

PN Junction

With silicon, you can create n-type material and p-type material by doping the silicon crystal. When you place a p-type material adjacent to n-type material, you get a pn junction - also known as a diode.

Diodes allow current to flow in only one direction. For example, assume the doping in the n and p region is 10^{16} atoms per cc. In the p-type region,

$$p_p \approx 10^{16}$$

$$n_p = \left(\frac{n_i^2}{p}\right) = 2.25 \cdot 10^4$$

In n-type silicon, the doping is Phosphorus, resulting in

$$n_n \approx 10^{16}$$

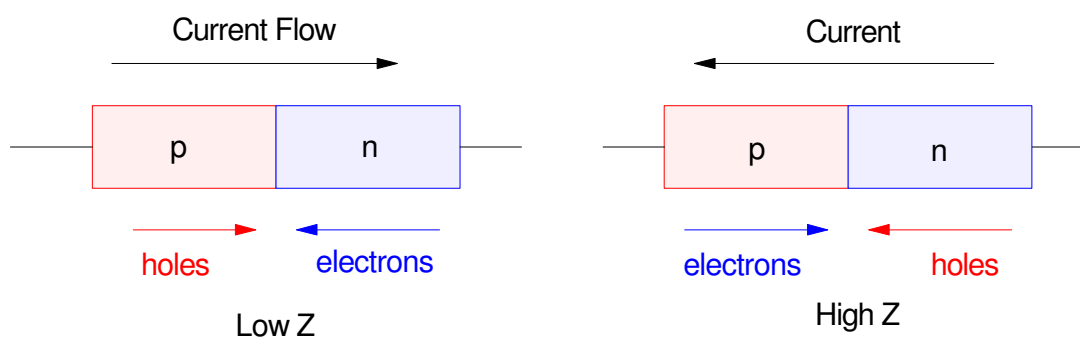
$$p_n = 2.25 \cdot 10^4$$

At the pn junction, some electrons and holes are created through thermal electrons. This is a small number, however, so assume it's zero. If electrons and holes are not created, current can only flow by electrons and holes flowing towards the pn junction (where they combine and disappear).

Assume the current is flowing from the p-side to the n-side (the left figure below.) In this case, you are using majority carriers, so there are approximately 10^{16} charge carriers.

Now, assume the current is flowing from the n-side to the p-side. In this case, you are using minority carriers, resulting in only $2.25 \cdot 10^4$ charge carriers. If the diode has a resistance of 1 when using majority carriers (current p to n), it has a resistance of $444\text{G}\Omega$ when using minority carriers (!).

A diode only allows current to flow from p to n



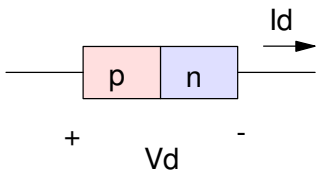
majority carriers: # charge carriers = 10^{16}

minority carriers: # charge carriers = 10^4

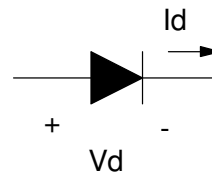
The net result is a diode has low resistance in one direction (p to n) but high resistance in the other direction (n to p). The difference in resistance is huge: $10^{12} : 1$ in this case. As a result, a good approximation for a diode is to say

- A diode is a short (zero ohms) when current flows p to n
- A diode is an open circuit (infinite ohms) when current tries to flow n to p

This is reflected in the symbol for a diode: an arrow reminding you which direction current can flow in a diode:



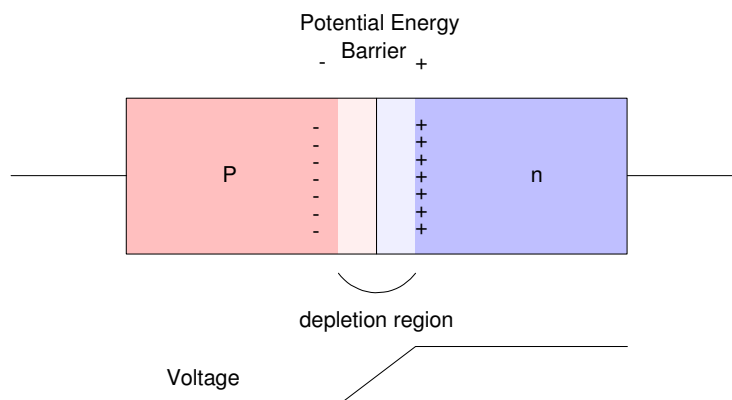
Diode pn junction



Diode symbol

In addition, you get diffusion current. Above 0K, all electrons are in motion. At the pn junction, there are lots of holes on the p-side, and almost none on the n-side. Due to random motion of the holes, holes will drift from the p-side to the n-side, with almost no holes drifting back. The same occurs for electrons in the n-type region. The net result is current flow across the pn junction, termed drift current.

