Circuits I Review

ECE 320 Electronics I (Digital)

Jake Glower - Lecture #1

Circuits I Review

EE 206 Circuits I covers steady-state analysis of circuits with DC inputs.

Three techniques are used:

- Current Loops
- Voltage Nodes
- Thevenin Equivalent

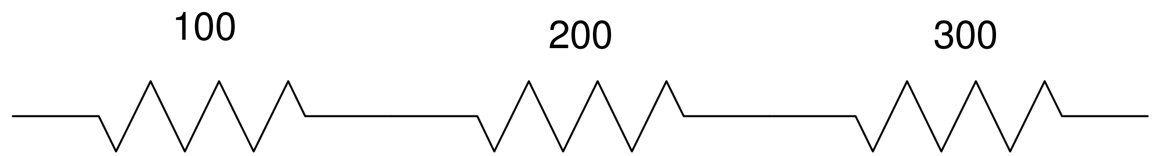
All of these techniques are used in Electronics I and II. There are shortcuts we'll introduce in this course - but you can analyze pretty much any circuit just using these techniques.

Resistors in Series and Parallel:

Resistors in series add:

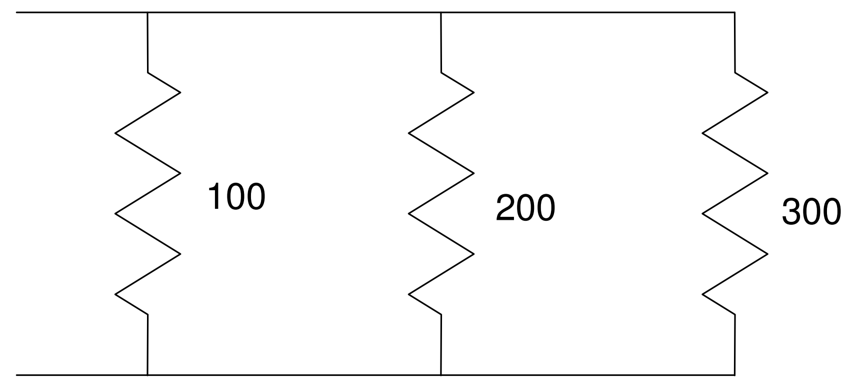
$$R = 100 + 200 + 300$$

$$R = 600\Omega$$



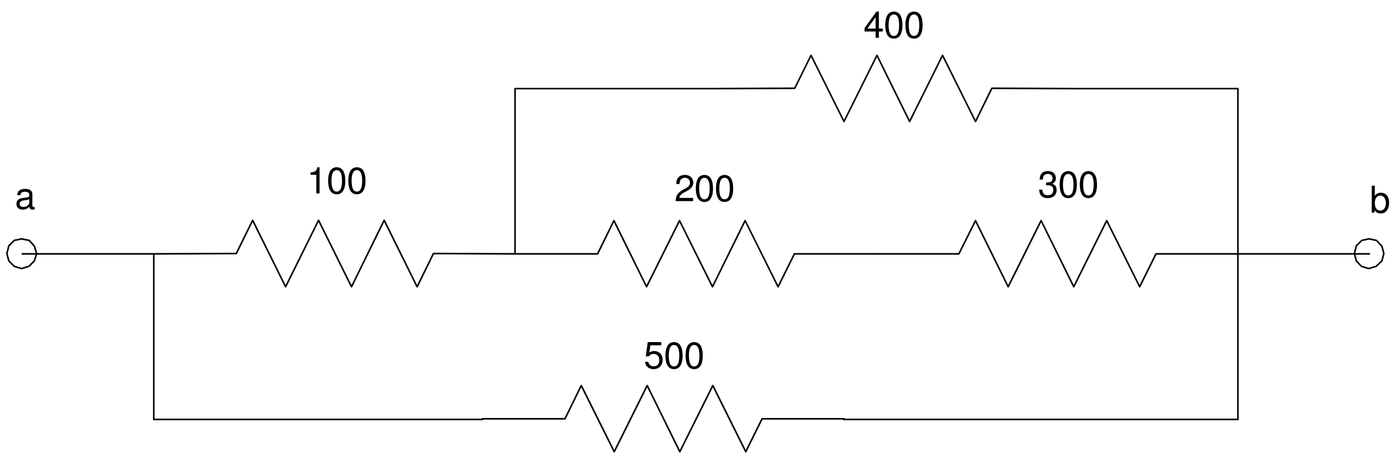
Resistors in parallel add as the sum of the inverses inverted.

$$R = \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{300}\right)^{-1} = 54.54\Omega$$



Example (Handout)

Find Rab



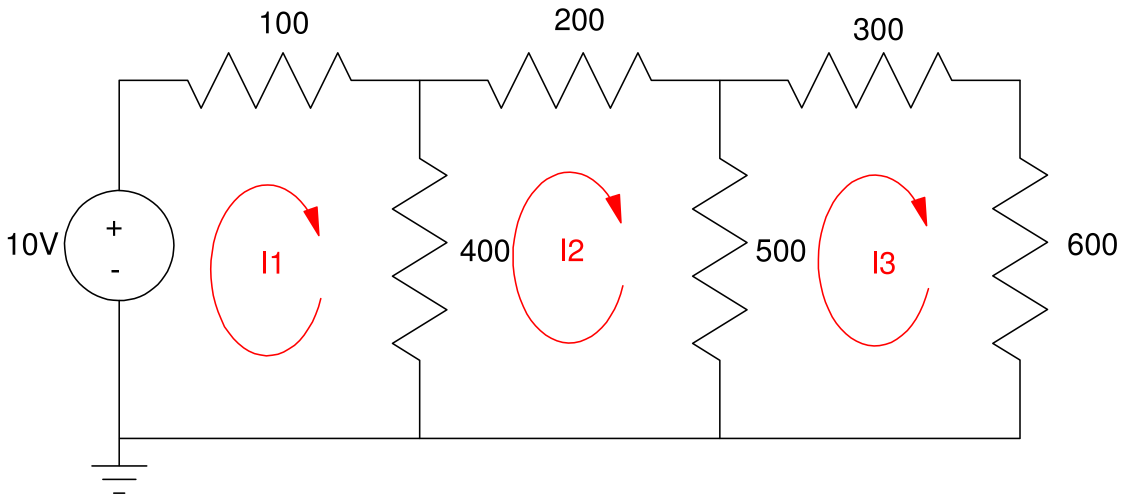
Current Loops

Conservation of Voltage:

- The sum of the voltages around any closed path must be zero.

Step 1: Determine the number of "windows" the circuit has. This is how many loop equations you'll need to write.

