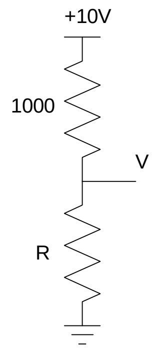
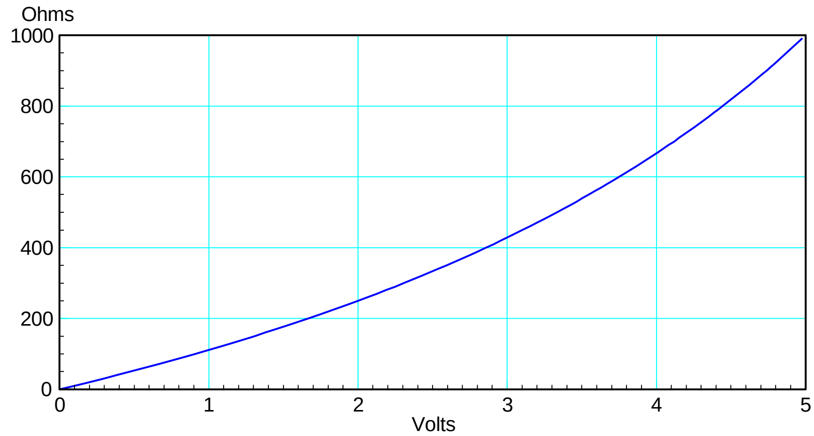
Examples: Regression using MATLAB:

Problem: Determine the resistance from the voltage over the range of 0 < R < 1000 Ohms





Solution (Least Squares): Express the problem as

$$Y = BA$$

where Y is the output, X is a known function matrix, and A is a constant but unknown matrix. The least squares solution for A will then be

$$A = (B^T B)^{-1} B^T Y$$

The estimated output is then

$$Y_e = BA$$

and the error in this estimate is

$$E = Y - Y_e$$

Zero-Based Calibration: First, set this up as

$$R \approx aV = AB$$

Let the basis function be R:

$$B = [V]$$

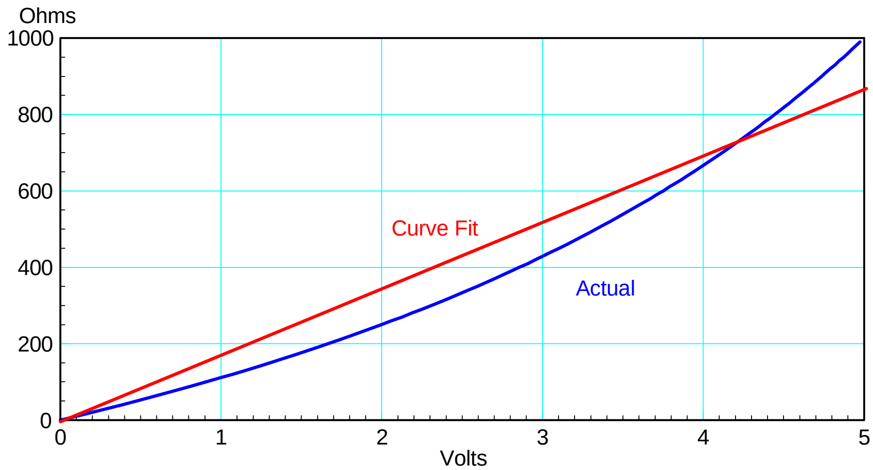
Solve for A as

In Matlab:

```
R = [0:1:1000]';
V = R ./ (1000+R) * 10;
B = V;

A = inv(B'*B)*B'*R

 A  = 169.53

plot(V,R,'b',V,B*A,'r');
xlabel('Volts');
ylabel('Ohms');
title('Zero Based Calibration');
```

so the best you can do is

$$R \approx 169.53R$$



Accuracy:

```
x = mean(R - B*A)
      x  = -20.967
```

Precision:

```
s = std(R - B*A)
```

s = 68.82

Endpoint Calibration: Next try

$$R \approx aV + b$$

$$R = \begin{bmatrix} V & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = BA$$

