

ECE 321 - Homework #3

Active Filters. Due Wednesday, April 20th

Please make the subject "ECE 321 HW#3" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Filters

1) Assume X and Y are related by the following transfer function:

$$Y = \left(\frac{40}{(s+2)(s+7)} \right) X$$

a) What is the differential equation relating x and y?

Cross multiply

$$(s^2 + 9s + 14)Y = 40X$$

'sy' means 'the derivative of y'

$$\frac{d^2y}{dt^2} + 9\frac{dy}{dt} + 14y = 40x$$

or using prime notation where y' means dy/dt

$$y'' + 9y' + 14y = 40x$$

b) Determine y(t) assuming

$$x(t) = 4 + 5 \cos(7t) + 6 \sin(7t)$$

Treat this as two problems (x(t) contains two frequencies)

(i) $x(t) = 4$

$$s = 0$$

$$Y = \left(\frac{40}{(s+2)(s+7)} \right)_{s=0} \cdot (4)$$

$$Y = 11.429$$

(ii) $x(t) = 5 \cos(7t) + 6 \sin(7t)$

$$X = 5 - j6$$

$$s = j7$$

$$Y = \left(\frac{40}{(s+2)(s+7)} \right)_{s=j7} \cdot (5 - j6)$$

$$Y = -4.259 - j0.809$$

$$y(t) = -4.259 \cos(7t) + 0.809 \sin(7t)$$

The total answer is the sum of the two

$$y(t) = 11.429 - 4.259 \cos(7t) + 0.809 \sin(7t)$$

Filter Design

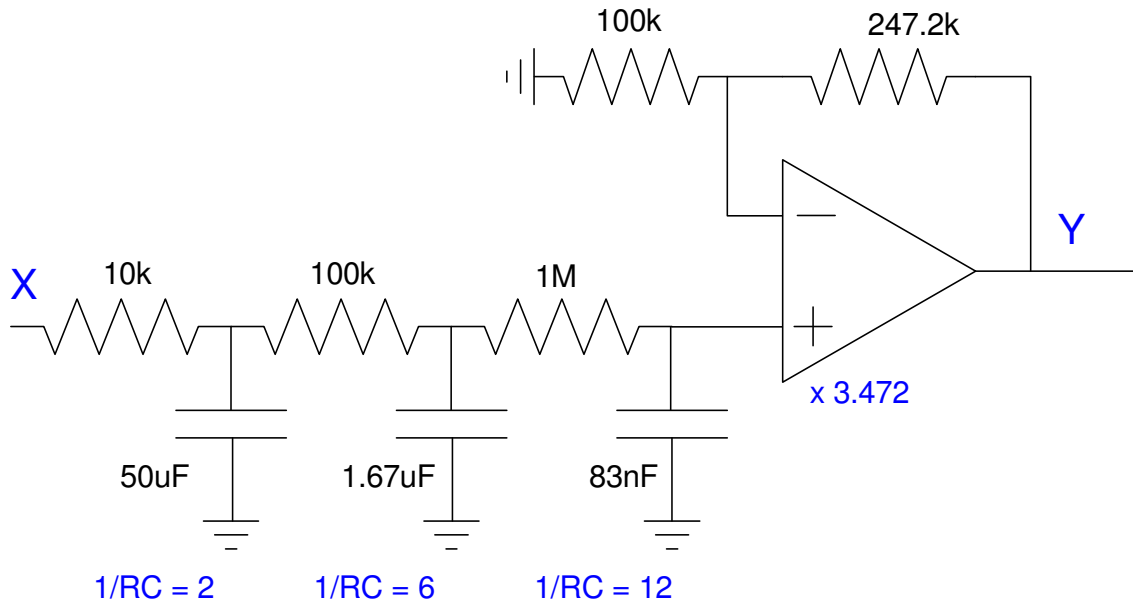
2) Give an op-amp circuit to implement the following filter

$$Y = \left(\frac{500}{(s+2)(s+6)(s+12)} \right) X$$

Rewrite as

$$Y = \left(\frac{2}{s+2} \right) \left(\frac{6}{s+6} \right) \left(\frac{12}{s+12} \right) (3.472) X$$