- N = 3



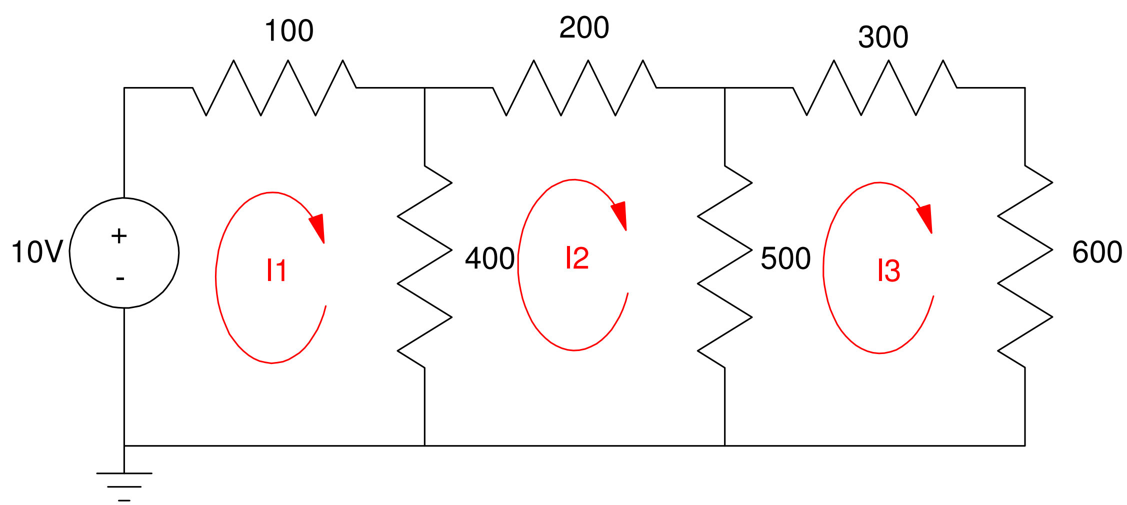
Step 2: Define the currents. I prefer to place a current in each window, each going clockwise. (shown in red).

Step 3) Write 3 equations for 3 unknowns:

$$-10 + 100I_1 + 400(I_1 - I_2) = 0$$

$$400(I_2 - I_1) + 200I_2 + 500(I_2 - I_3) = 0$$

$$500(I_3 - I_2) + 300I_3 + 600I_3 = 0$$

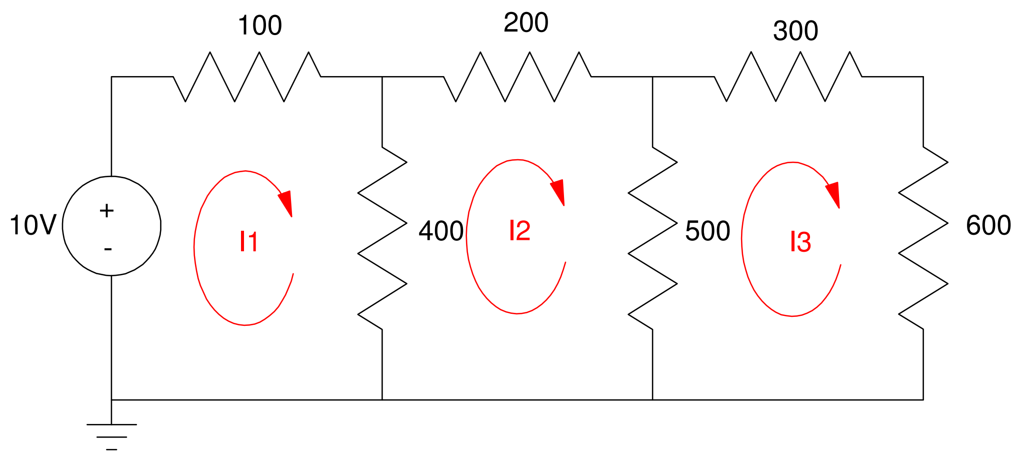


Step 4: Solve. First group terms:

$$500I_1 - 400I_2 = 10$$

$$-400I_1 + 1100I_2 - 500I_3 = 0$$

$$-500I_2 + 1400I_3 = 0$$

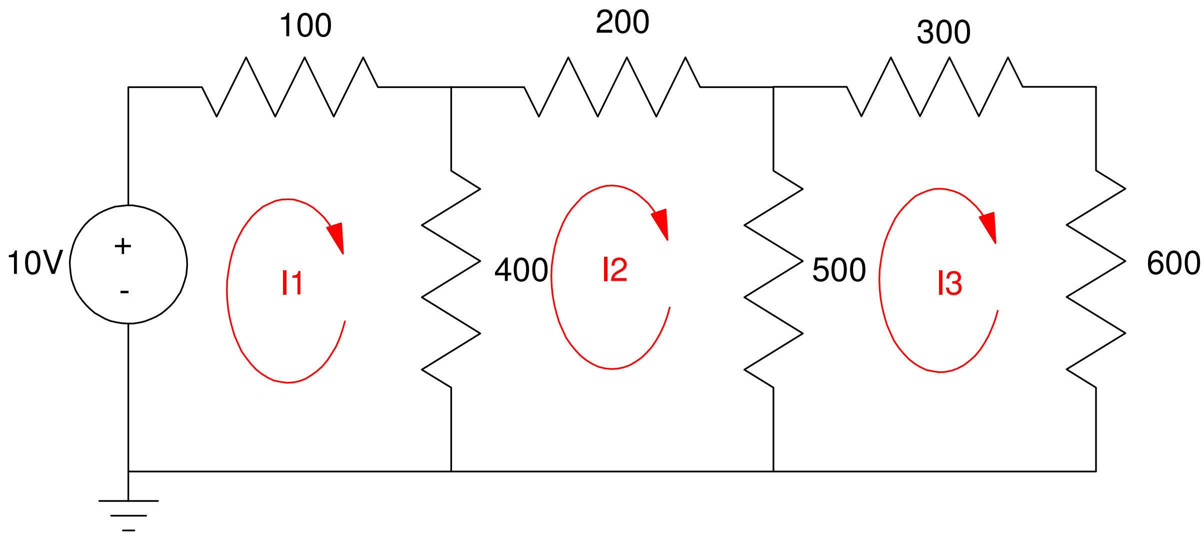


Put in matrix form:

$$\begin{bmatrix} 500 & -400 & 0 \\ -400 & 1100 & -500 \\ 0 & -500 & 1400 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