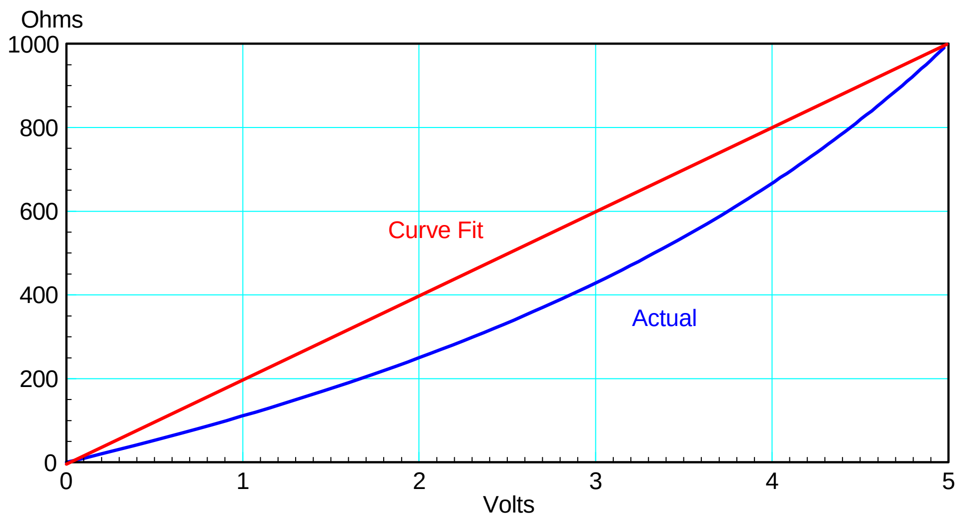
subject to the constrain that it passes through the endpoints.

Option 1) Repeat but just use the endpoints to find a and b.

Option 2) Use weighted least squares.

$$R \approx 200V$$

Plotting the results



Accuracy:

```
x = mean(V - B*A)
     x  = -113.69
```

Precision

```
s = stdev(V - B*A)
```

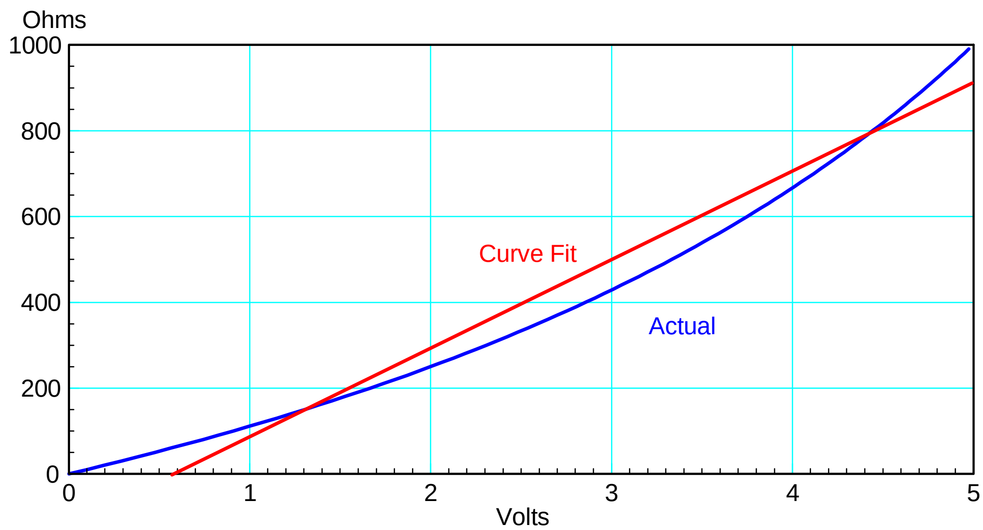
s = 51.65

Linear Interpolation: Repeat, but use all points (uniform weighting)

$$V \approx aR + b$$

```
B = [V, V.^0];
A = inv(B'*B)*B'*R

a    201.715
b    -118.913
```

$$V \approx 201.715V - 118.913$$



Note that

- R(V=0) is no longer zero.
- In return, you have a more accurate calibration scheme (on average)

