Active Filters

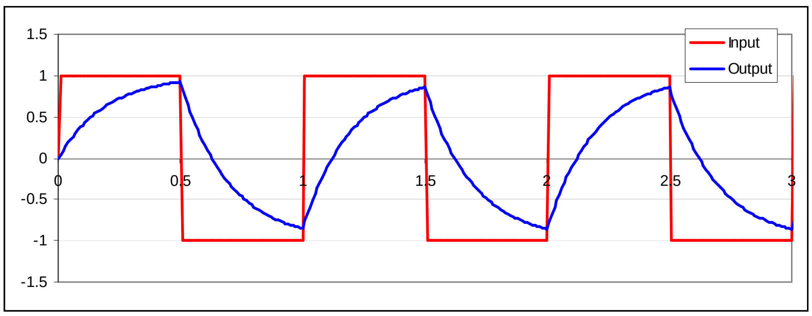
Background:

Filters are circuits whose behaviour changes with frequency. Essentially, all circuits with capacitors and/or inductors are filters.

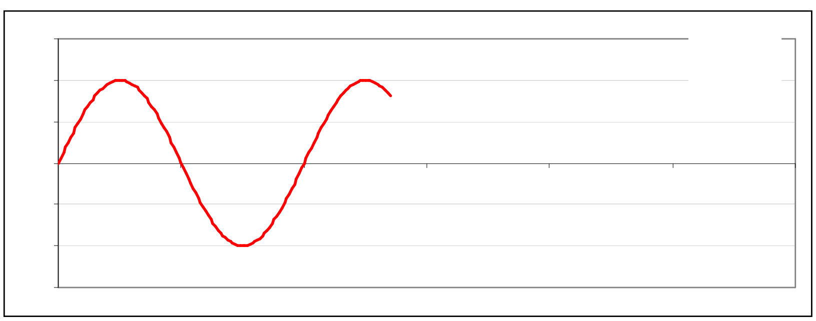
When analyzing a filter, sinusoids are used for inputs and outputs. Sinusoids are very special signals. For example, assume you apply a 1Hz signal to an RC filter which satisfies the following differential equation

$$\frac{dy}{dt} + 5y = 5x$$

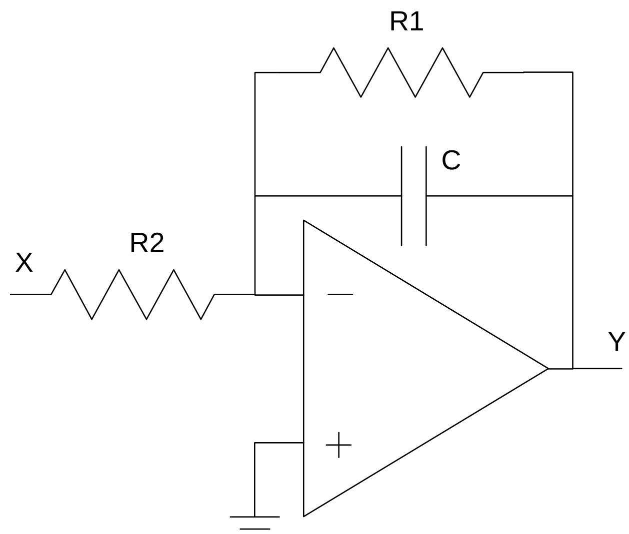
where y is the output of the RC filter and x is the input. If the input is a 1Hz square wave, the output will not be a square wave. The relationship between the two is not a simple one.



If the input is a 1Hz sine wave, however, the output is also a 1Hz sine wave.



Real Poles, No Zeros (take 2)