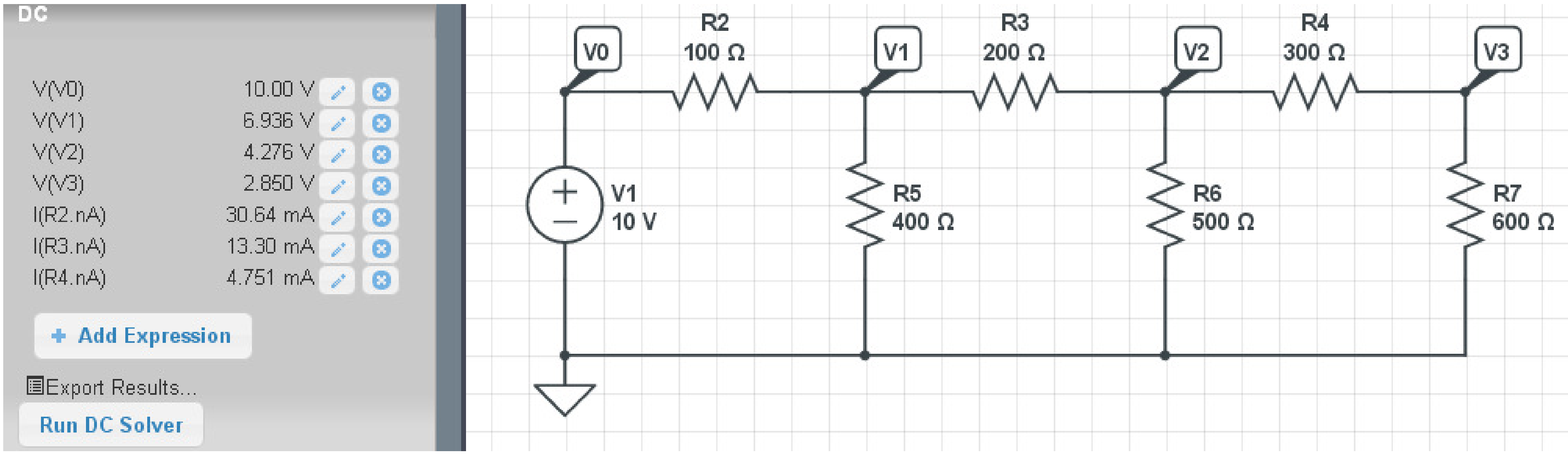
Solve in Matlab:

```
A = [500,-400,0 ; -400,1100,-500 ; 0,-500,1400]

         500          -400             0
        -400          1100          -500
           0          -500          1400

B = [10;0;0]

    10
     0
     0

I = inv(A)*B

I1 =     0.0306
I2 =     0.0133
I3 =     0.0048
```



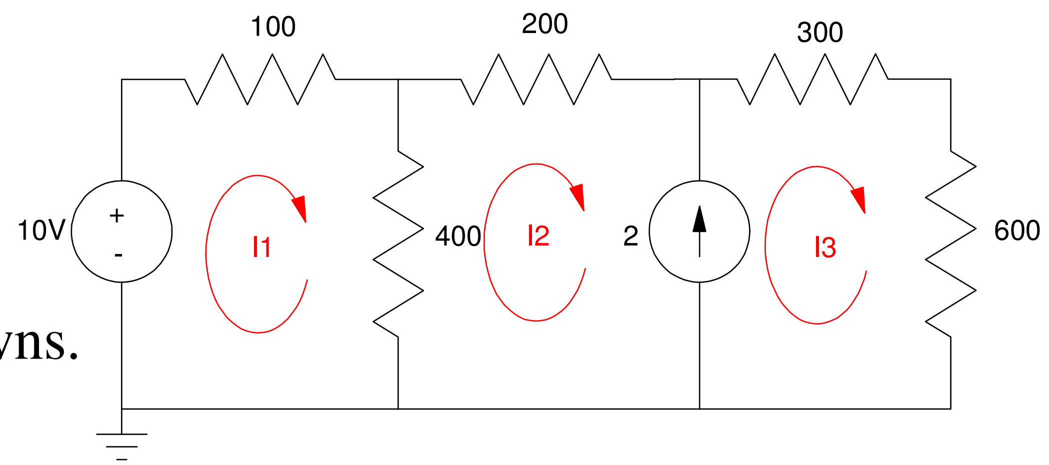
Testing (CircuitLab)

- Add Expression
- Click on the left side of each resistor.



SuperLoops

- If you need more equations, choose another closed path
- A path which passes over several loops is called a *superloop*



Example:

Step 1: N = 3 (3 windows)

Step 2: Define currents

Step 3: Write N equations for N unknowns.

$$I_3 - I_2 = 2$$

$$-10 + 100I_1 + 400(I_1 - I_2) = 0$$

SuperLoop: Outer Loop

$$-10 + 100I_1 + 200I_2 + 300I_3 + 600I_3 = 0$$

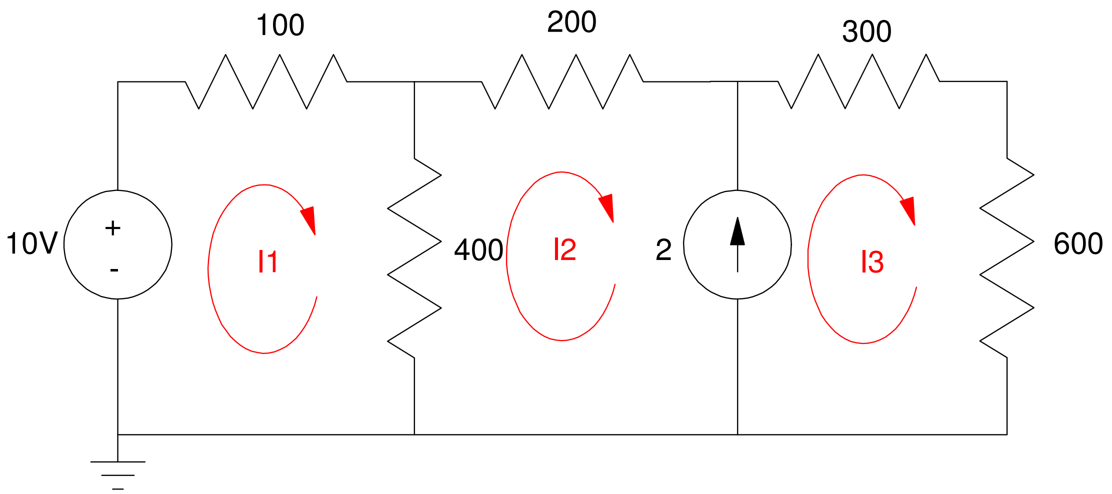
Step 4: Solve N equations for N unknowns.

