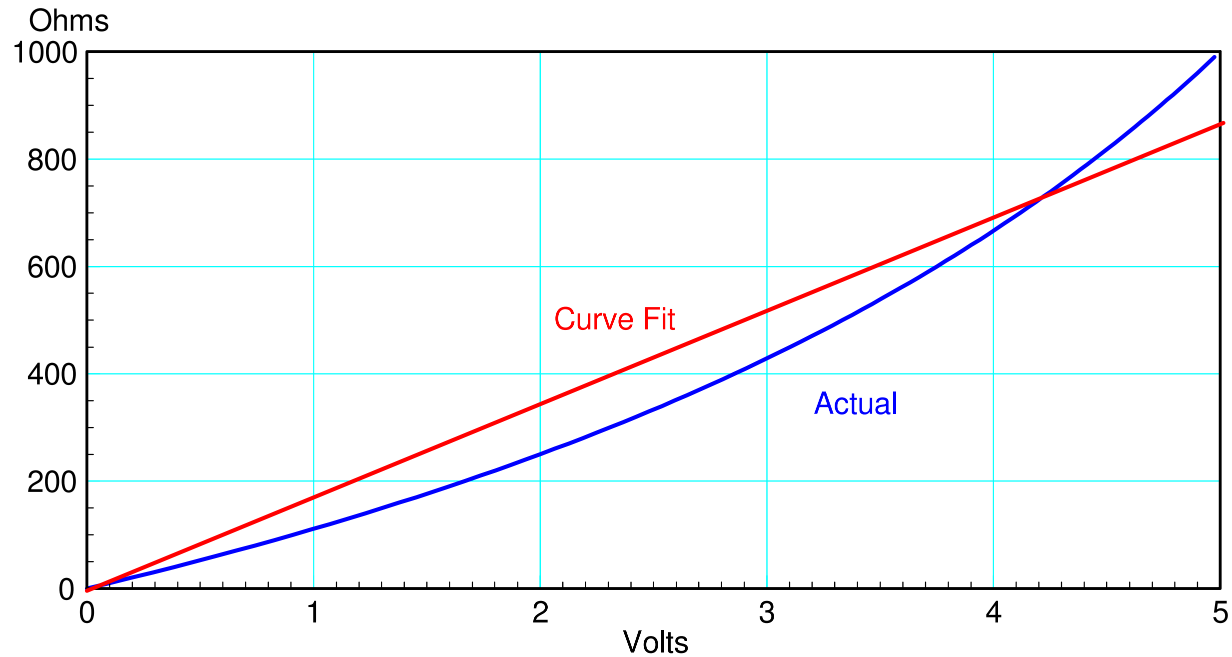
$$R \approx 169.53 \cdot V$$

Accuracy:

```
x = mean(R - B*A)
      x  = -20.967
```

Precision:

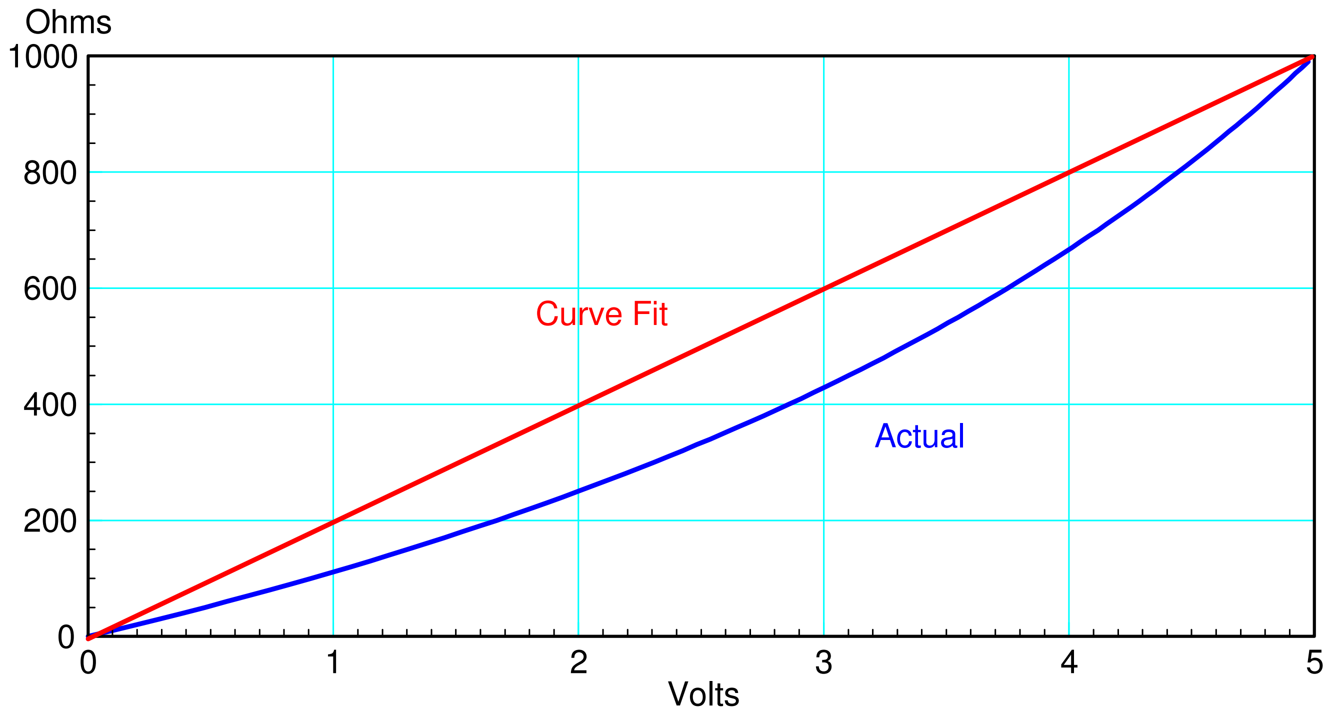
```
s = std(R - B*A)
      s  =  68.82
```



