Analog Computers

Problem:

Design a circuit to implement a generic proper transfer function

$$Y = \left(\frac{b_{n-1}s^{n-1}+b_{n-2}s^{n-2}+...+b_1s+b_0}{s^n+a_{n-1}s^{n-1}+a_{n-2}s^{n-2}+...+a_1s+a_0}\right)U$$

Solution:

There are many. This is one way to do it. Just to make it more managable, assume a 3rd-order system

$$Y = \left(\frac{b_2s^2+b_1s+b_0}{s^3+a_2s^2+a_1s+a_0}\right)U$$

Step 1: Change the problem. Create a dummy state, X

$$X = \left(\frac{1}{s^3+a_2s^2+a_1s+a_0}\right)U$$

$$Y = (b_2s^2+b_1s+b_0)X$$

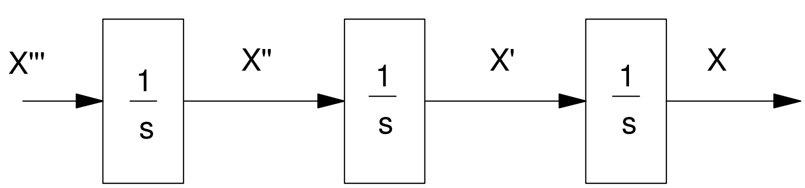
Step 2: Cross multiply and solve for the highest derivative of X:

$$X = \left(\frac{1}{s^3+a_2s^2+a_1s+a_{00}}\right)U$$

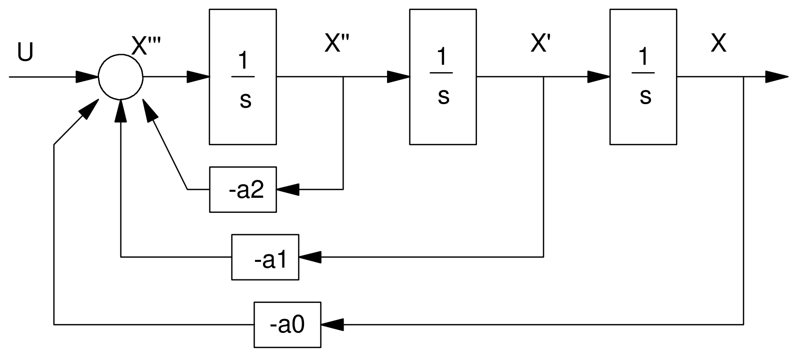
$$(s^3+a_2s^2+a_1s+a_0)X = U$$

$$s^3X = -(a_2s^2+a_1s+a_0)X+U$$

Step 3: Given snX, solve for X by integrating n times (notation: X' means dx/dt)

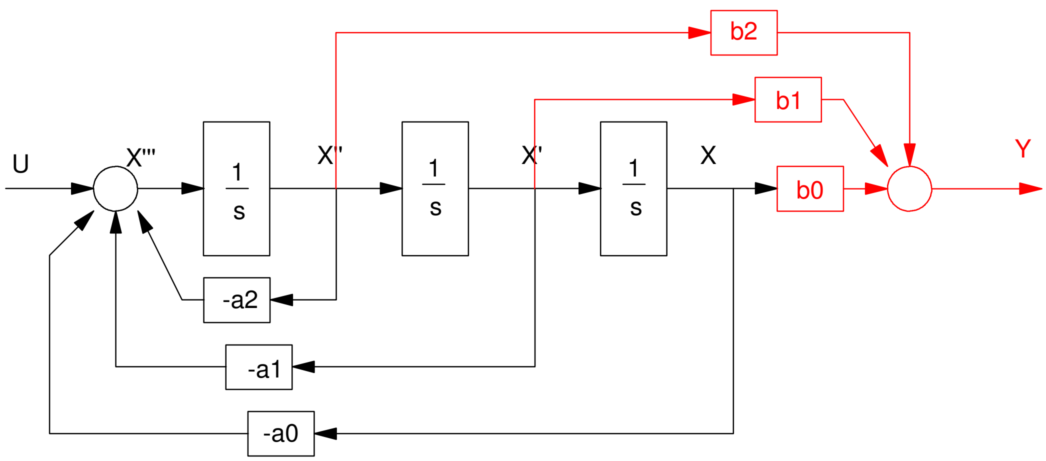


Step 4: Create X''' using the differential equation from step 2:

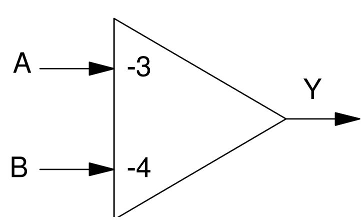


Step 5: Now that you know X and its detivatives, create Y:

$$Y = (b_2 s^2 + b_1 s + b_0) X$$

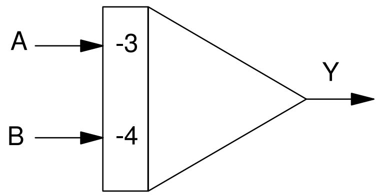


Step 6) Convert to analog computer notation. Here, a triangle means an amplifuer:



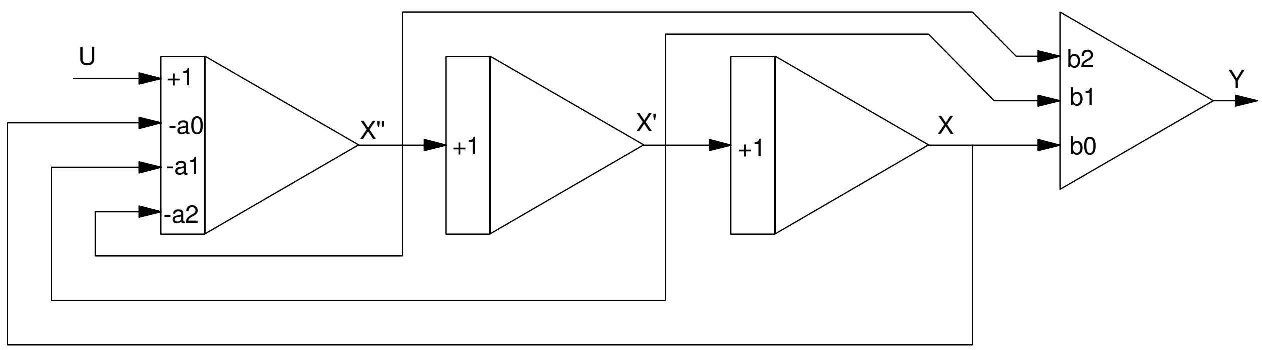
Y = -3A - 4B

whereas a triangle with a box means integrator:



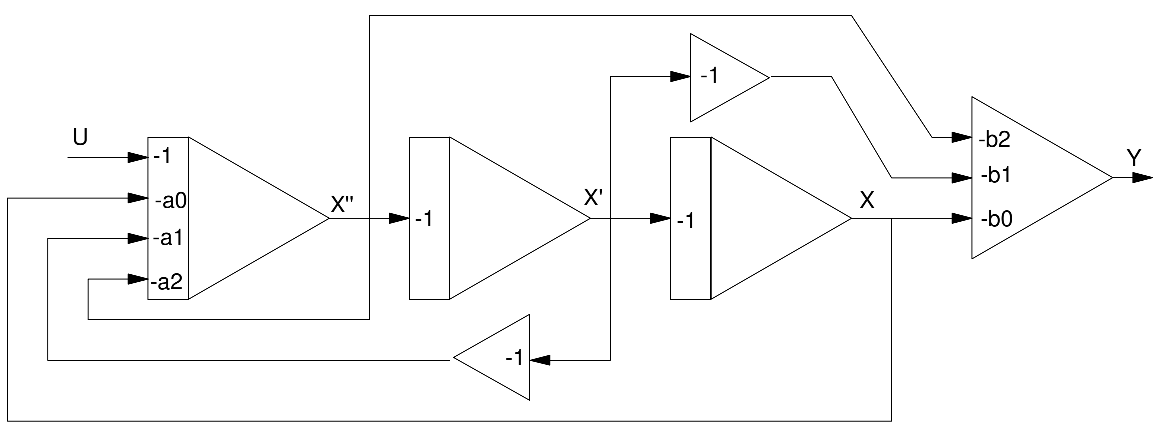
Y = (1/s) (-3A - 4B)