Group terms:

$$I_3 - I_2 = 2$$

$$500I_1 - 400I_2 = 10$$

$$100I_1 + 200I_2 + 900I_3 = 10$$

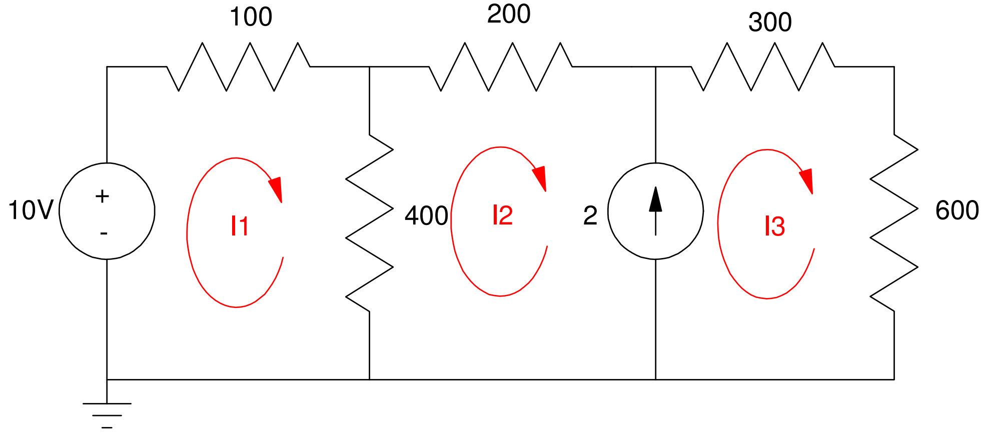


Put in matrix form:

$$\begin{bmatrix} 0 & -1 & 1 \\ 500 & -400 & 0 \\ 100 & 200 & 900 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 10 \end{bmatrix}$$

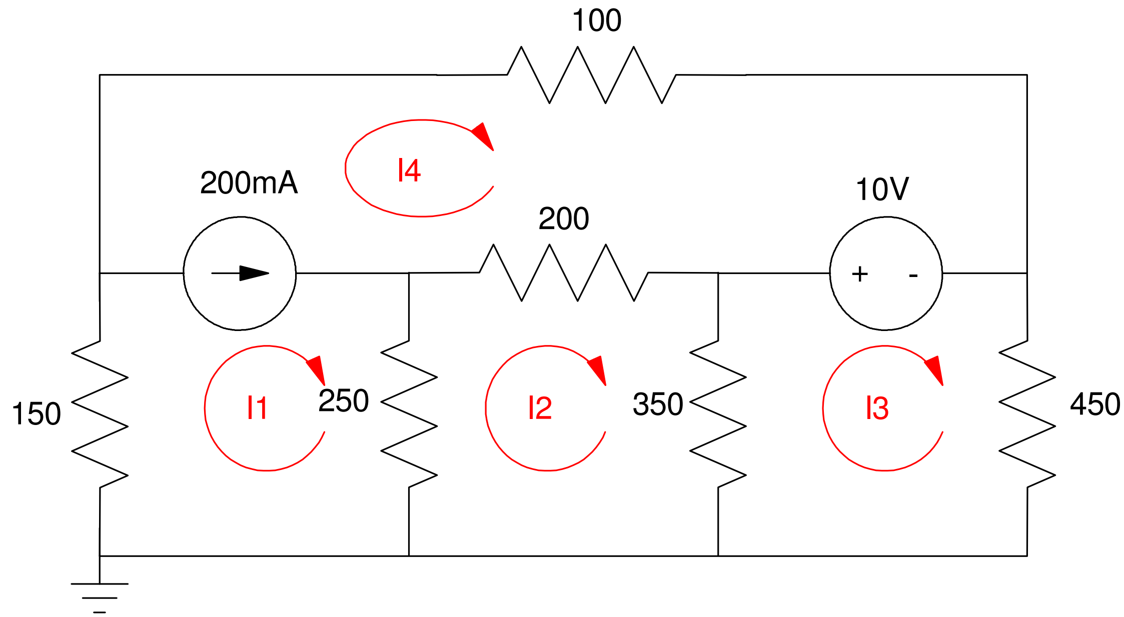
Solve in Matlab:

```
>> A = [0,-1,1 ; 500,-400,0 ; 100,200,900]

      0    -1     1
    500  -400     0
    100   200   900

>> B = [2;10;10]

      2
     10
     10

>> I = inv(A)*B

I1 =    -1.1949
I2 =    -1.5186
I3 =     0.4814
```



Example (Handout)

Write the current loop equations



Voltage Nodes:

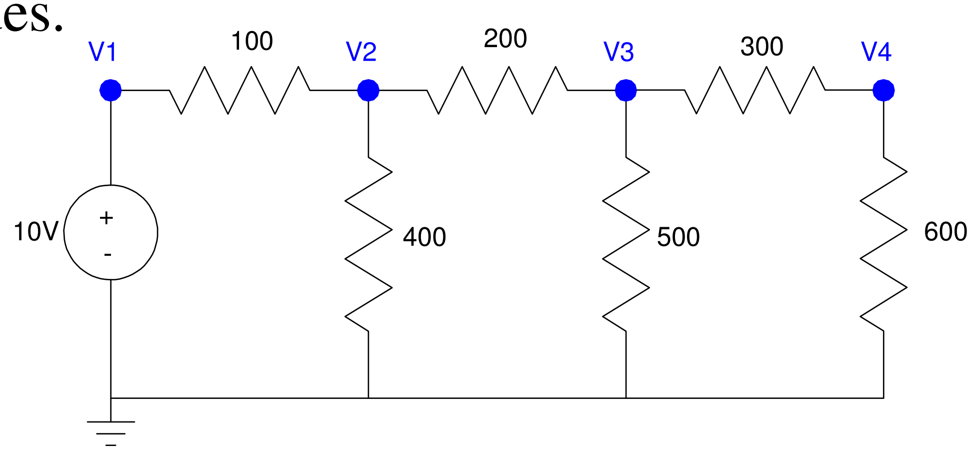
Conservation of Current

- The total current flowing to a node must be zero
- For N unknown voltages, write N voltage node equations

Step 1: Define circuit ground.

Step 2: Define the remaining N voltage nodes.