Endpoint Calibration: Use two endpoints

$$R \approx aV + b$$

$$R \approx 200V + 0$$



Accuracy:

```
x = mean(R - B*A)
     x  = -113.69
```

Precision

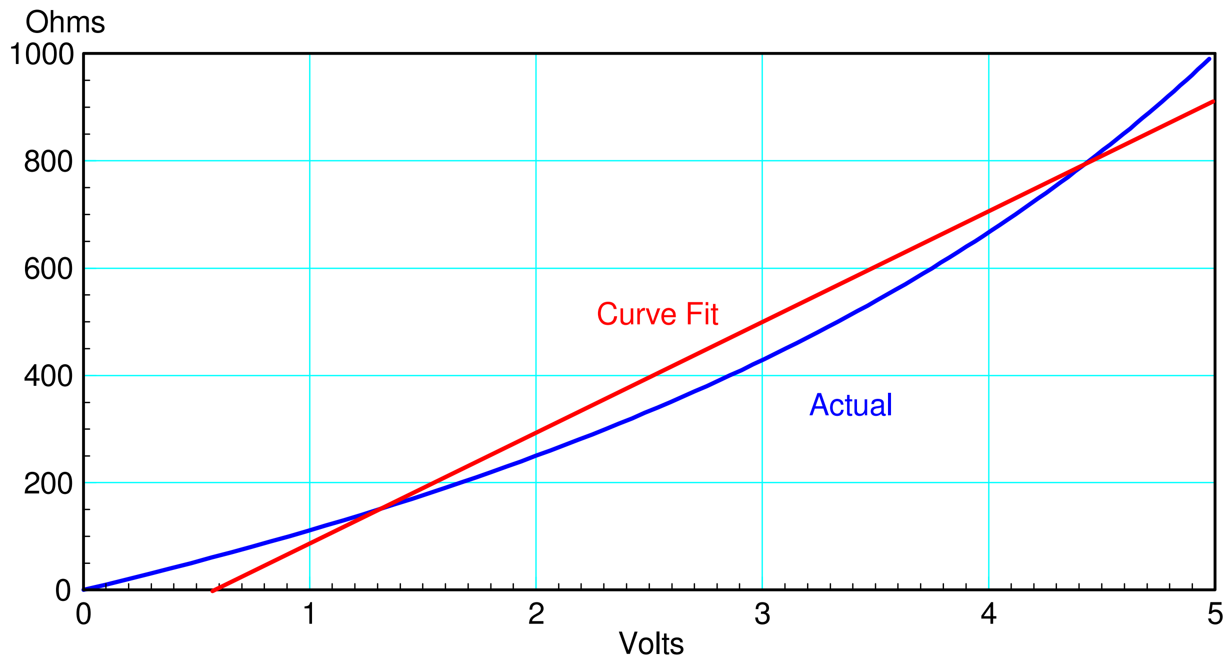
```
s = stdev(R - B*A)
     s  =   51.65
```