Implement as three RC filters along with a non-inverting amplifier



3) Give an op-amp circuit to implement the following filter

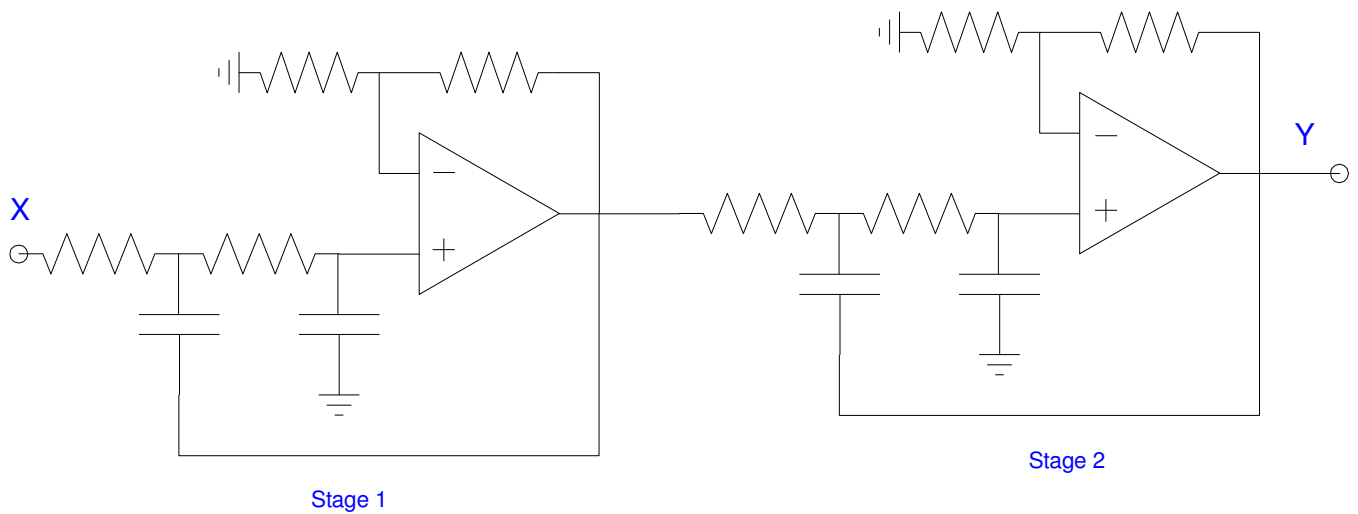
$$Y = \left(\frac{500}{(s^2+2s+10)(s^2+4s+20)} \right) X$$

Rewrite this as

$$Y = \left(\frac{500}{(s+1\pm j3)(s+2\pm j4)} \right) X$$

$$Y = \left(\frac{-}{s+3.162\angle\pm 71.57^\circ} \right) \left(\frac{-}{s+4.472\angle\pm 63.43^\circ} \right) X$$

Implement this in two stages



Stage 1:

$$G_1(s) = \left(\frac{k}{s+3.162\angle\pm 71.57^\circ} \right)$$

$$\frac{1}{RC} = 3.162$$

Let R = 100k. C = 3.163uF

$$3 - k = 2 \cos(71.57^\circ)$$

$$k = 2.368$$

Stage 2:

$$G_2(s) = \left(\frac{-}{s+4.472 \angle \pm 63.43^\circ} \right)$$

$$\frac{1}{RC} = 4.472$$

Let $R = 100k$. $C = 2.236\mu F$

$$3 - k = 2 \cos(63.43^\circ)$$

$$k = 2.105$$

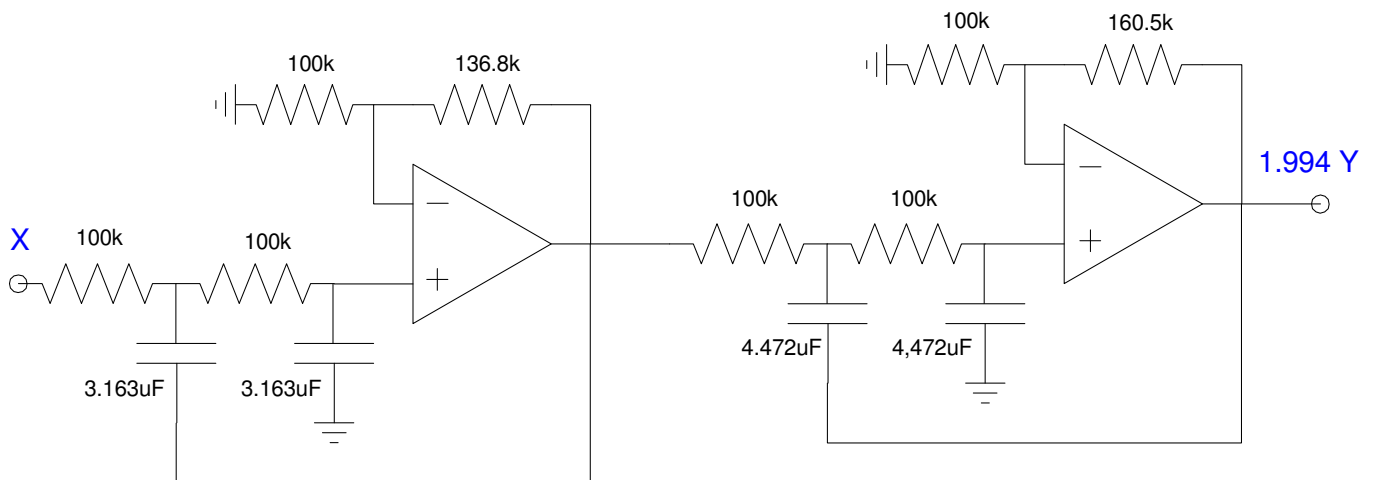
The resulting DC gain is

$$DC = 2.368 \cdot 2.105 = 4.985$$

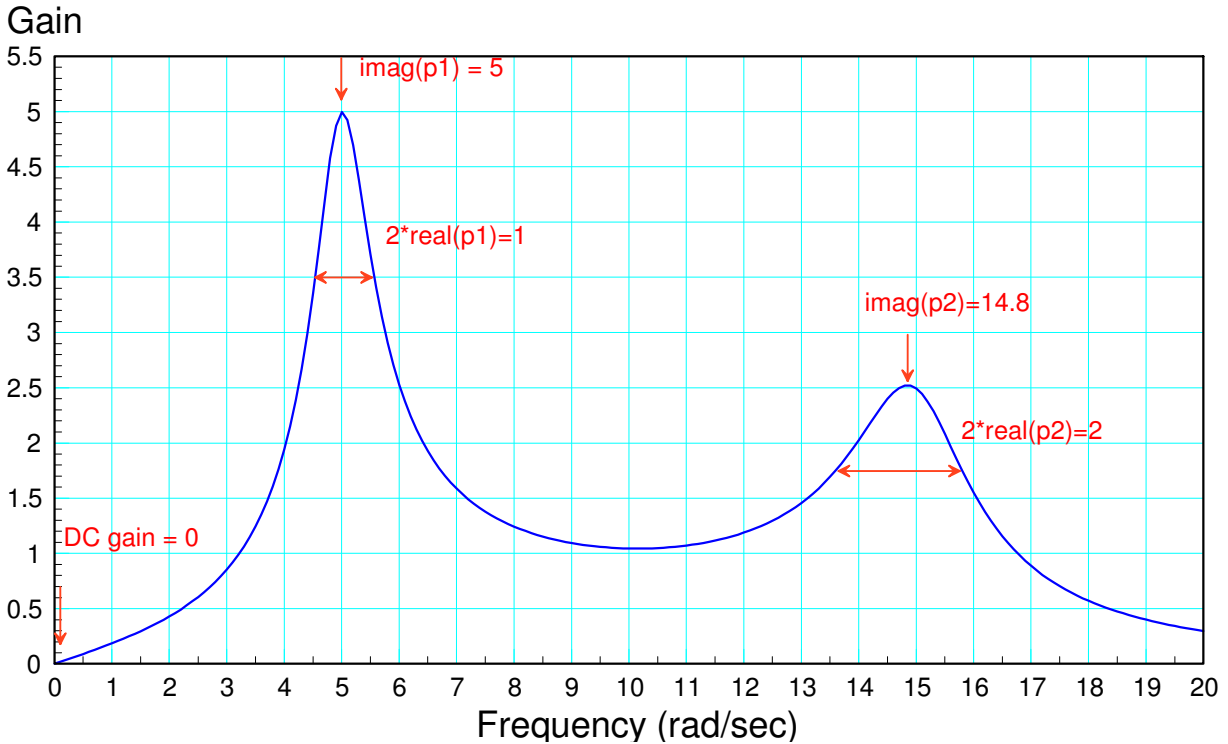
The DC gain should be 2.50

$$\left(\frac{500}{(s^2+2s+10)(s^2+4s+20)} \right)_{s=0} = 2.50$$

meaning the DC gain is 1.994x too much. Call the output 1.994y



4) Give the transfer function of a filter with the following gain vs. frequency



The DC gain is zero.

There must be a zero at $s = 0$

There is a resonance at 5 rad/sec

$\text{imag}(\text{pole \#1}) = \text{resonance frequency} = 5 \text{ rad/sec}$

$2 * \text{real}(\text{pole \#1}) = \text{bandwidth} = 1 \text{ rad/sec}$

$\text{real}(\text{pole \#1}) = 0.5$

$$p_1 = -0.5 \pm j5$$

There is a resonance at 14.8 rad/sec

$\text{imag}(\text{pole \#2}) = \text{resonance frequency} = 14.8 \text{ rad/sec}$

$2 * \text{real}(\text{pole \#2}) = \text{bandwidth} = 2 \text{ rad/sec}$

$\text{real}(\text{pole \#2}) = 1 \text{ rad/sec}$

$$p_2 = -1 \pm j14.8$$

So

$$G(s) \approx \left(\frac{ks}{(s+0.5 \pm j5)(s+1 \pm j14.8)} \right)$$

To find k, match the gain at some frequency. Using Matlab, make the maximum gain equal to 5

```
>> w = [0:0.01:20]';  
>> s = j*w;  
>> p1 = -0.5 + j*5;  
>> p2 = -0.5 - j*5;  
>> p3 = -1 + j*14.8;  
>> p4 = -1 - j*14.8;  
>>  
>> G = s ./ ( (s-p1).*(s-p2).*(s-p3).*(s-p4) );  
>> k = 5 / max(abs(G))
```