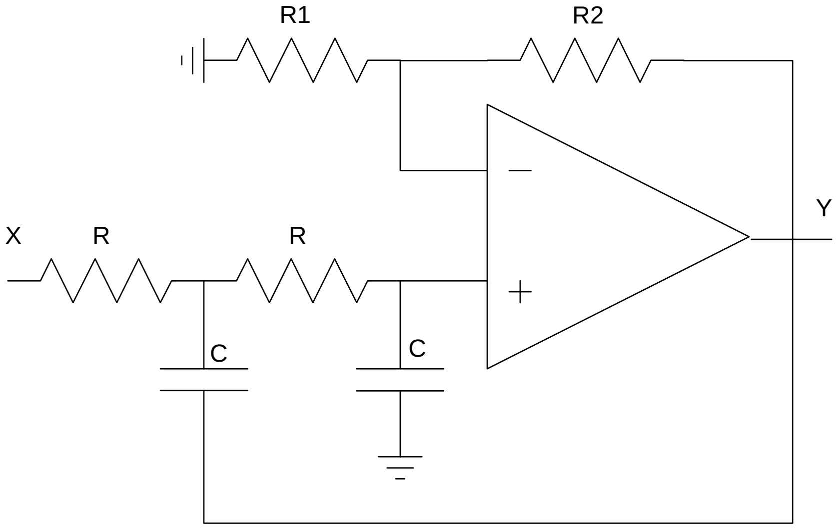
$$Y = \frac{a}{s+b} X$$

where

$$a = \frac{1}{R_2 C}$$

$$b = \frac{1}{R_1 C}$$

Complex Poles, No Zeros



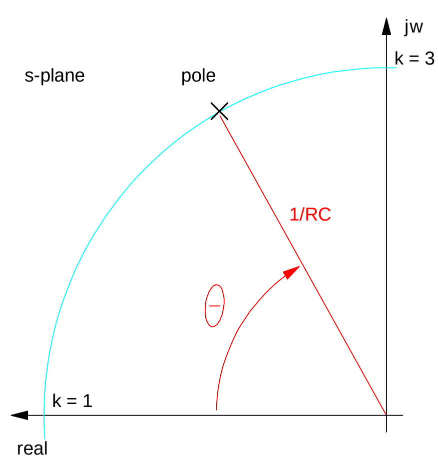
$$Y = \frac{k \times \left(\frac{1}{RC}\right)^2}{s^2 + \left(\frac{3\ k}{RC}\right)s + \left(\frac{1}{RC}\right)^2} X$$

This filter has two complex poles with

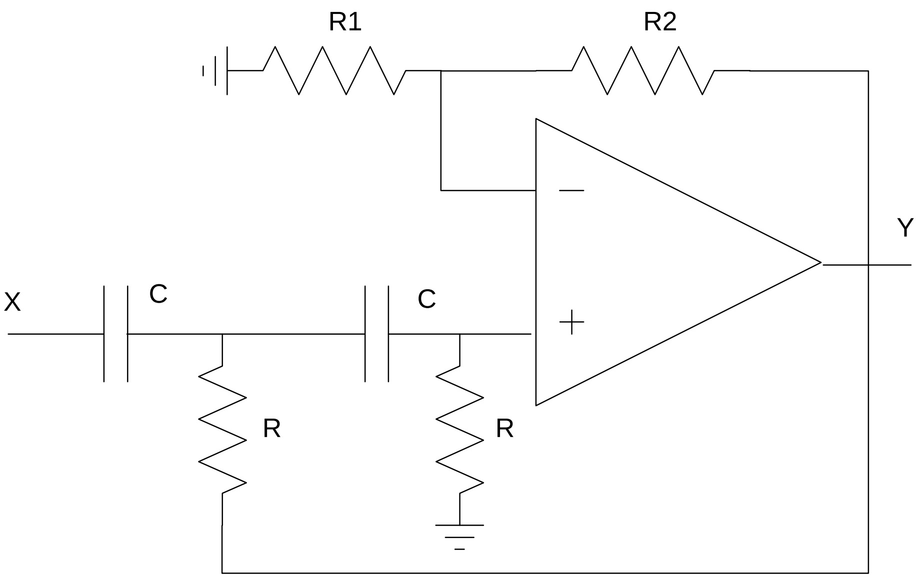
- Amplitude = $\frac{1}{RC}$
- Angle: $3 \quad k = 2\cos$
- DC gain $k = \ 1 + \frac{R_2}{R_1}$

Note that the angle of the poles goes from

- 0 degrees when k =1
- 90 degrees when k = 3 (an oscillator)



Comples Polex, Two Zeros at s = 0

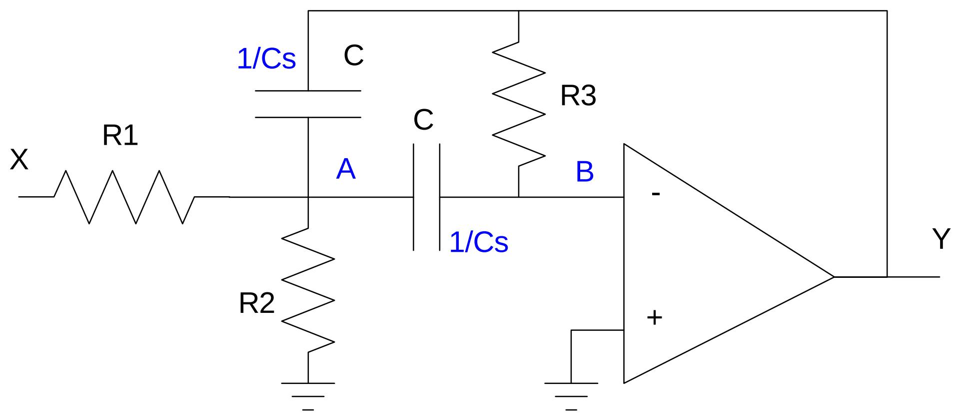


$$Y = \frac{k s^2}{s^2 + \left(\frac{3-k}{RC}\right)s + \left(\frac{1}{RC}\right)^2} X$$

