Ideal Diodes

Diodes are useful electronic devices. One major problem with them, however, is they are nonlinear devices. This requires you to solve N nonlinear couples equations when solving a diode circuit.

The Ideal Diode model is a way to come up with an approximation to the nonlinear diode equations which

- Are simple, and
- Are fairly accurate.

For example, assume the diode equations for a Silicon diode are

$$I_d = 10^{-7} \cdot \left(\exp\left(\frac{V_d}{0.052} \right) - 1 \right)$$

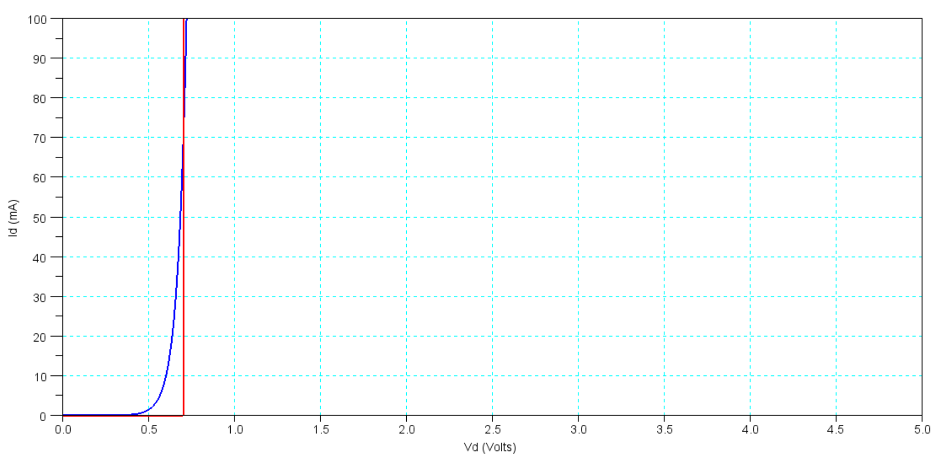
or equivalently

$$V_d = 0.052 \cdot \ln\left(\frac{I_d}{10^{-7}} + 1 \right).$$

One way to simplify this is to use a piecewise linear model with two states

State	Model	Condition
On	Vd = 0.7V	Id > 0
Off	Id = 0	Vd < 0.7V

The comparison with the ideal diode model and the actual VI characteristics are shown in the following figure.

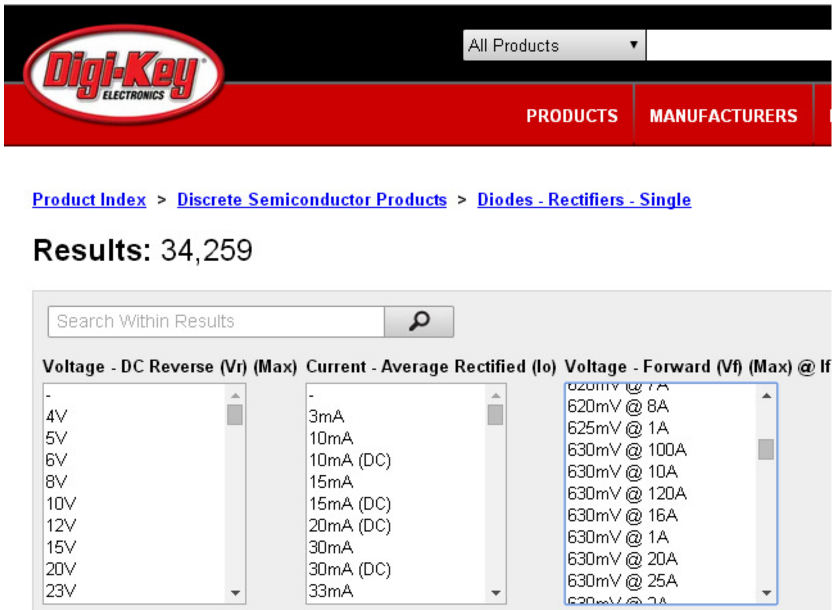


VI Characteristics for a Nonlinear Diode Model (blue) and Ideal Diode Model (red)

Note that

- The ideal diode model is only accurate at one current - 70mA in this case
- The ideal diode model isn't perfect - but is fairly accurate otherwise.