Linear Calibration: Use all points

$$R \approx aV + b$$

```
B = [V, V.^0];
A = inv(B'*B)*B'*R
```

$$R \approx 201.715V - 118.913$$



Accuracy:

```
x = mean(R - B*A)
     x  = 0
```

Precision

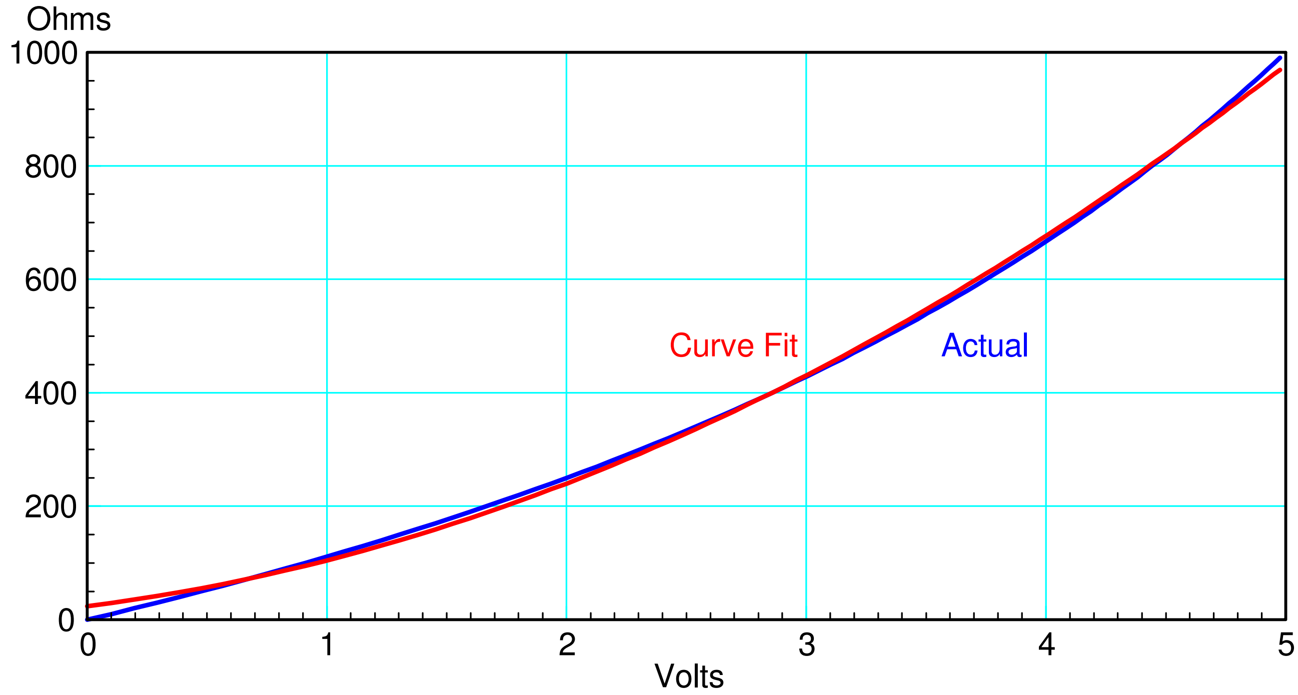
```
s = stdev(R - B*A)
    s  =  51.59
```

Polynomial Calibration: Add terms to the basis

$$R \approx aV^2 + bV + c$$

```
B = [V.^2, V, V.^0];
A = inv(B'*B)*B'*R
```

$$R \approx 27.54V^2 + 52.98V + 24.01$$



Accuracy

```
x = mean(R - B*A)
    x  =   0
```

Precision

```
s = stdev(R - B*A)
    s  =  9.000
```

Summary

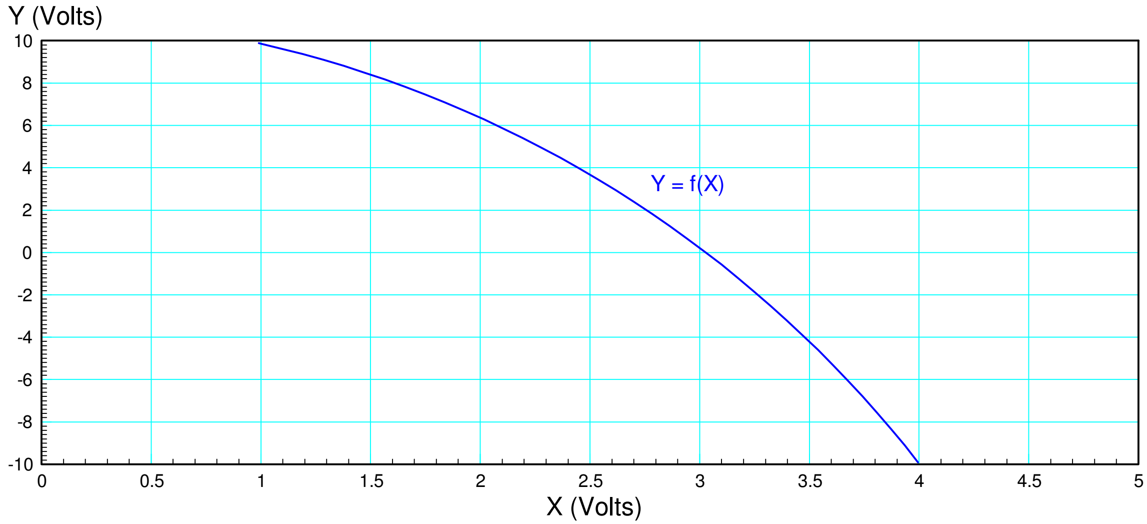
Calibration Scheme	R = f(V)	Accuracy mean(error)	Precision std(error)
Zero-Based	R = 169.53 V	-20.97	68.82
Endpoint	R = 200 V	-113.69	51.65
Linear	R = 201.7 V - 118.9	0	51.6
Polynomial	$R = 27.54V^2 + 52.98V + 24.01$	0	9.0