Eventually, the holes accumulate in the n-side and the electrons accumulate on the p-side:



This has several effects:

- (a) A region with no charge carriers is created near the pn junction, termed the depletion region. Since there are no charge carriers, there is no current flow.
- (b) A capacitor is created: you have charge across an insulator. The capacitance is small, but it's non-zero. If you have a high-speed switching circuit, you need to take this capacitance into account when calculating the rise times for signals.
- (c) A voltage is created between the n-side and the p-side. If you try to get current to flow from p-to-n, you have a voltage drop across the diode equal to

$$V_d = V_T \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

For Silicon at 273C, this works out to

$$V_d = (0.026V) \ln \left(\frac{10^{16} \cdot 10^{16}}{(1.5 \cdot 10^{10})^2} \right) = 0.697V \approx 0.7V$$

Silicon diodes have about a 0.7V drop across them when current flows.

(d) The voltage varies with different types of materials. These are approximately as follows:

- Silicon: 0.7V
- Germanium: 0.3V
- Red LED: 1.9V (varies)
- Blue LED: 3.3V (varies)

(e) The voltage varies with temperature. Since the intrinsic carrier concentration is:

$$n_i^2 = A_o T^3 e^{-E_g/kT}$$

$$V_T = \frac{T}{11,600}$$

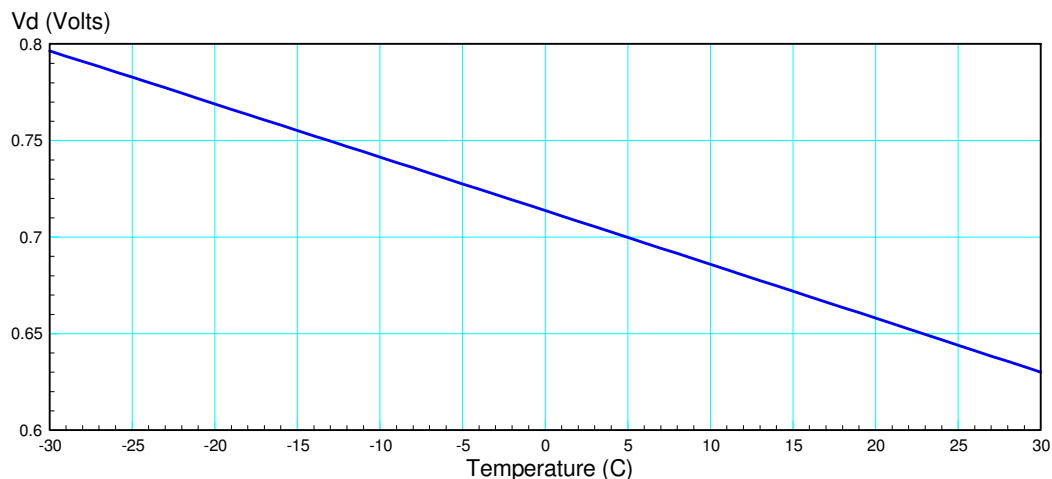
as T varies, Vd varies. Plotting this in Matlab:

```
C = [-30:30]';           temperature in Celcius
T = C + 273;           temperature in Kelvin
Ao = 2.33e33;
k = 8.62e-5;           Boltzman's constant
Eg = 1.4;              Energy gap for Silicon

ni = (Ao * (T.^3) .* exp(-Eg ./ (k*T) ) ) ^ 0.5;
Vt = T / 11600;

Vd = Vt .* log(1e28 ./ (ni .^ 2) );

plot(C, Vd)
xlabel('Temperature (Celcius)');
ylabel('Vd');
```



Voltage drop across a pn junction vs. temperature

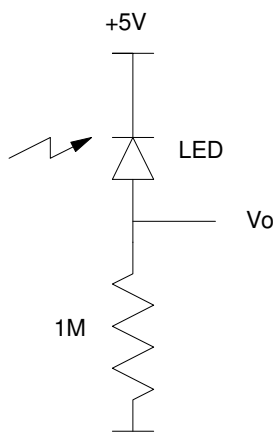
You can use the voltage drop across a diode to measure temperature. The change in voltage isn't large, but it's there. It's also almost a linear relationship. This allows you to use a forward biased diode to measure temperature (termed a *thermal diode*).