Accuracy:

```
x = mean(R - B*A)
     x  = 0
```

Precision

```
s = stdev(V - B*A)
    s  =  51.59
```

Polynomial Calibration: The data looks like a quadratic function. Hence, a relationship like

$$R \approx aV^2 + bV + c$$

may work better. To do this, change the basis funciton (B) to

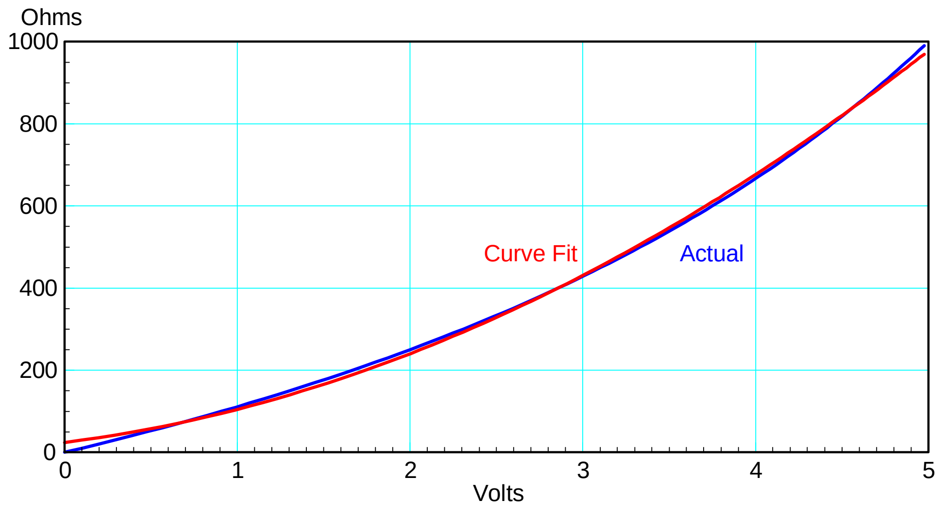
$$R = \begin{bmatrix} V^2 & V & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = BA$$

and solve just like before:

```
B = [V.^2, V, V.^0];
A = inv(B'*B)*B'*R

a     27.54
b     52.98
c     24.01
```

meaning

$$R \approx 27.54V^2 + 52.98V + 24.01$$



Accuracy

```
x = mean(V - B*A)
    x  =   0
```

Precision

```
s = stdev(V - B*A)
    s  =  9.000
```

Summary:

Calibration Scheme	R = f(V)	Accuracy mean(error)	Precision std(error)
Zero-Based	R = 169.53 V	-20.97	68.82
Endpoint	R = 200 V	-113.69	51.65
Linear Interpolation	R = 201.7 V - 118.9	0	51.6
Polynomial Interpolation	$R = 27.54V^2 + 52.98V + 24.01$	0	9.0

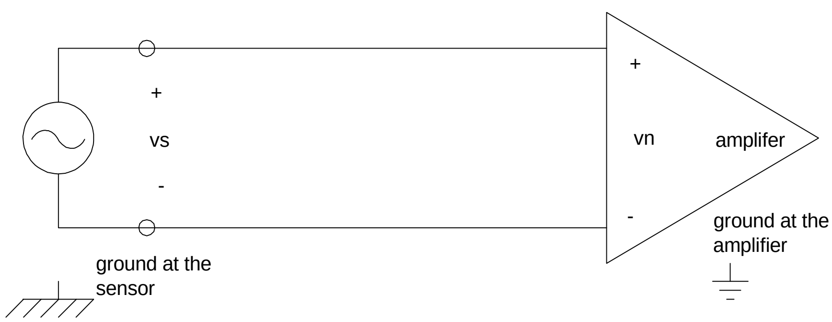
Noise

One problem commonly encountered is how to measure a remote location. For example, if you wanted to measure the temperature at the top of a smoke stack, it would be much more convenient to place the monitoring equipment on the ground rather than at the top of the smoke stack. In order to do this, wires are used to transfer the voltage from the sensor to the data recording instruments.

One problem with using wires to transfer data is that wires also act as antennas. By using a long stretch of wire, one might wind up with more noise than signal at the base of the smoke stack. During this week, different connections of wires from a sensor to an amplifier will be investigated in terms of their susceptibility to noise. By the end of the week, the student should be able to identify sources of noise in a given transmission line set up and suggest changes to reduce the noise level in the circuit.

Definitions:

Consider the problem of trying to measure a voltage, Vs, remotely.



Ideally, the output voltage is proportional to Vs.

Common-Mode Gain: The output proportional to the sum (or average) of Va and Vb. Common mode gain is ideally zero so that noise along the length of the line cancels out.

Differential Gain: The output proportional to the difference of Va and Vb.