This filter has two complex poles with

- Amplitude = $\frac{1}{RC}$
- Angle: $3 - k = 2\cos$
- High Freq gain $k = 1 + \frac{R_2}{R_1}$

Comples Polex, One Zeros at s = 0: $Y = \frac{as}{s^2+bs+c} X$



$$Y = \frac{\frac{1}{R_1 C} s}{s^2 + \frac{2}{R_3 C} s + \frac{R_1+R_2}{R_1 R_2} \frac{1}{R_3 C^2}} X$$

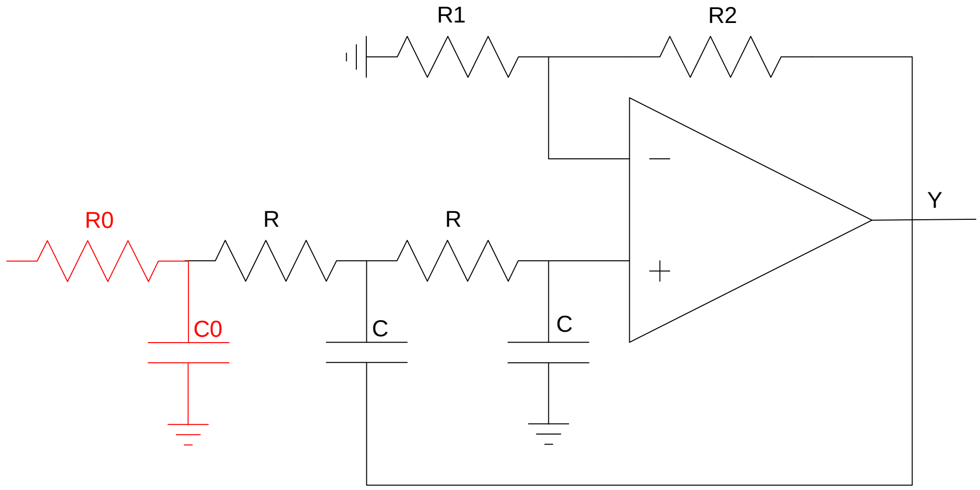
Example: Design a circuit to implement

$$Y = \frac{1,244,485}{(s+85)(s+121\ \ 69.5^0)(s+121\ \ \ 69.5^0)}\ X$$

Rewrite this as

$$Y = \left(\frac{85}{s+85}\right)\left(\frac{14,641}{(s+121\ \ 69.5^0)(s+121\ \ \ 69.5^0)}\right) X$$

Use the previous filters



$$\left(\frac{\frac{1}{R_0C_0}}{s+\frac{1}{R_0C_0}}\right)\left(\frac{k\times\left(\frac{1}{RC}\right)^2}{s^2+\left(\frac{3\ k}{RC}\right)s+\left(\frac{1}{RC}\right)^2}\right)$$

To avoid loading, let

- R0 = 10k
- R = 100k

Matching terms in the denominator:

$$\frac{1}{R_0C_0} = 85 \qquad C_0 = 1.17\ F$$

$$\frac{1}{RC} = 121 \qquad C = 0.082\ F$$

$$3\ \ k = 2\cos(69.5^0)$$

$$k = 2.3$$

$$1+\frac{R_2}{R_1} = 2.3$$

R1 = 100k, R2 = 1.3k

Note: This circuit has a DC gain of 2.3 (instad of 1.0). Just note this and call the output 2.3Y.