If you look up diodes at Digikey, the specifications look like the following:



Diode's Available from Digikey as of September 1, 2016

- Vr: If you apply enough voltage to anything, it will eventually conduct. The diode is guaranteed to work (i.e. block current when reverse biased) for voltages up to Vr. No guarantees beyond this.
- Io: How much current the diode is able to withstand and not burn out under normal conditions.
- Vf: The voltage drop across the diode when current is flowing.

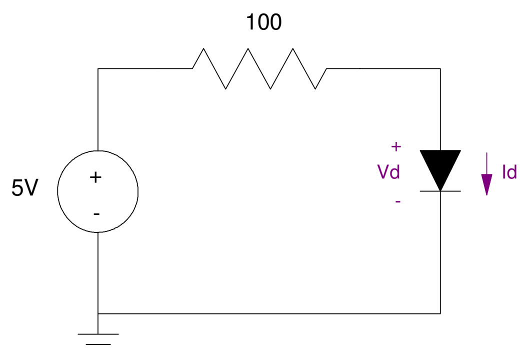
Note that Vf is defined at a specific current. This is necessary since the diode's VI characteristic are nonlinear - and the voltage changes as the current changes, However, if you assume an ideal diode model:

Vf is the voltage across the diode when the diode is "on"

The current through the diode when off is zero. That kind of goes without saying.

Solving Diode Circuits Using the Ideal Diode Model: Take 1

Problem: For the following circuit, determine Vd and Id



Single Diode Circuit

Nonlinear Solution: Write 2 equations for 2 unknowns

Diode Equation:

$$I_d = 10^{-7} \cdot \left(\exp\left(\frac{V_d}{0.052} \right) - 1 \right)$$

Node Equation:

$$\left(\frac{V_d - 5}{100} \right) + I_d = 0$$

Solving in Matlab results in

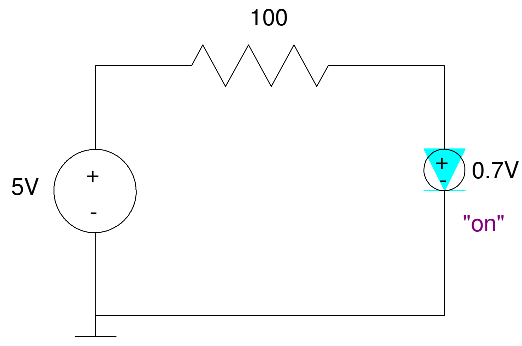
Vd = 0.6748V

Id = 43.25mA

Ideal Diode Model Solution: First, determine which model to use: on or off. The 5V source is trying to turn on the diode, so assume "on". When the diode is on,

Vd = 0.7V.

Redraw the circuit using the ideal diode model ("on" in this case):



Diode Circuit with the Diode Replaced with its Model in the On State

The current is then

$$I_d = \left(\frac{5-0.7}{100}\right) = 43mA$$

Check: This solution is correct if my assumption that the diode is "on" is correct. For a diode to be "on",

Id > 0

Id is +43mA, so the diode is on.