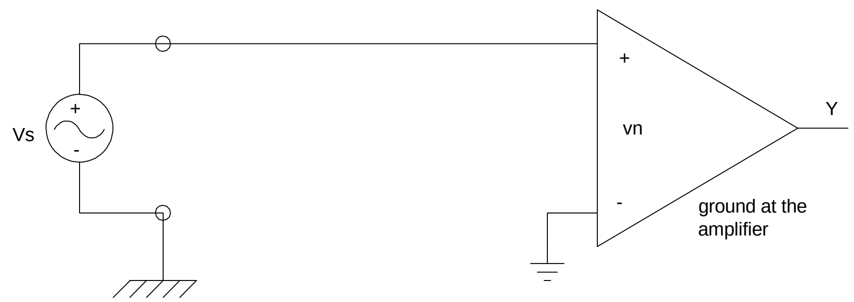
Common Mode Rejection Ratio: The ratio of the differential gain to the common-mode gain. The larger this number is, the better the amplifier is.

Ground Loops: All grounds are not equal. The potential at one point may be different than another point. (For example, there's about a 4V difference between Cincinnati and Columbus. If you take a wire, ground one end in Cincinnati and one end in Columbus, you'll likewise get some current flow. Over smaller distances, noise (from radio signals, transformers, etc.) may affect the potential at one location and not another.

Signal-to-Noise Ratio: The ratio of the energy at Vn due to the signal (Vs) to the energy of Vn due to other sources.

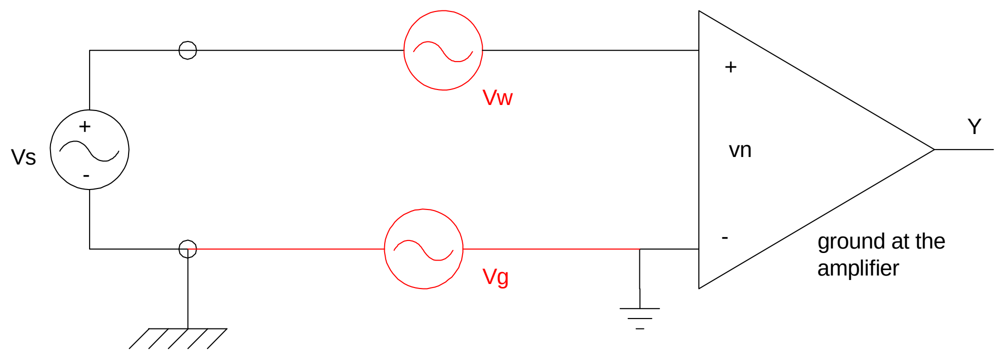
Basic Circuit

Case 1: The simplest circuit - and also the worse - for measuring a voltage remotely is as follows:

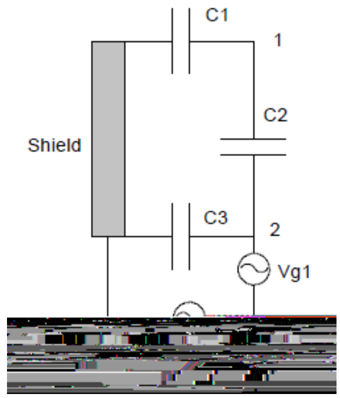


In order to save wire, the sensor is grounded locally. A single wire is then used to transfer this data to the data recorder. This voltage is then compared with its local ground.

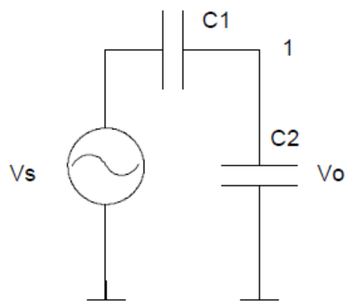
The problem with this circuit is several. This can be seen by adding two more voltages to this circuit to signify noise sources.



Since the two grounds may be at different potentials, a voltage source, Vg is added between nodes b and d i Td [8.32776 14647(r)-5.19369(c)-3.28667(u)-.24 45(e)-3.28667(s)-3.73946( )-0.91622(4(e)-3.28534( )-0.9162.)-5.1936



Here, C1, C2, and C3 signify the small capacitance between the wires. The voltage V12, is then found using voltage division for capacitors:



$$V_o = \frac{C_1}{C_1+C_2} V_s$$

and is

$$V_{12} = \frac{C_1}{C_1+C_2} (V_{g1} + V_{g2})$$

Connection C:

Connecting the ground to the ground wire of the instrumentation amplifier results in the noise sources having no effect on the signal.