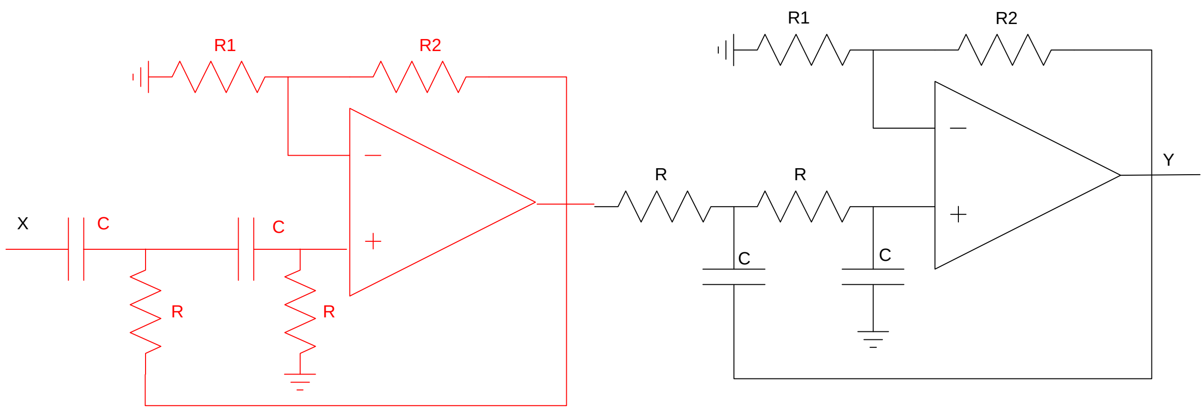
Example: Design a filter to implement

$$Y = \frac{100,000 s^2}{(s^2+14s+100)(s^2+100s+10,000)} X$$

Solution: Rewrite this as the product of two filters:

$$Y = \left(\frac{s^2}{s^2+14s+100}\right)\left(\frac{10,000}{s^2+100s+10,000}\right) X$$

Using the previous circuits (building blocks),



$$\frac{k \cdot s^2}{s^2 + \left(\frac{3-k}{RC}\right)s + \left(\frac{1}{RC}\right)^2} \qquad \frac{k \cdot \left(\frac{1}{RC}\right)^2}{s^2 + \left(\frac{3-k}{RC}\right)s + \left(\frac{1}{RC}\right)^2}$$

Matching the poles:

$$\left(\frac{1}{RC}\right)^2 = 100 \qquad \left(\frac{1}{RC}\right)^2 = 10,000$$

R = 100k | R = 100k

C = 0.1uF | C = 0.001uF

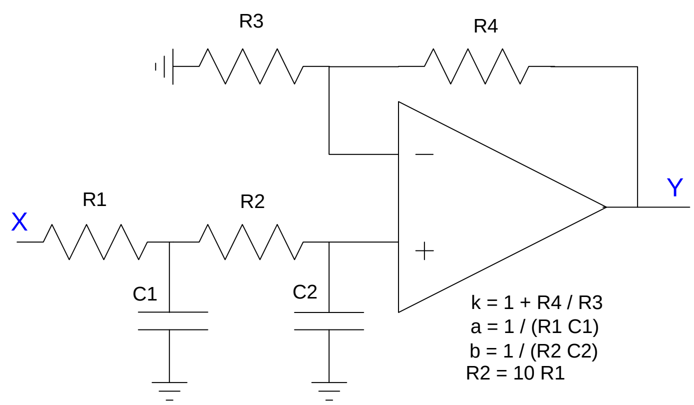
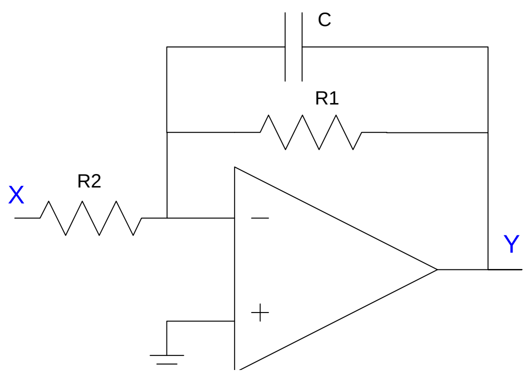
$$(3-k) = 2\cos(45^0) \qquad (3-k) = 2\cos(60^0)$$

k = 1.5858 | k = 2

R1 = 100k | R1 = 100k

R2 = 58k | R2 = 100k

Note: This circuit has a DC gain of 3.17 (instead of 1.0). Just note this and call the output 3.17Y.

Filter Description	G(s)	Circuit
Real Poles No Zeros DC gain > 1	$\frac{kab}{(s+a)(s+b)}$; $a = \frac{1}{R_1C_1}$ $b = \frac{1}{R_2C_2}$ $R_2 = 10R_1$ $k = 1 + \frac{R_4}{R_3}$	
Single Real Pole No Zeros	$\frac{a}{s+b}$; $b = \frac{1}{R_1C}$ $\frac{a}{b} = \frac{R_1}{R_2}$	
Complex Poles No Zeros	$\frac{ka^2}{(s+a\quad)(s+a\quad)}$; $a = \frac{1}{RC}$ $k = 1 + \frac{R_4}{R_3}$ 3 $k = 2\cos$