Applying this to the above block diagram:

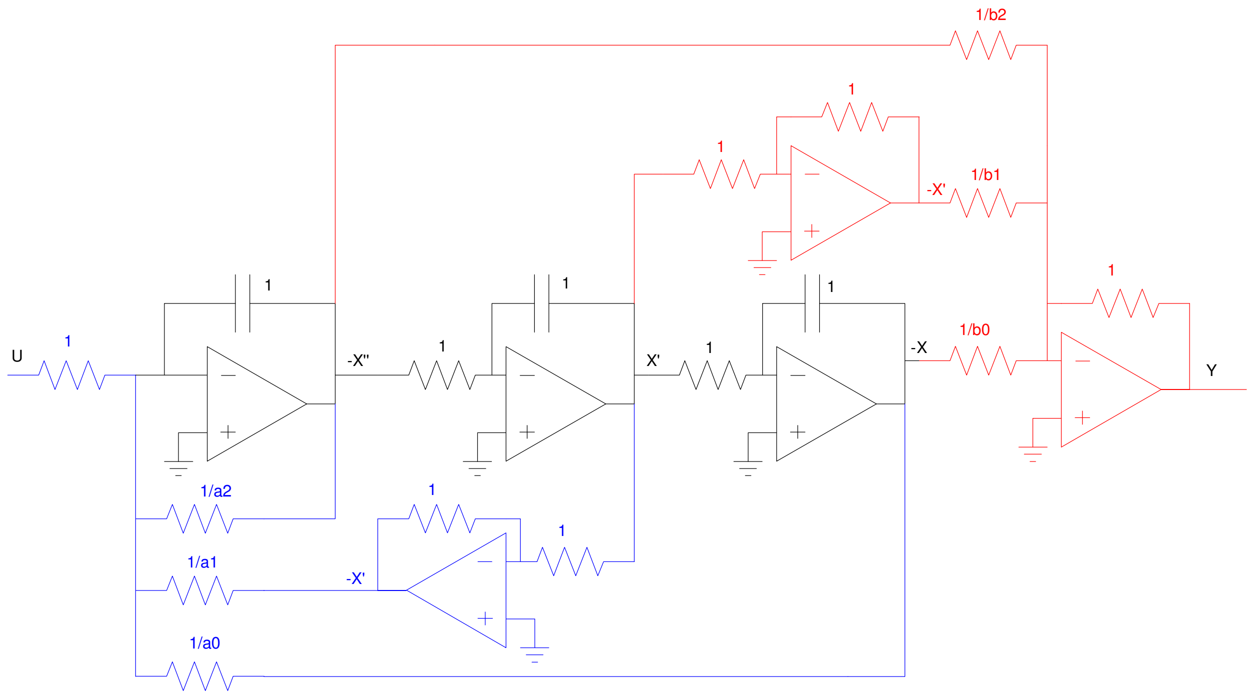


Now comes the tricky part: Play with the gains so that they are all negative (we'll be using inverting amplifiers). You might have to add in a inverter to get the gains to balance out.

- For the feedback loops, there net gain should be negative. Make sure there are an odd number of op-amps in each loop.
- For the output gains, if the net gain should be positive, make sure there are an even number of op-amps in each path from U to Y



Now convert to an op-amp circuit.



Final Op-Amp Circuit: All units M Ohms and uF.

This technique works well if the poles are close to 1.000. If the poles are not close to 1,

- Scale the poles so that they are close to 1.000
- Design the analog computer using the previous techniques
- Scale the circuit by making C larger (slower) or smaller (faster) to return to the original pole locations.