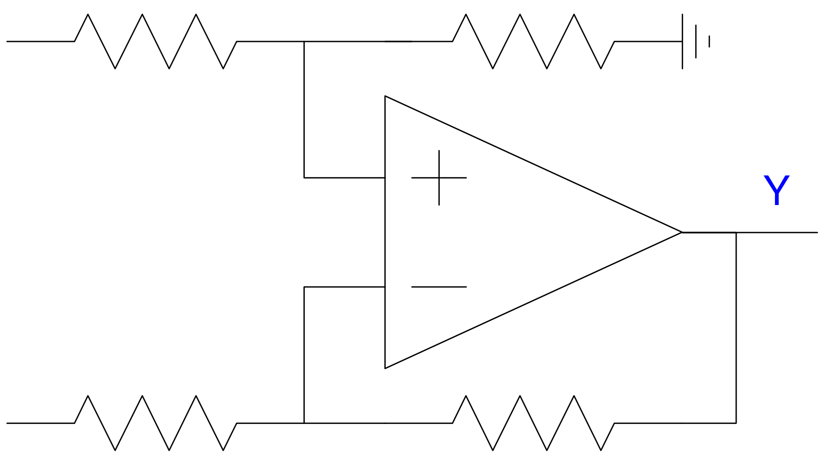
Handout

Design a circuit to approximate y(x)

Determine Y if X = 2.5V

- Actual
- Curve Fit

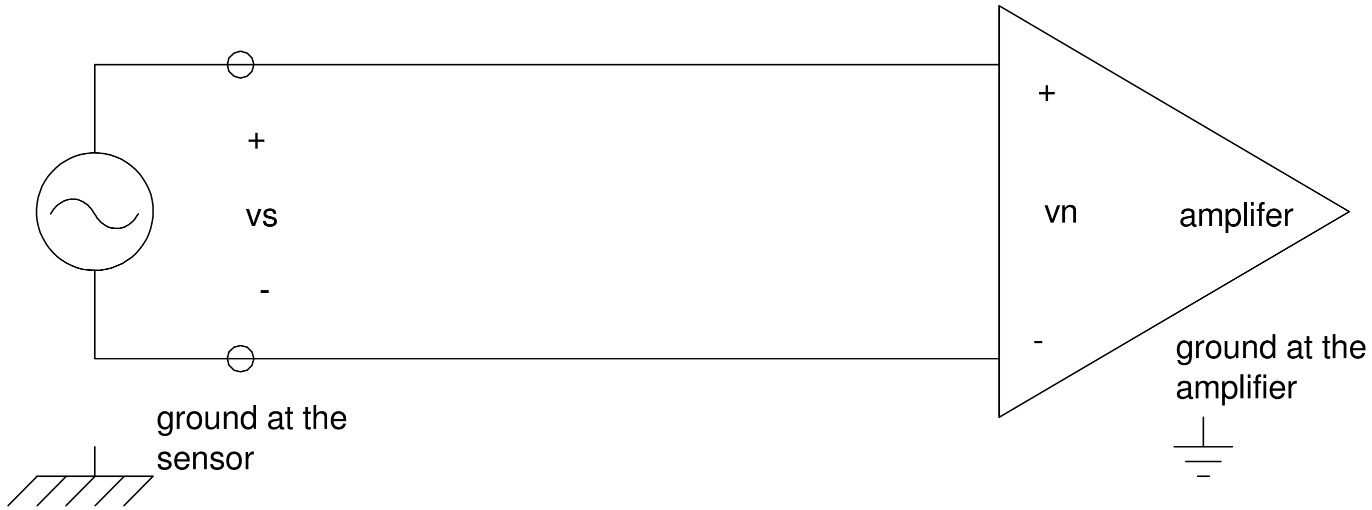




Noise

Consider the problem of trying to measure a voltage, Vs, remotely.

Ideally, the output voltage is proportional to Vs.