Step 3: Write N equations for N unknowns



$$V_1 = 10$$

$$\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{400}\right) + \left(\frac{V_2 - V_3}{200}\right) = 0$$

$$\left(\frac{V_3 - V_2}{200}\right) + \left(\frac{V_3}{500}\right) + \left(\frac{V_3 - V_4}{300}\right) = 0$$

$$\left(\frac{V_4 - V_3}{300}\right) + \left(\frac{V_4}{600}\right) = 0$$

Step 4: Solve. Group terms:

$$V_1 = 10$$

$$\left(\frac{-1}{100}\right)V_1 + \left(\frac{1}{100} + \frac{1}{400} + \frac{1}{200}\right)V_2 + \left(\frac{-1}{200}\right)V_3 = 0$$

$$\left(\frac{-1}{200}\right)V_2 + \left(\frac{1}{200} + \frac{1}{500} + \frac{1}{300}\right)V_3 + \left(\frac{-1}{300}\right)V_4 = 0$$

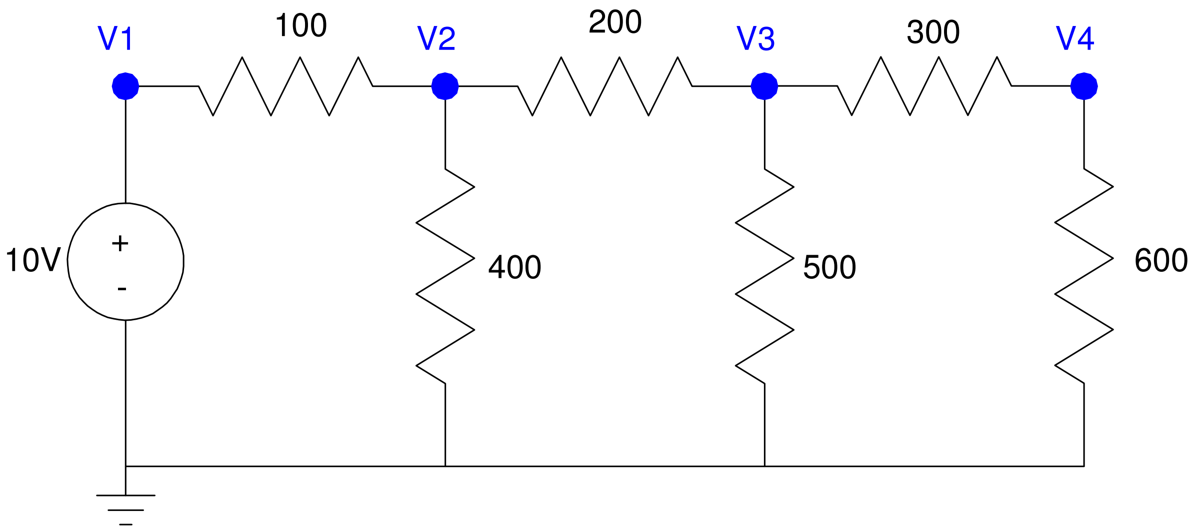
$$\left(\frac{-1}{300}\right)V_3 + \left(\frac{1}{300} + \frac{1}{600}\right)V_4 = 0$$

Place in matrix form:

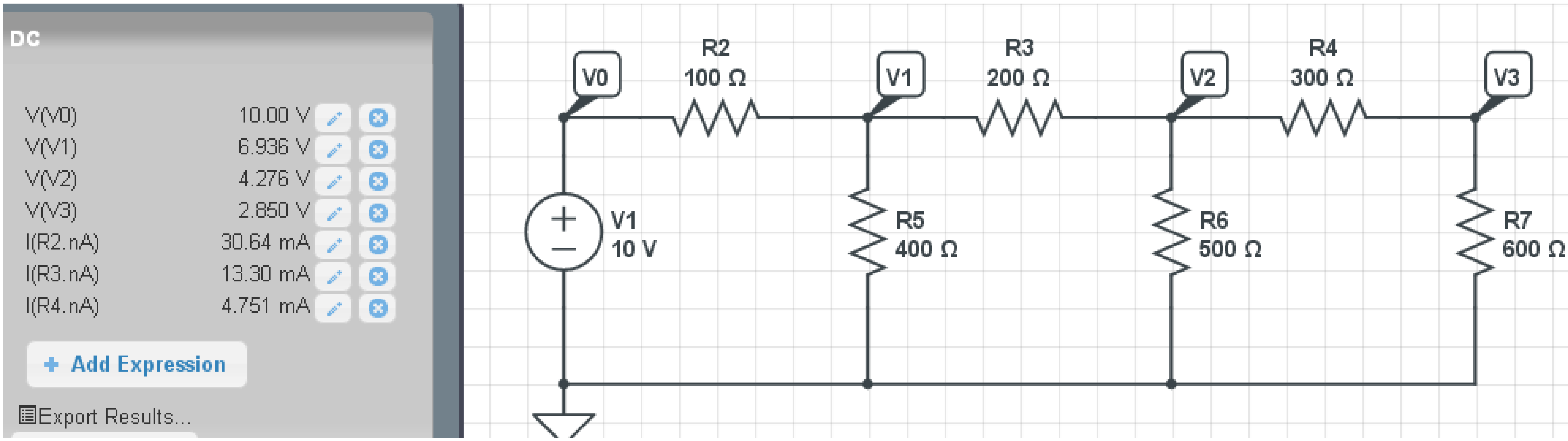
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{400} + \frac{1}{200}\right) & \left(\frac{-1}{200}\right) & 0 \\ 0 & \left(\frac{-1}{200}\right) & \left(\frac{1}{200} + \frac{1}{500} + \frac{1}{300}\right) & \left(\frac{-1}{300}\right) \\ 0 & 0 & \left(\frac{-1}{300}\right) & \left(\frac{1}{300} + \frac{1}{600}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
A = [1,0,0,0 ; -1/100, 1/100+1/400+1/200,-1/200,0];
A = [A ; 0,-1/200,1/200+1/500+1/300,-1/300 ; 0,0,-1/300,1/300+1/600]

    1.0000         0         0         0
   -0.0100    0.0175   -0.0050         0
         0   -0.0050    0.0103   -0.0033
         0         0   -0.0033    0.0050

B = [10;0;0;0]