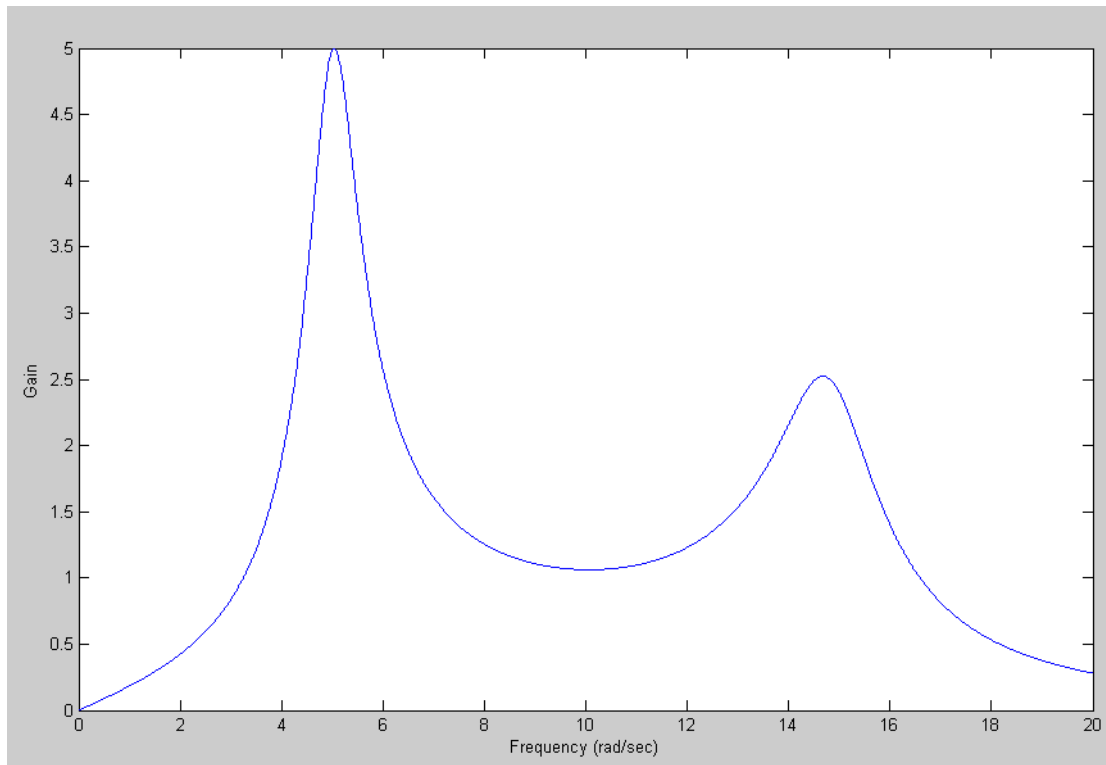
k =

974.9369

```
>> plot(w, abs(k*G))
```

and

$$G(s) \approx \left(\frac{974.94s}{(s+0.5 \pm j5)(s+1 \pm j14.8)} \right)$$



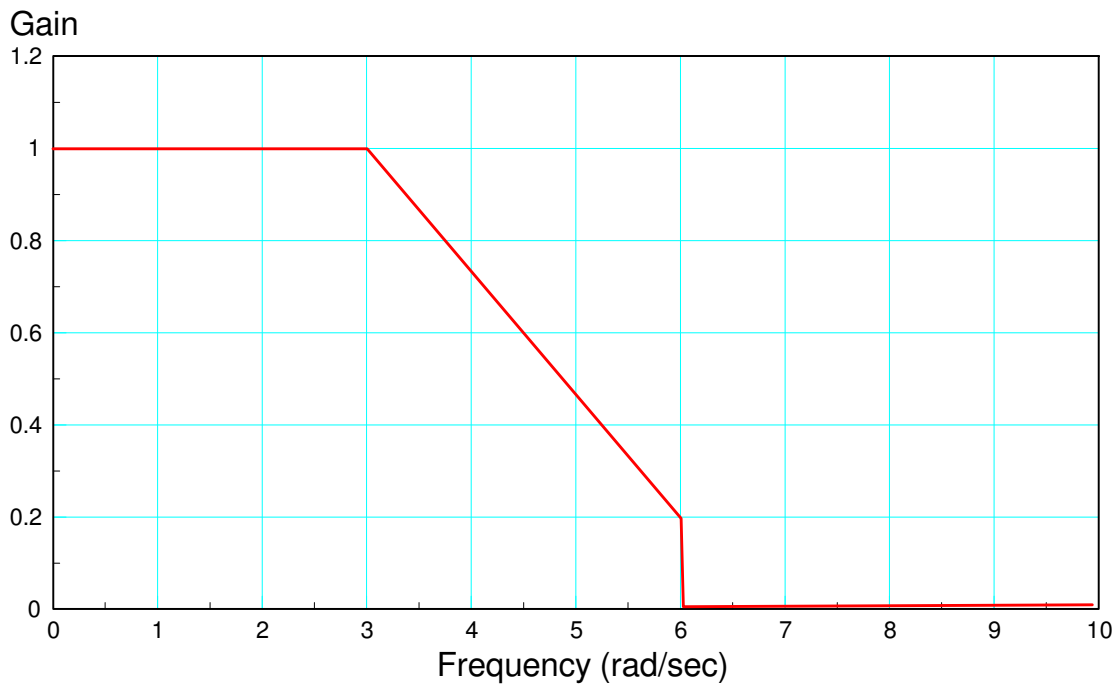
Filter Design using *fminsearch()*

5) Design a filter of the form

$$Y = \left(\frac{ace}{(s+a)(s^2+bs+c)(s^2+ds+e)} \right) X$$

to give a gain vs. frequency as close to the following plot as possible over the range of (0, 10) rad/sec.

Plot your filter's actual frequency response vs. it's ideal response (red line).



