

Max, Min, Average Circuits

Fuzzy Logic:

Fuzzy Logic is a type of logic which can deal with statements being partially true. For example, if

- 0.0V means false and
- 10.0V means true

then 5.0V means 50% true. With fuzzy logic, you can implement logic AND and OR with $\min()$ and $\max()$ functions. For example, with Boolean logic, AB is false (0V) if either A or B is low. A consistent interpretation of this is

$$AB = \min(A, B)$$

With this interpretation, you can deal with cases where A and B are partially true. For example

$$(70\% \text{ true}) \text{ and } (30\% \text{ true}) = \min(70\%, 30\%) = 30\%$$

Similarly, $A+B$ is true if either A or B is true. A consistent interpretation of OR is then $\max()$

$$A+B = \max(A, B)$$

$$(70\% \text{ true}) \text{ or } (30\% \text{ true}) = \max(70\%, 30\%) = 70\%$$

Not is just the inverse.

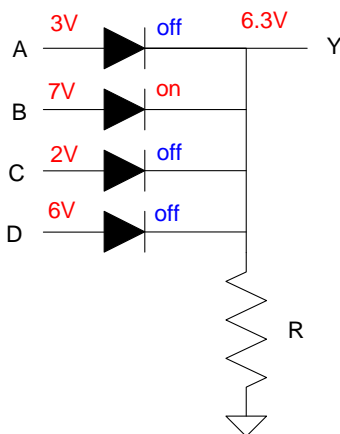
$$\text{not}(A) = 100\% - A$$

$$\text{not}(70\% \text{ true}) = 30\% \text{ true}$$

Max Circuit: (Logic OR)

- Design a circuit which outputs the maximum of four voltages.
- Design a circuit which computes OR function in Fuzzy Logic

Solution: Use four diodes as switches. Whichever one has the highest input voltage turns on.



$$Y = \max(A, B, C, D) = A + B + C + D \text{ (using fuzzy logic)}$$

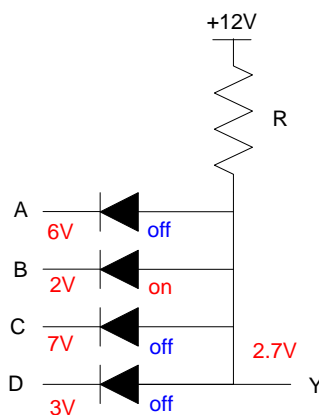
For example, assume the voltages shown above. B has the highest voltage, so it turn on its diode, raising the voltage at the output, Y, to 7V minus the voltage drop across the diode (0.7V). The other three diodes are now reverse biased and turn off.

Note: Diodes are either on or off. With the above circuit, there are 2^4 combinations - meaning you could analyze 16 different circuits with various diodes on and off. Only one of these 16 will work, however: the other 15 will have a contradiction, such as a diode is forward biased by more than 0.7V but off. If you can 'guess' the right combination of on and off, you can save yourself a lot of time with diode circuits.

Min Circuit: (Logic AND)

- Design a circuit which outputs the minimum of four voltages.
- Design a circuit which computes the AND function in Fuzzy Logic

Solution: Use the previous solution but flip the diodes. Also replace the ground with a voltage that's more than whatever the inputs can be. (+12V assumed here). Whichever diode has the lowest input voltage turns on. This then turns off the other three diodes.



$$Y = \min(A, B, C, D) = ABCD \text{ (using fuzzy logic)}$$

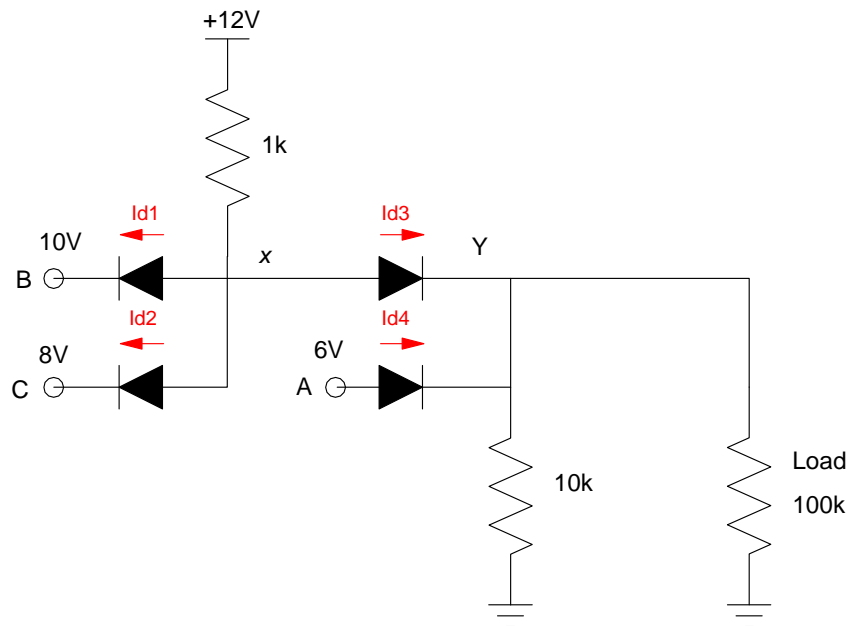
Min / Max Circuits:

You can combine these into a single circuit. For example, design a circuit which meets the following requirements:

Inputs: A, B, C. 0-10V signals, capable of 10mA.