    10
     0
     0
     0

V = inv(A)*B

V1   10.0000
V2    6.9359
V3    4.2755
V4    2.8504
```



Testing: Check with CircuitLab



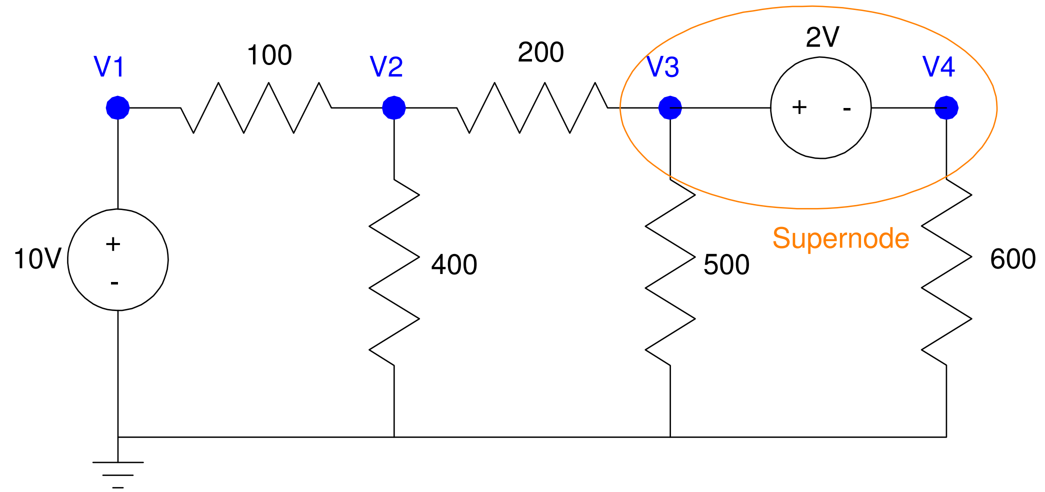
Super-Nodes:

- The current coming out of any closed-path must be zero
- Paths which enclose several nodes are called *supernodes*

Example:

Step 1&2: 4 Nodes

- Need 4 equations for 4 unknowns



Step 3:

$$V_1 = 10$$

$$V_3 - V_4 = 2$$

$$\left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{400}\right) + \left(\frac{V_2 - V_3}{200}\right) = 0$$

$$\left(\frac{V_3 - V_2}{200}\right) + \left(\frac{V_3}{500}\right) + \left(\frac{V_4}{600}\right) = 0 \quad \textit{supernode}$$

Step 4: Solve. Group terms

$$V_1 = 10$$

$$V_3 - V_4 = 2$$

$$\left(\frac{-1}{100}\right)V_1 + \left(\frac{1}{100} + \frac{1}{400} + \frac{1}{200}\right)V_2 + \left(\frac{-1}{200}\right)V_3 = 0$$

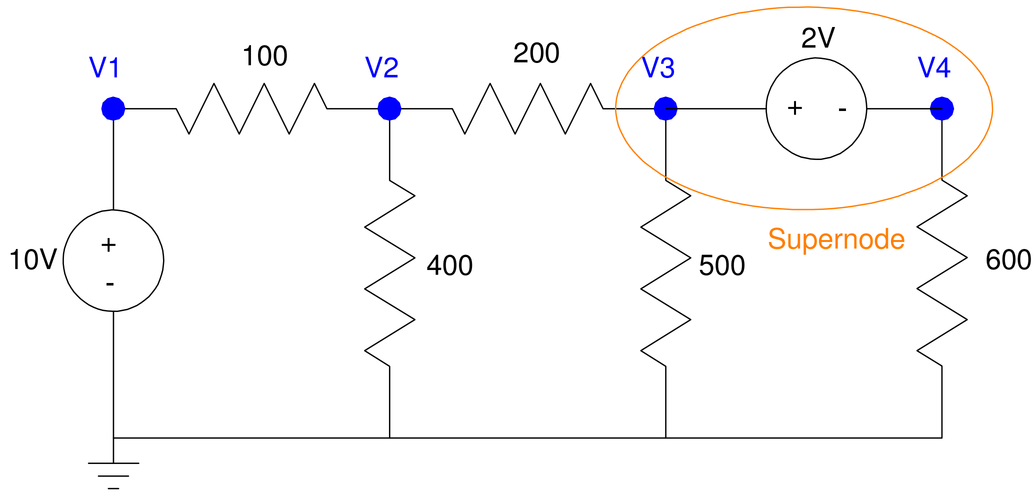
$$\left(\frac{-1}{200}\right)V_2 + \left(\frac{1}{200} + \frac{1}{500}\right)V_3 + \left(\frac{1}{600}\right)V_4 = 0$$

Place in matrix form:

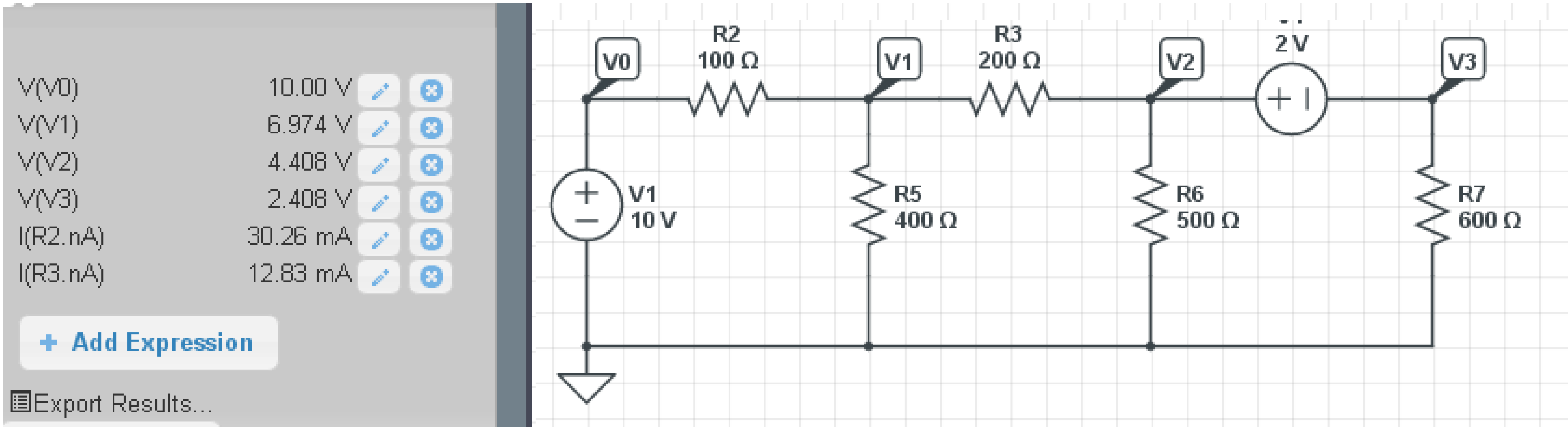
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{400} + \frac{1}{200}\right) & \left(\frac{-1}{200}\right) & 0 \\ 0 & \left(\frac{-1}{200}\right) & \left(\frac{1}{200} + \frac{1}{500}\right) & \left(\frac{1}{600}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab:

```
A = [1,0,0,0;0,0,1,-1;-1/100,1/100+1/400+1/200,-1/200,0;0,-1/200,1/200+1/500,1/600]

     1.0000         0         0         0
          0         0    1.0000   -1.0000
    -0.0100    0.0175   -0.0050         0
          0   -0.0050    0.0070    0.0017