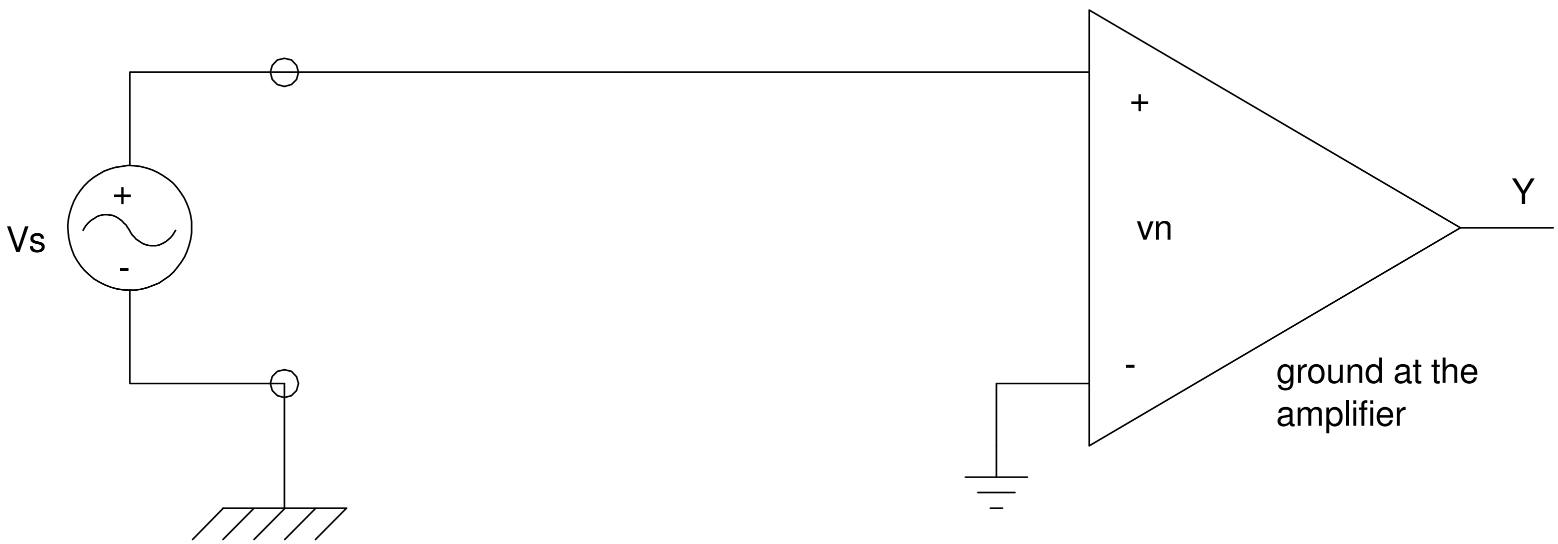
Definitions

- **Common-Mode Gain:** The output proportional to the sum (or average) of Va and Vb. Common mode gain is ideally zero so that noise along the length of the line cancels out.
- **Differential Gain:** The output proportional to the difference of Va and Vb.
- **Common Mode Rejection Ratio:** The ratio of the differential gain to the common-mode gain. The larger this number is, the better the amplifier is.
- **Ground Loops:** All grounds are not equal. The potential at one point may be different than another point. (For example, there's about a 4V difference between Cincinnati and Columbus. If you take a wire, ground one end in Cincinnati and one end in Columbus, you'll likewise get some current flow. Over smaller distances, noise (from radio signals, transformers, etc.) may affect the potential at one location and not another.
- **Signal-to-Noise Ratio:** The ratio of the energy at Vn due to the signal (Vs) to the energy of Vn due to other sources.

Basic Circuit

Case 1: The simplest circuit - and also the worse:

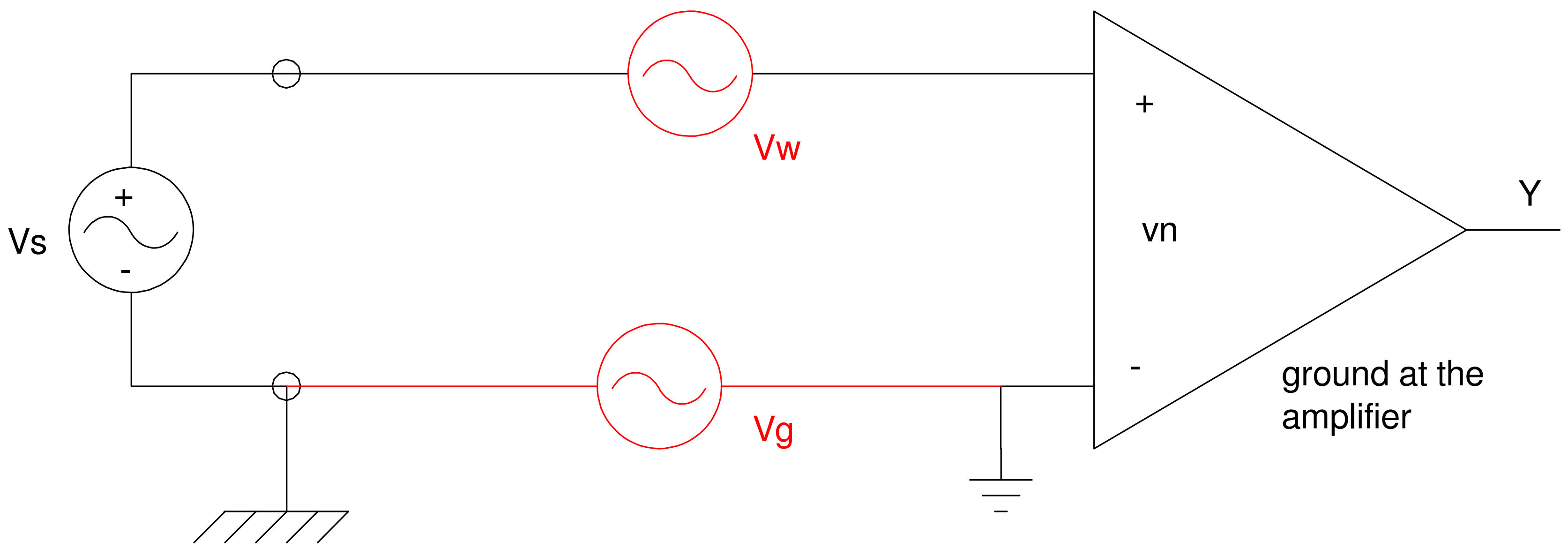
- In order to save wire, the sensor is grounded locally. A single wire is then used to transfer this data to the data recorder. This voltage is then compared with its local ground.