Outputs: Y, capable of driving a 100k load (0.1mA @ 10V)

Relationship: $Y = \max(A, \min(B,C)) = A + BC$ (using Fuzzy Logic)



$$Y = \max(A, \min(B, C)) = A + BC$$

For example, determine Y given the voltages are as shown in the above figure. First, determine which diodes are on and off. B and C are set up as a min() function, so

- Diode2 is probably on
- Diode1 is probably off

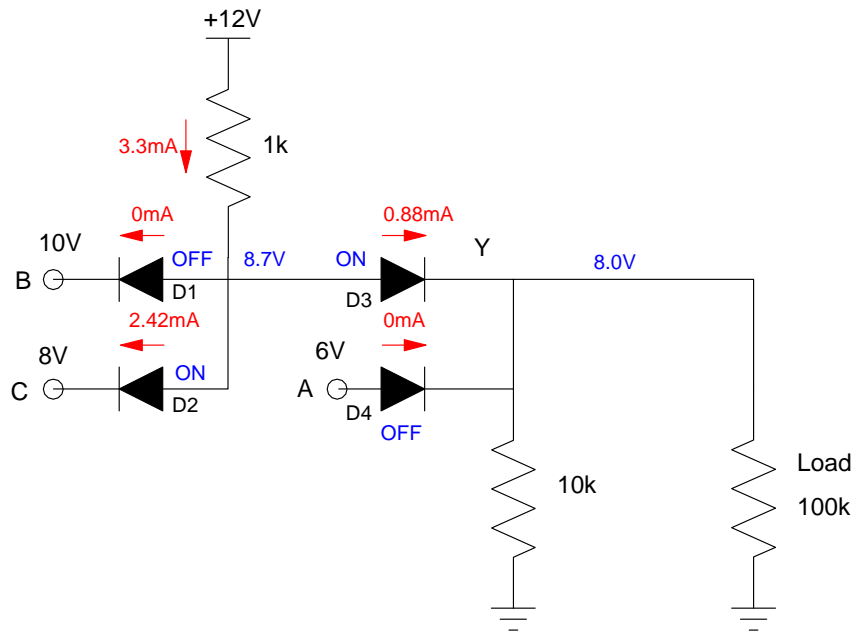
This results in the voltage at X being 8.7V (assuming ideal silicon diodes)

Diode 3 and 4 are set up as a max() function. The max of X and A is 8.7V, so

- Diode3 is probably on
- Diode4 is probably off

This results in $Y = 8.0V$ (X - 0.7V drop across diode D3). The currents are then

- $I_{d1} = 0mA$ (diode is off)
- $I_{d4} = 0mA$ (diode is off)
- $I_{d3} = 0.88mA$ (the 10k and 100k resistors draw 0.88mA @ 8V)
- $I_{d2} = 2.42mA$ (the 1k resistor has 3.3mA flowing ($(12V - 8.7V) / 1k$). 0.88mA flows right. The remaining current goes through I_{d2}).



Resulting currents and voltages for min/max circuit

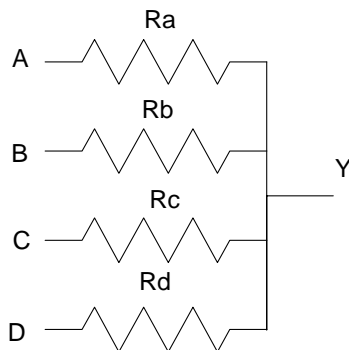
Average Circuit:

Problem: Design a circuit which computes the average of four voltages.

Solution: This is a linear function

$$Y = \frac{1}{4}(A + B + C + D)$$

You likewise need a circuit with linear elements, such as resistors, not nonlinear elements, such as diodes.



If all the resistors are the same, using voltage nodes at Y:

$$\left(\frac{Y-A}{R}\right) + \left(\frac{Y-B}{R}\right) + \left(\frac{Y-C}{R}\right) + \left(\frac{Y-D}{R}\right) = 0$$

Solving for Y:

$$4Y = A + B + C + D$$

$$Y = \frac{1}{4}(A + B + C + D)$$

If you want a weighted average, vary the resistors. A smaller resistor increases the contribution of that input.

Example: Design a circuit which converts +/- 10 V to 0..5V and draws less than 1mA.

Solution: Rewrite this as

$$Y = 0.25X + 2.5V$$

Assume you have a 5V source (A). Rewrite this as

$$Y = 0.25X + 0.5A$$

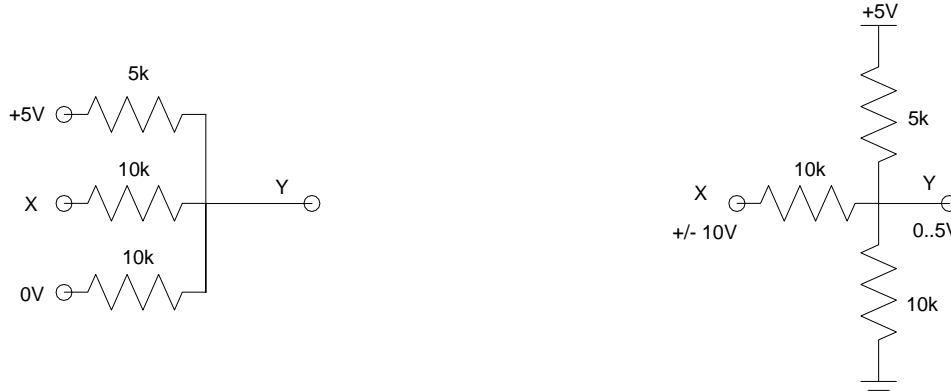
$$Y = \left(\frac{X+2A}{4} \right)$$

Pick a resistor (10k for 1mA @ 10V). The resistors tied to X are then

$$X = 10k \text{ (weight = 1)}$$

$$A = 10k / 2 \text{ (weight = 2)}$$

ground = 10k (the numerator doesn't add up to 4, meaning the remaining resistor is tied to 0V)



Circuit to convert +/- 10V to 0.5V (left) and the same circuit redrawn (right)