B = [10;2;0;0]

    10
     2
     0
     0

V = inv(A)*B

V1   10.0000
V2    6.9737
V3    4.4079
V4    2.4079
```

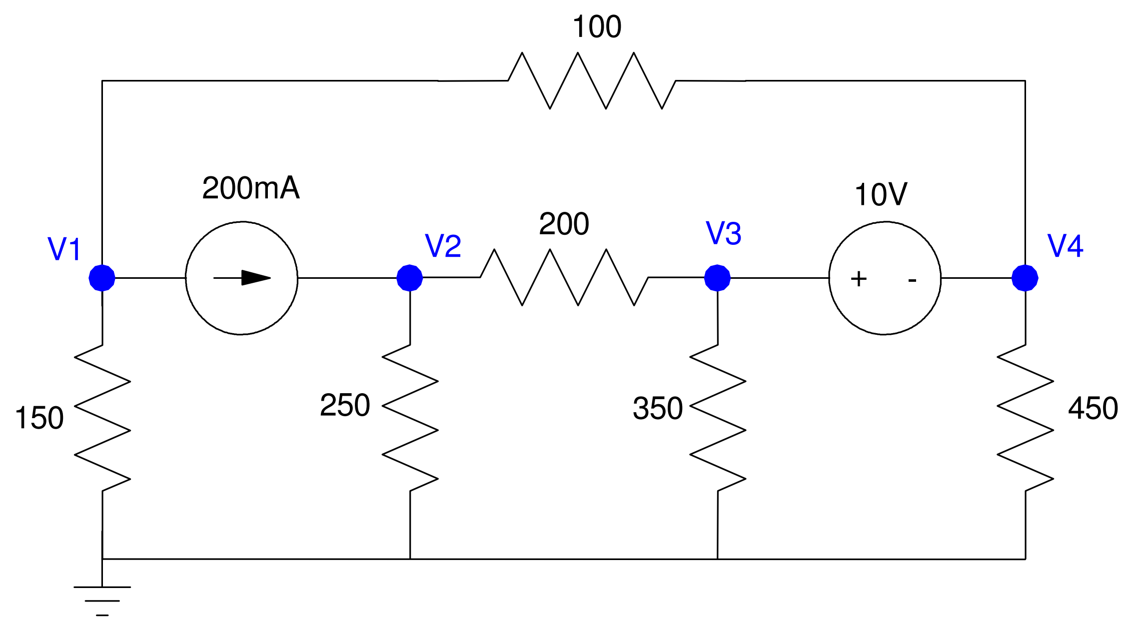


Checking in Circuitlab



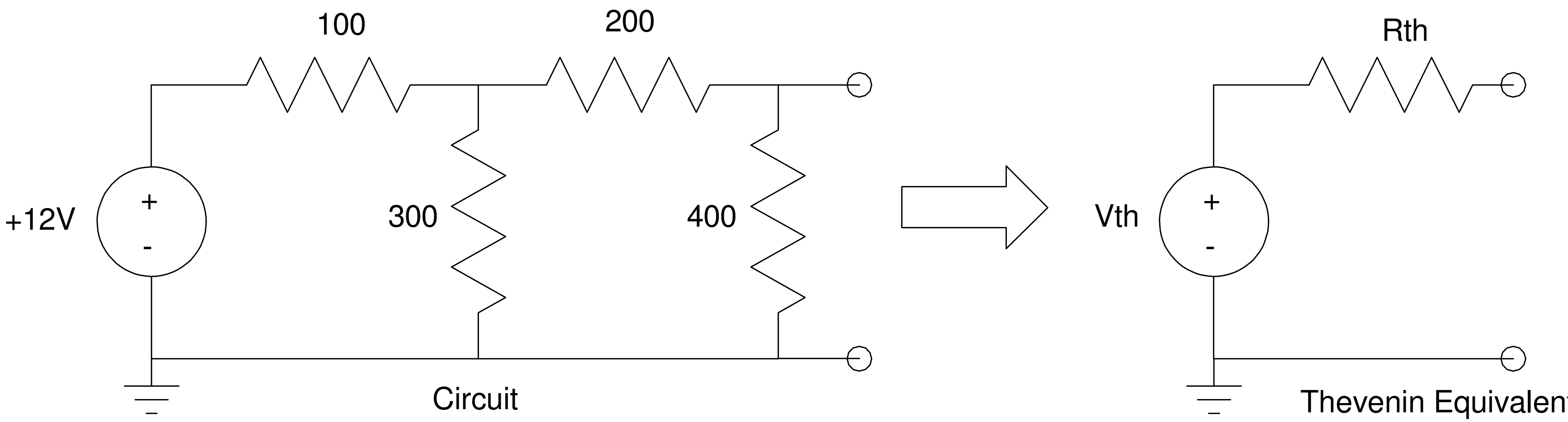
Example: (handout)

Write the voltage node equations



Thevenin Equivalents

- If a circuit is linear (i.e. only contains sources, resistors, capacitors, and inductors), its voltage / current relationship will follow a straight line, termed the load line.
- Any circuit which has the same load line is indistinguishable from the original circuit.
- The simplest circuits which produce a load line are a voltage source and resistor (Thevenin equivalent) and a current source and a resistor (Norton equivalent).

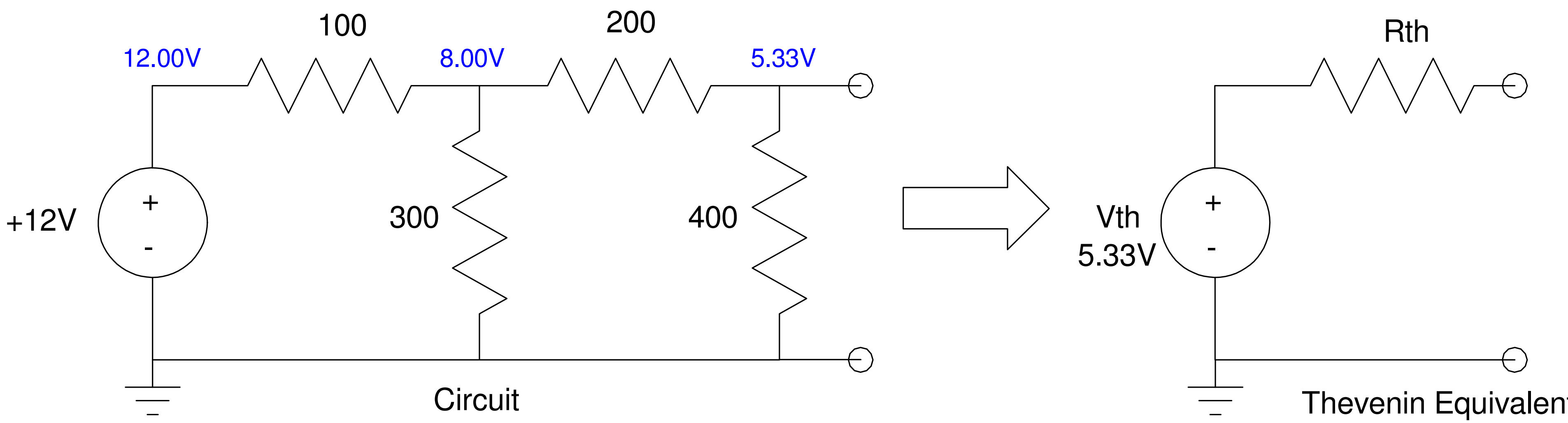


Since the two circuits are equivalent, whatever test you do on the circuit to the right to find Vth and Rth, do the same test to the original circuit.

Vth: Measure the open-circuit votlage.

- The original circuit gives 5.33V