Step 1: Create an m-file where you

- Guess {a,b,c,d,e}
- It computes $G(j\omega)$
- It compares it to the desired $G(j\omega)$, and
- Returns the sum-squared error

```

function [ J ] = costf( z )
a = z(1);
b = z(2);
c = z(3);
d = z(4);
e = z(5);

w = [0:0.01:10]';
s = j*w;

Gideal = (1)*(w<3) + (1.8 - 0.0.267*w).*(w>=3).*(w<6);

G = a*c*e ./ ( (s+a).*(s.^2 + b*s + c) .* (s.^2 + d*s + e) );
G = abs(G);
E = abs(Gideal) - abs(G);

J = sum(E .^ 2);

plot(w,Gideal,w,abs(G),'r');
ylim([0,1.4]);
pause(0.01);
end

```

Optimizing using fminsearch

```

>> [Z,e] = fminsearch('costF',[1,2,3,4,5])

Z =      a      b      c      d      e
    1.8061  2.4968  9.5605  1.7538  24.9067

e = 1.4611

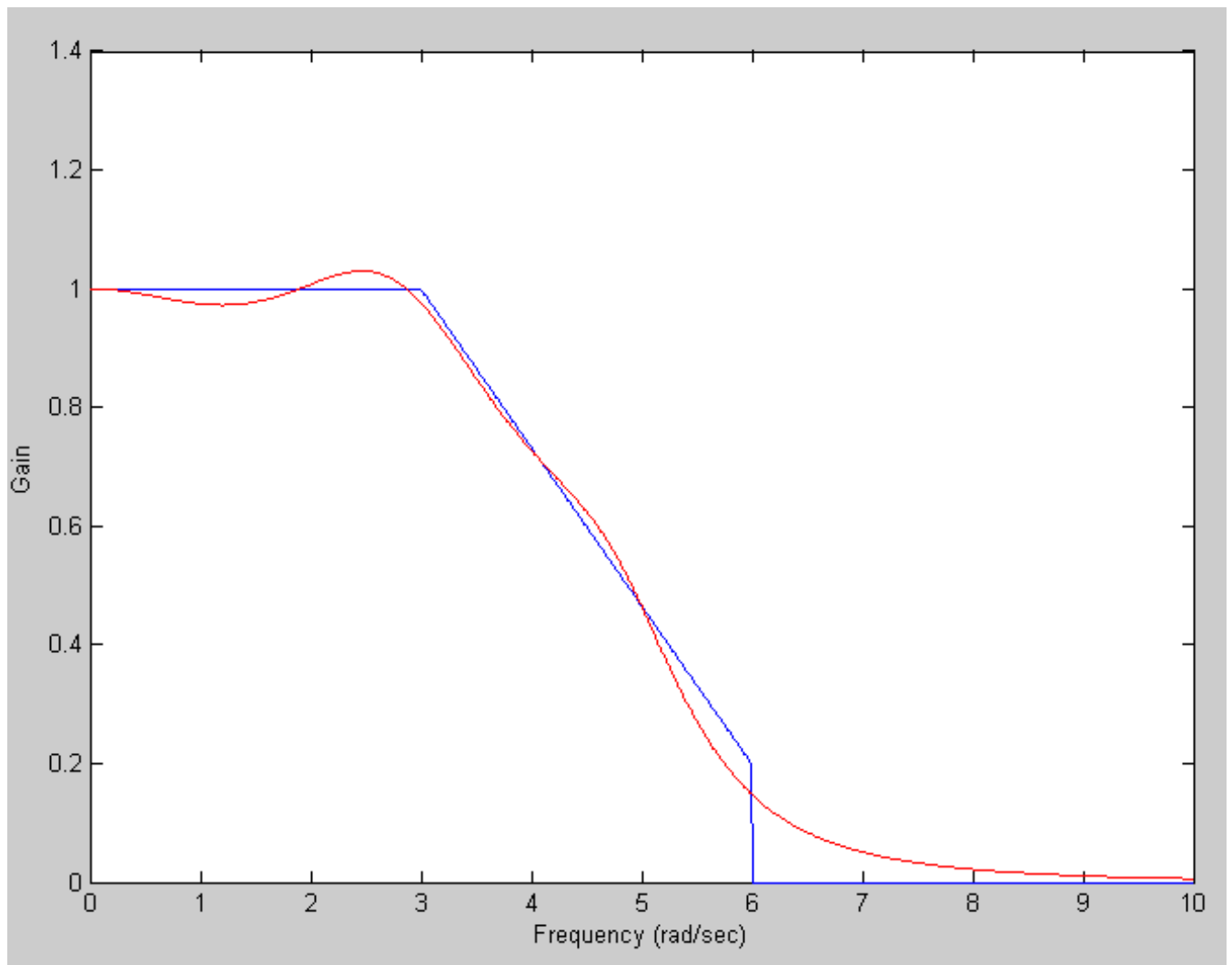
```