Note that if you assume the diode was "off", you would get

- Vd = +5.0V
- Id = 0mA

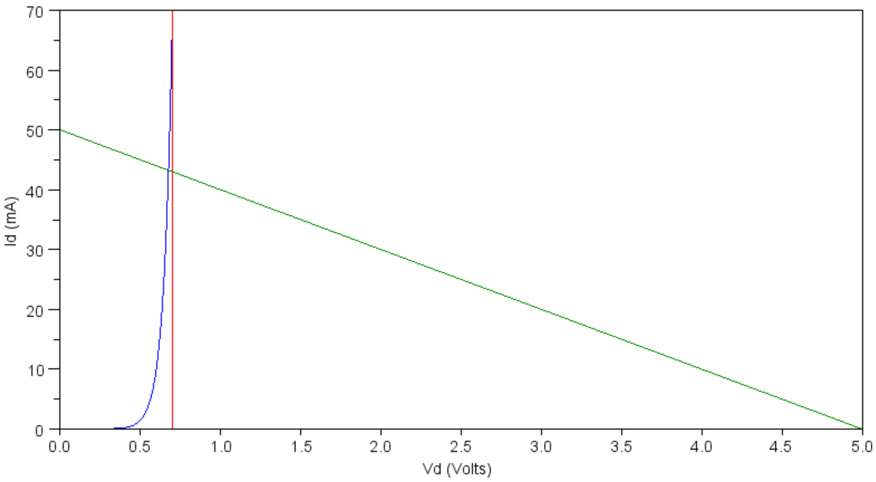
To be "off", the condition is

Vd < 0.7V

This is false - meaning the diode is not off. It must be on.

Graphical Solution:

Draw the diode models along with the equation from voltage nodes (termed the load line) and you get the following:



Graphical Solution using the Nonlinear Diode Model (blue) and the Ideal Diode Model (red)

Note that the ideal diode model is slightly off - but close.

This results in three different solutions for the same problem:

	Vd	Id
Nonlinear Model	0.6748 V	42.25 mA
Ideal Diode	0.7000 V	43.00 mA
CircutiLab (4004 diode)	0.7517 V	42.48 mA

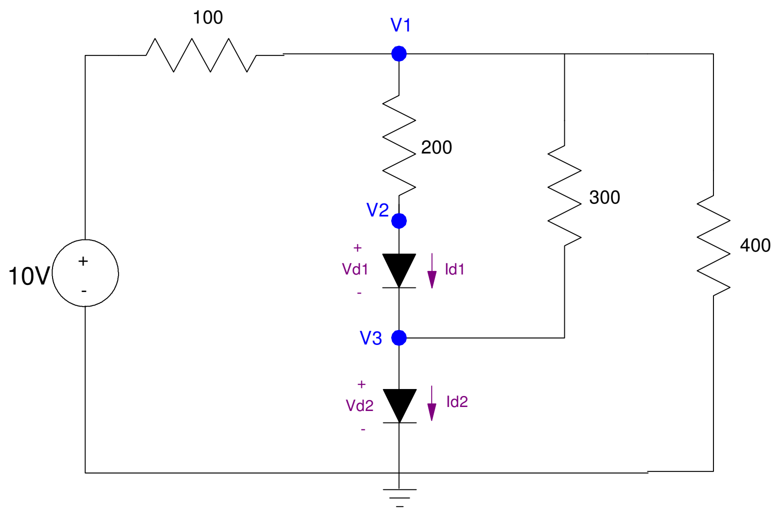
If you built this circuit in lab, you'd get a 4th answer. This is typical in electronics

- All four answers should be similar: it's the same circuit
- All four answers should be slightly different: each uses a slightly different model for the diode

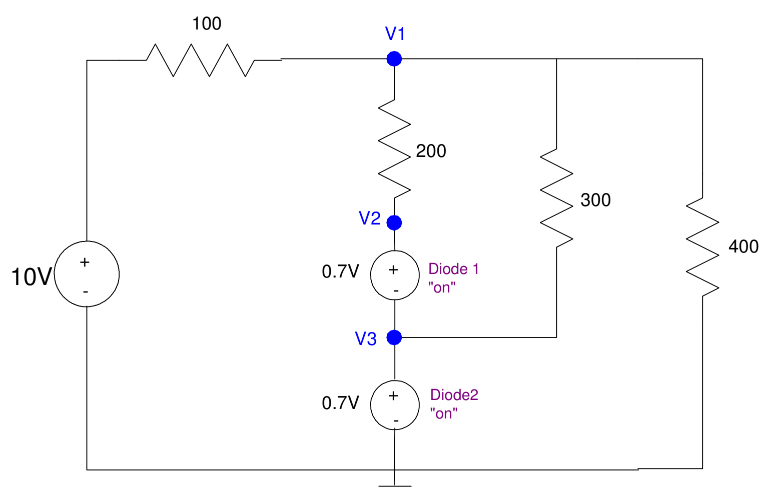
Likewise, if you get answers which are close, you probably solved the problem correctly.