- The Thevenin equivalent gives Vth

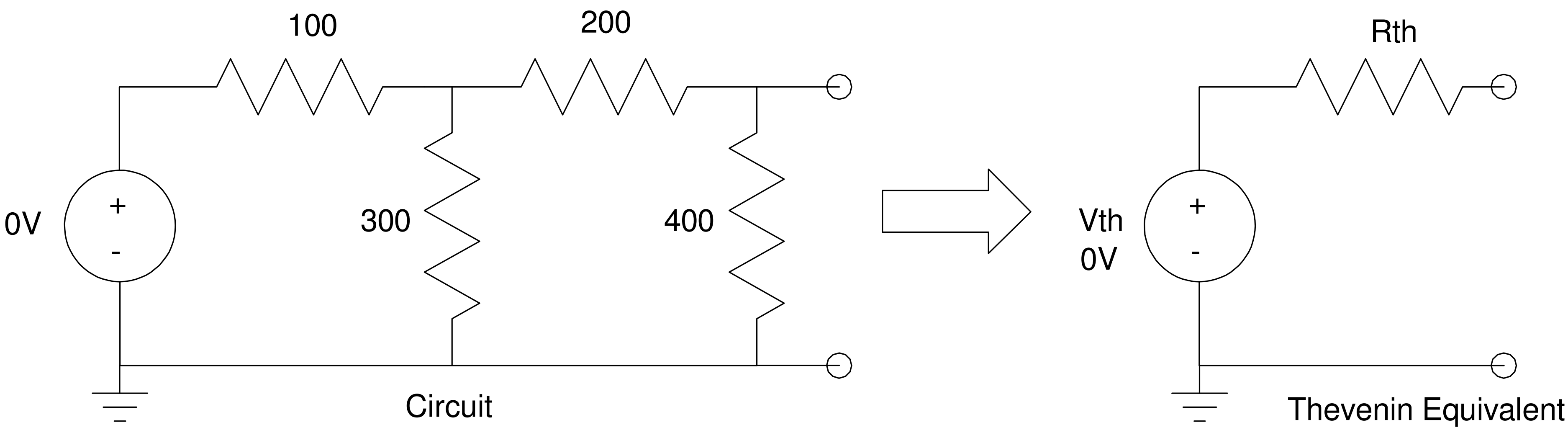
So Vth = 5.33V



Rth: Turn off the source and measure the resistance looking in. This gives

- R = 162.96 Ohms (original circuit)
- R = Rth (circuit to the right)

so Rth = 162.96 Ohms.



Rth (take 2): Sometimes the resistance looking in isn't obvious. In that case, apply a 1V test votlage and compute the current. The Thevenin resistance is then

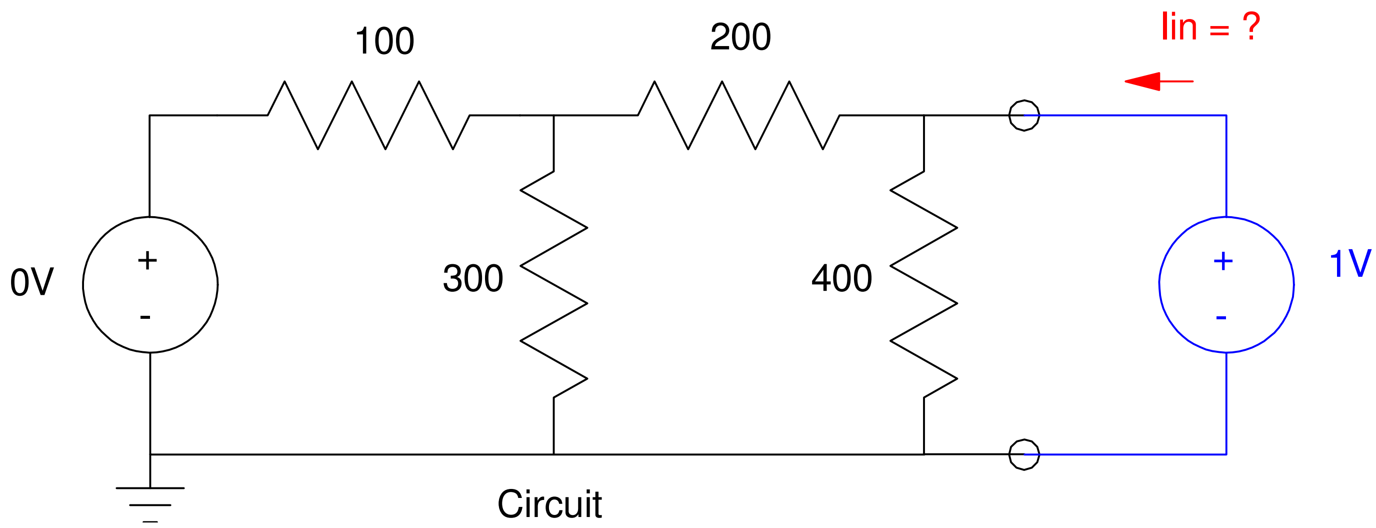
Rth = 1V / Iin

For example with this circuit

Iin = 6.136mA

meaning

Rth = 1V / 6.136mA = 162.96 Ohms



Summary

Resistors

- Add in series
- Add as $\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_2}+\ldots\right)^{-1}$ in parallel

Current Loops

- The total current coming from a closed area must be zero
- Write N equations for N unknown currents

Voltage Nodes

- The voltages around a closed path must sum to zero
- Write N equations for N unknown voltages

Thevenin Equivalents

- Replace a ciruit with a simpler circuit that has the same load line