meaning

$$G(s) = \left(\frac{1.8061 \cdot 9.5605 \cdot 24.9067}{(s+1.8061)(s^2+2.4968s+9.5605)(s^2+1.7538s+24.9067)} \right)$$

or

$$G(s) = \left(\frac{430.069}{(s+1.8061)(s^2+2.4968s+9.5605)(s^2+1.7538s+24.9067)} \right)$$



6) Design circuit to implement the filter you designed in problem #5

$$G(s) = \left(\frac{430.069}{(s+1.8061)(s^2+2.4968s+9.5605)(s^2+1.7538s+24.9067)} \right)$$

Write this as

$$G(s) = \left(\frac{1.8061}{(s+1.8061)} \right) \left(\frac{9.5605}{s^2+2.4968s+9.5605} \right) \left(\frac{24.9067}{s^2+1.7538s+24.9067} \right)$$

$$G(s) = \left(\frac{1.8061}{(s+1.8061)} \right) \left(\frac{9.5605}{s+3.092 \angle \pm 66.19^\circ} \right) \left(\frac{24.9067}{s+4.99 \angle 79.88^\circ} \right)$$

Implement as

- An RC filter
- An active low-pass filter, and
- An active low-pass filter

Stage 1

$$\frac{1}{RC} = 1.8061$$

Let R = 10k, C = 55.36uF

Stage 2

$$\frac{1}{RC} = 3.092$$

Let R = 100k, C = 3.234uF

$$3 - k = 2 \cos (66.19^\circ)$$

$$k = 2.193$$

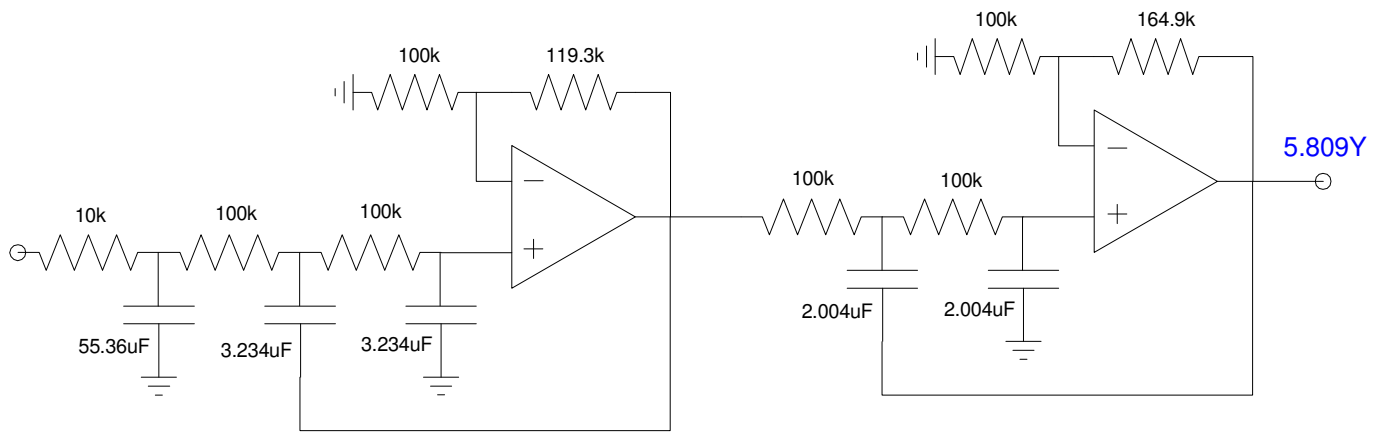
Stage 3

$$\frac{1}{RC} = 4.99$$

Let R = 100k, C = 2.004uF

$$3 - k = 2 \cos (79.88^\circ)$$

$$k = 2.649$$



7) Check your filter using CircuitLab