The problem with this circuit is several. This can be seen by adding two more voltages to this circuit to signify noise sources.

$$Vn = Vs + Vg + Vw$$

This circuit is sensitive to common mode noise (Vg+Vw).

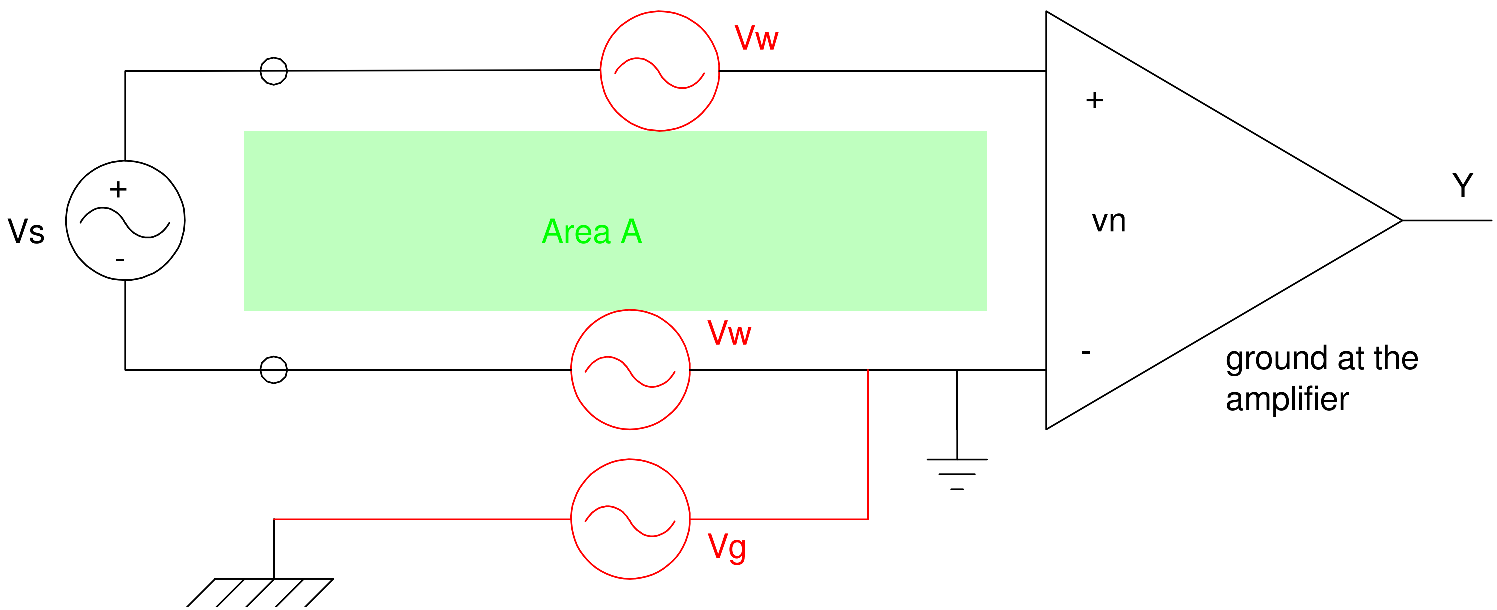


Case 2: In order to eliminate this common-mode noise, a pair of wires could be used:

Better: Vw cancels.