Solving Diode Circuits Using the Ideal Diode Model: Take 2

Consider next a 2-diode circuit



With an ideal diode model, first determine (guess) if the diodes are on or off. +10V is trying to push current through the diodes, so let's assume both are on. Redrawing the circuit with the diodes replaced with their "on" model gives:



Diode Circuit with Both Diodes Replaced with their On Model

This results in

- V3 = +0.7V
- V2 = +1.4V

V1 is solved using voltage nodes:

$$\left(\frac{V_1-10}{100}\right)+\left(\frac{V_1-1.4}{200}\right)+\left(\frac{V_1-0.7}{300}\right)+\left(\frac{V_1}{400}\right)=0$$

which gives

V1 = 5.248V

Check: For the diodes to be on, Id > 0.

Diode 1:

$$I_{d1}=\left(\frac{V_1-1.4}{200}\right)=19.2mA>0$$

Diode2:

$$I_{d2}=I_{d1}+\left(\frac{V_1-0.7}{300}\right)=32.1mA>0$$

so the diodes are actually on.

Like before, this gives three different answers for the same circuit

	Vd	V2	V3
Nonlinear Model	5.2057 V	1.2678 V	0.6339 V
Ideal Diode	5.248 V	1.4000 V	0.7000 V
CircutiLab (4004 diode)	5.270 V	1.464 V	0.7423 V

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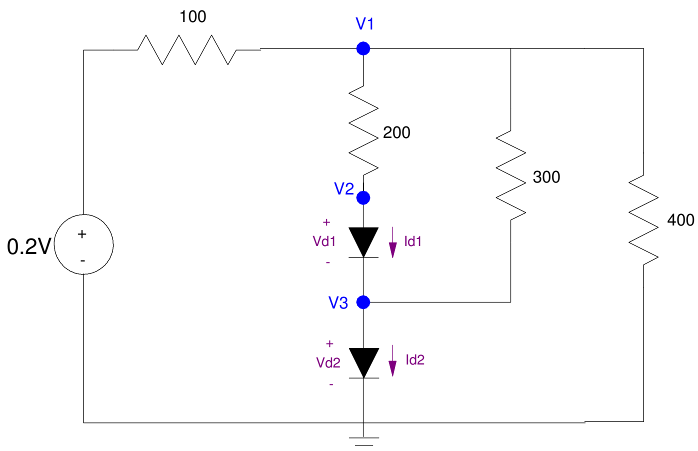
Solving Diode Circuits Using the Ideal Diode Model: Take 3

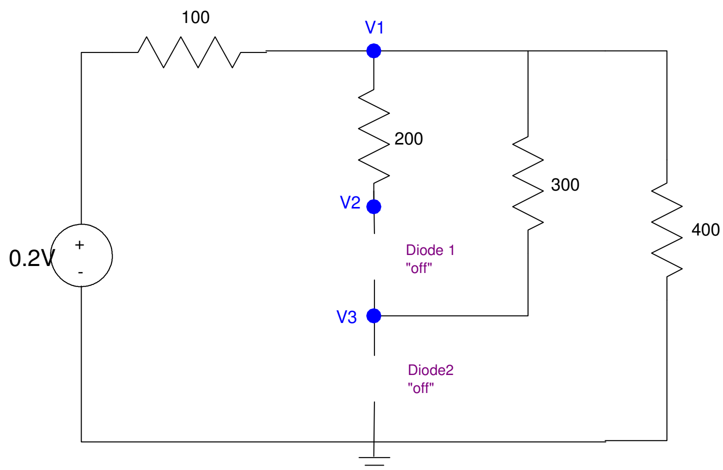
Repeat if the input voltage drops to 0.2V



Diode Circuit with a 0.2V Source

Solution: Ideal silicon diodes need at least 0.7V to turn on. Since the input voltage is so small, the diodes probably are off.