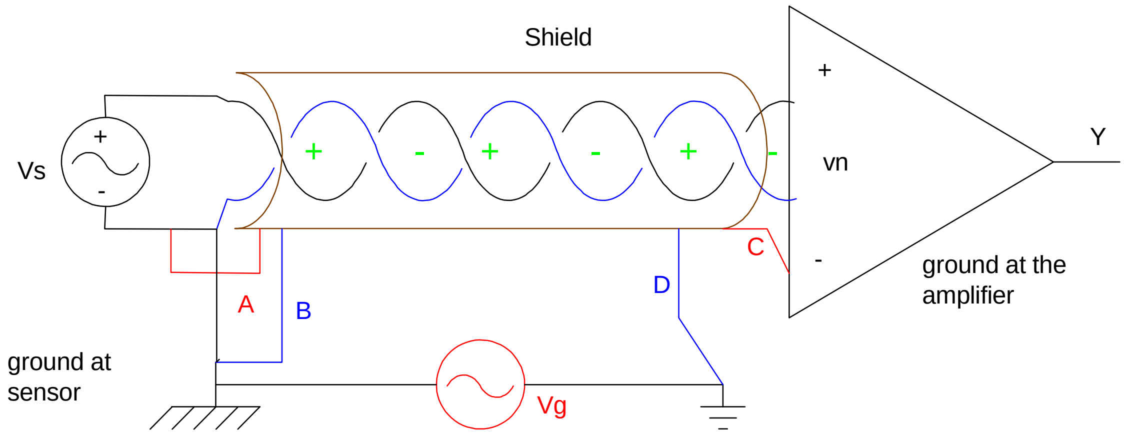
Connection D:

Grounding the shield to the ground at the amplifier's side results in the signal picked up by the amplifier's ground wires appearing at the output with a gain of

$$V_{12} = \left(\frac{C_1}{C_1+C_2}\right) V_{g1}$$

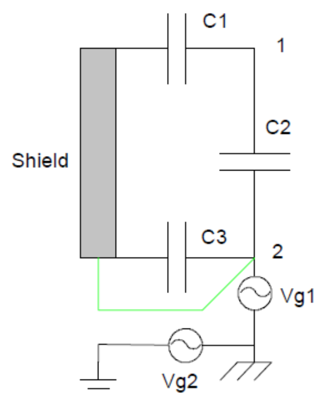
Note that Vg1 is typically very small - especially if short leads are used for the amplifier's ground wires. Hence, case C and D will not differ that much, although connection C is theoretically the better of the two.

Case 5: Shielded Twisted Pair with a Grounded Sensor: If the sensor is designed such that it has to be grounded, the circuit for the sensor - wiring - amplifier becomes as follows:



The goal is to ground the shield (at A, B, C, D) such that the signal at Vn is affected by Vg1 and Vg2 as little as possible.

Connection A: If the ground is connected to the sensor's ground wire on the sensor side, the circuit looks like the following:

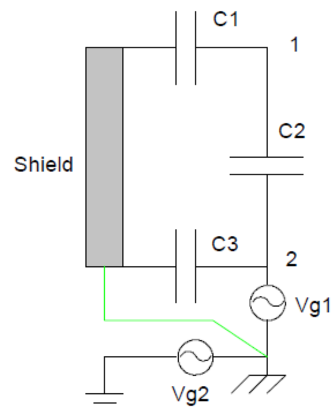


In this case,

V12 = 0

The shield will prevent electric fields from affecting the signal wires and will not add any voltages to the measured signal.

Connection B:

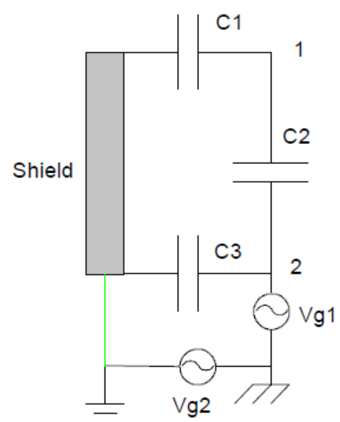


$$V_{12} = \left(\frac{C_1}{C_1+C_2}\right) V_{g1}$$

Connection B is only slightly inferior to connection A. Only the voltages which are induced on the sensor's grounding wire will be added to the amplifier's input.

Connection C: In this case, Connection C is clearly bad. As before, this uses the shield to take all the induced voltages and apply them to one of the lines and not the other - maximizing the noise received.

Connection D:



$$V_{12} = \left(\frac{C_1}{C_1+C_2}\right) (V_{g1} + V_{g2})$$