Worse: Enclosed area picks up noise (Faraday's law)

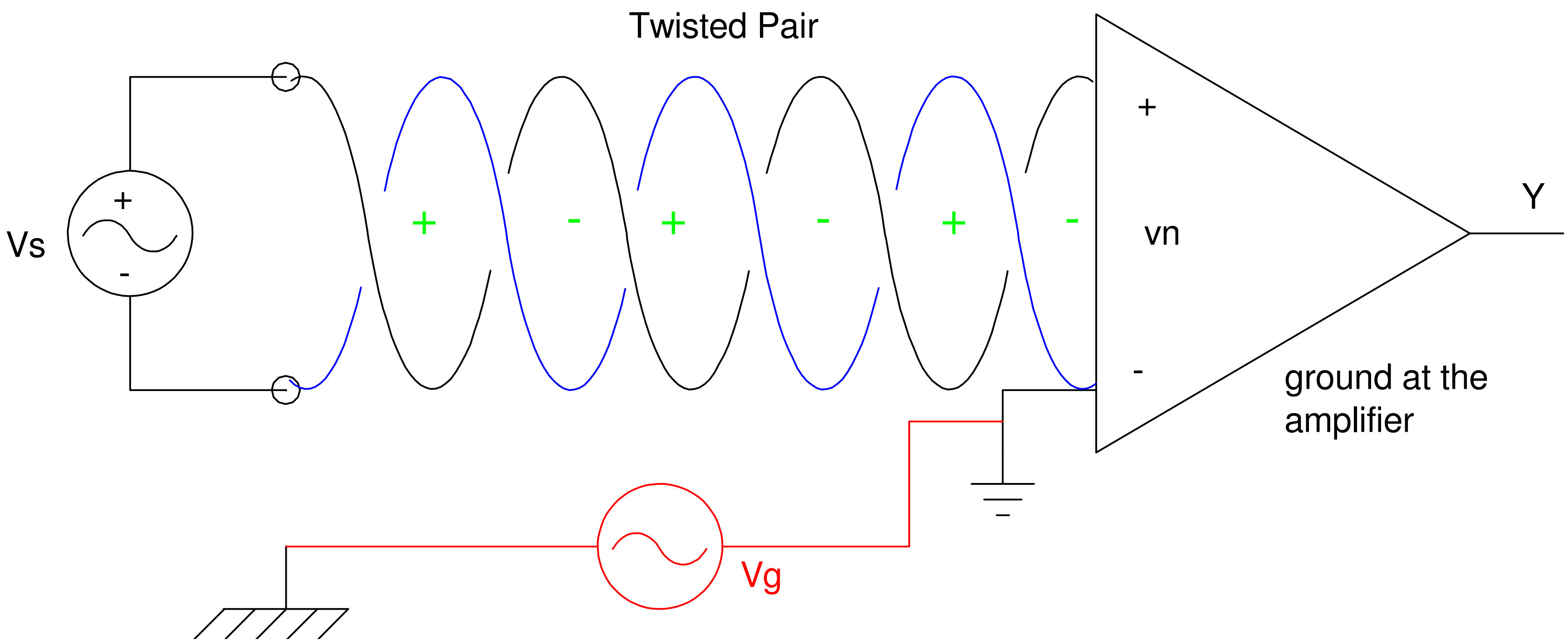
$$Vn = Vw + Vs - Vw + VA$$



Case 3: Twisted Pair:

- Twist the wires.
- Areas cancel (almost)