Assume both diodes are off (open circuit). Redrawing the circuit with the off model gives:



Same circuit with the diodes replaced with their "off" models

Solve for V1 using voltage nodes:

$$\left(\frac{V_1 - 0.2}{100}\right) + 0 + 0 + \left(\frac{V_1}{400}\right) = 0$$

- V1 = 0.16V

Since Id = 0

- V2 = 0.16V
- V3 = 0.16V

Check: For the diodes to be off, Vd < 0.7V

Diode 1:

$$V_{d1} = V_2 - V_3 = 0V < 0.7V$$

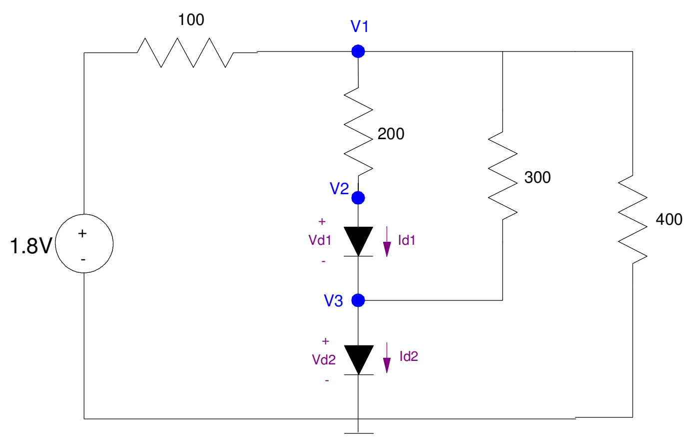
Diode 2:

$$V_{d2} = V_3 - 0 = 0.16V < 0.7V$$

Yes, both diodes are off.

Solving Diode Circuits Using the Ideal Diode Model: Take 4

Repeat when the input becomes 1.8V

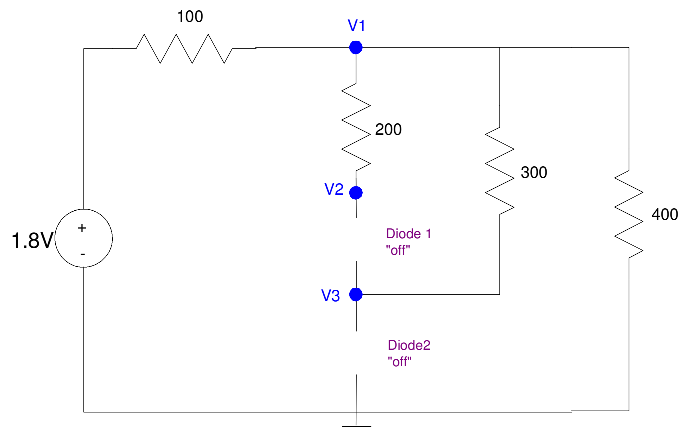


Diode Circuit with a 1.8V Source

This is where it gets hard: you have to determine whether the diodes are on or off to solve.

One solution is to try every permutation. With 2 diodes, there are 4 possible on/off conditions.

Guess 1: Assume Both Diodes Off:



1.8V Source with both diodes assumed to be off (incorrect assumption)

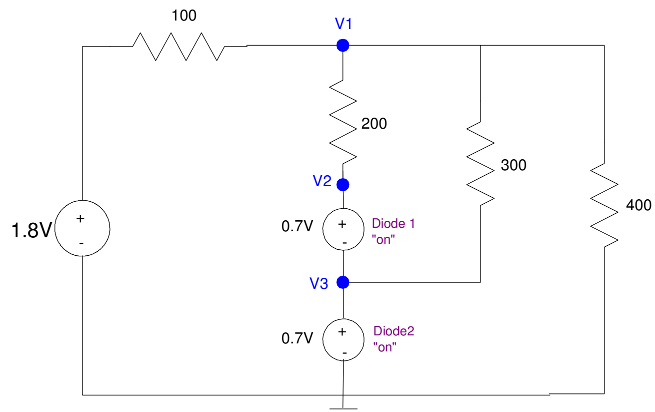
By voltage division

$$V_1 = \left(\frac{400}{400+100}\right)1.8V = 1.44V$$

$$V_{d2} = 1.44V$$

This is too much: 1.44V will turn on diode 2. So, both diodes are not off.