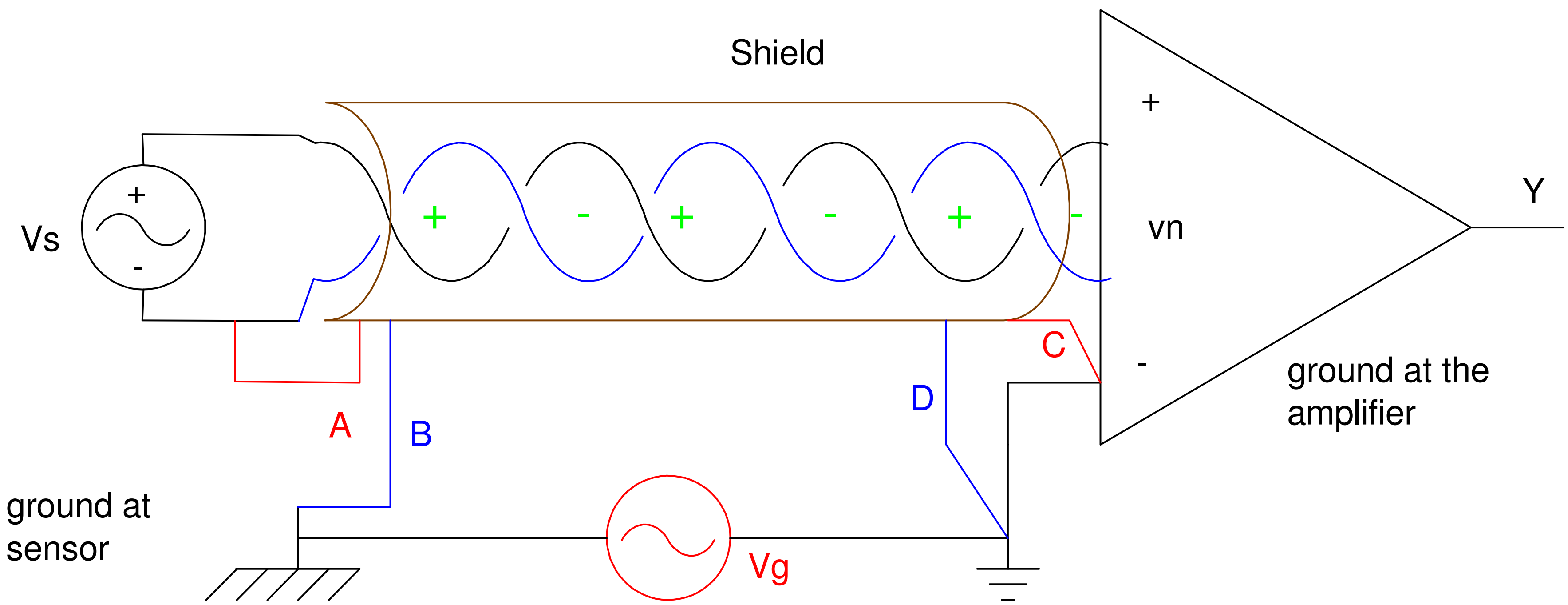
Good for short runs of wires



Case 4: Shielded Twisted Pair with a Grounded Amplifier:

Best: C (D close second)

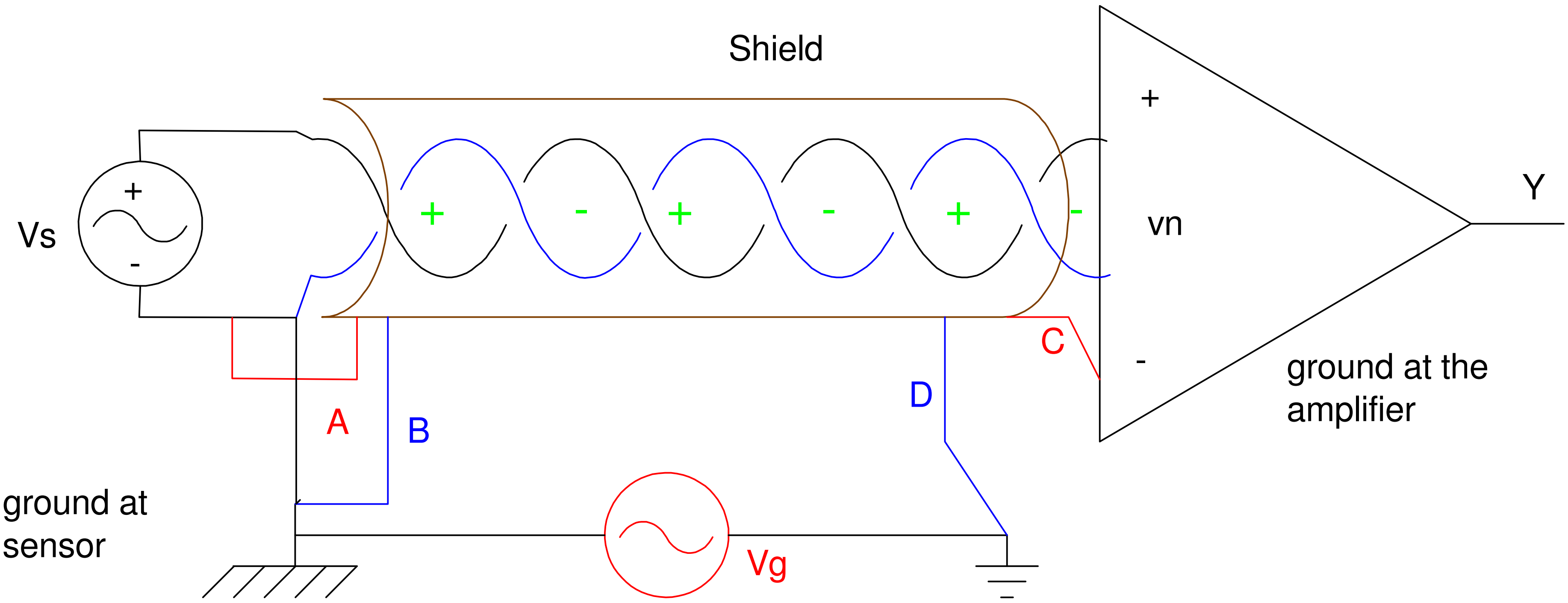
Worst: A or B



Case 5: Shielded Twisted Pair with a Grounded Sensor:

Best: A (B close second)

Worst: C and D



Summary

Calibration: Approximate y = f(x) with

- A line (linear)
- A parabola
- Other

Higher-order functions are more accurate

- but more complex to use

Noise:

- Avoid ground loops: Use a single ground
- Use twisted pair (reduces the area enclosed)
- Use shielding (reduces the field strength)
- Filters are band-aids: it's better to prevent the noise than to filter