Guess 2: Assume Both Diodes On:



1.8V Source with both diodes assumed to be on (incorrect assumption)

This results in

- V3 = 0.7V
- V2 = 1.4V

By voltage nodes:

$$\left(\frac{V_1-1.8}{100}\right) + \left(\frac{V_1-1.4}{200}\right) + \left(\frac{V_1-0.7}{300}\right) + \left(\frac{V_1}{400}\right) = 0$$

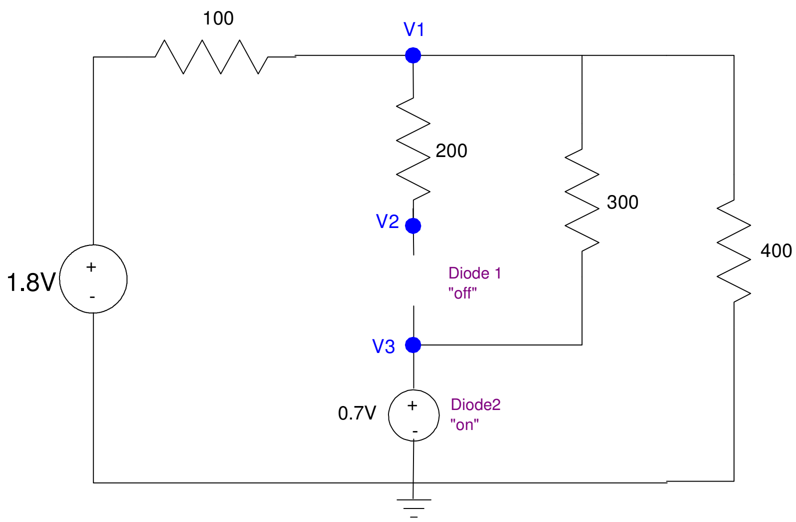
$$V_1 = 1.312V$$

Check: For both diodes to be on, Id > 0

Diode 1:

$$I_{d1} = \left(\frac{V_1 - V_2}{200}\right) = -0.44mA$$

Guess 3: Assume Diode2 is on, Diode1 is off:



Circuit with the assumption that D1 is off, D2 is on

Solve using voltage nodes:

$$\left(\frac{V_1 - 1.8}{100}\right) + 0 + \left(\frac{V_1 - 0.7}{300}\right) + \left(\frac{V_1}{400}\right) = 0$$

This gives

- V1 = 1.2842 V
- V2 = 1.2842 V
- V3 = 0.7 V

Check: Diode 1 is off:

Vd1 = V2 - V3 = 0.5842V < 0.7V (check)

Diode2 is on

$$I_{d2} = \left(\frac{V_1 - 0.7}{300}\right) = 0.263mA$$

which is positive. So, the correct answer is

- **V1 = 1.2842V**
- **V2 = 1.2842V**
- **V3 = 0.7V**

Once again, there are three solutions for the same circuit

	Vd	V2	V3
Nonlinear Model	1,1182 V	0.7154 V	0.5153 V
Ideal Diode	1.2842 V	1.2842 V	0.7000 V
CircutiLab (4004 diode)	1.255 V	1.202 V	0.6415 V

Comment:

- A circuit with N diodes has 2^N permutations of "on" and "off".
- One of these permutations will be correct (off diodes have Vd < 0.7V and on diodes have Id > 0)

It really helps if you can "guess" which of the 2^N permutations is correct. That's not always easy to do.