Diode Radiation Sensors

You can also use a diode to measure radiation by trying to push current backwards through the diode. If you try to send current through a diode from the n-side to the p-side, there are actually two sources of current flow:

- Drift current: current flow due to the resistance of the device. This is almost zero due to the large resistance of a diode to reverse current flow ($444G\Omega$).
- Thermal Current: Above 0K, there will be a few electrons and holes created in the depletion zone due to random motion of electrons. Once an electron is created, it flows into the n-region and the hole flows into the p-region. This is actually the main contributor to current flow in a reverse biased diode.

Similarly, if you expose a diode to radiation, the radiation can kick an electron out of its covalent bond, creating an electron-hole pair. By measuring the current flow, you also have a radiation sensor. You can verify this by applying light to a reverse-biased LED as follows:



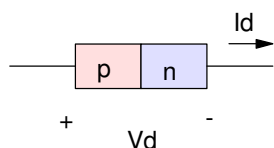
In a dark room, there should be no current flow through the diode. If you shine a light on it, however, current is created by the light kicking electrons out of their covalent bond at the pn junction. This current is converted to a voltage by the 1M resistor as

$$V = IR$$

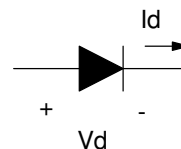
The current isn't a lot - hence you need to use a current amplifier - a.k.a. a transistor. More on this later...

Diodes

Assume you have a pn junction creating a diode:



Diode pn junction

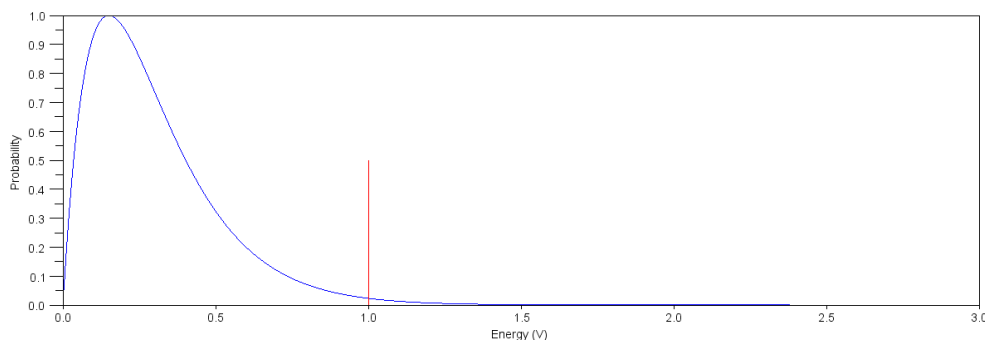


Diode symbol

With a pn junction, a potential energy barrier is created - meaning only electrons with enough energy are able to cross this barrier.

At a given temperature, the energy of an electron can be approximated by a Gamma distribution:

$$p(V) = \frac{\beta^\alpha}{\Gamma(\alpha)} V^{\alpha-1} e^{-\beta V}$$



Gamma Distribution for the Energy of a Given Electron.

With the above distribution, the electrons with an energy above the energy gap are able to cross the gap and flow across the pn junction. As the voltage applied increases, the above curve shifts right to higher voltages and more and more current flows. You can also think of this as the threshold the electrons need to exceed getting smaller and smaller (as one over the applied voltage) - with the area to the right of the threshold determining the current.

From this curve, you can see that

- When the applied voltage is small, the current flow (area under the curve) is almost zero.
- If you move the energy gap (red line) to the left - corresponding to raising the voltage applied to the pn junction - more and more electrons will have enough energy to cross the pn junction, creating more and more current flow.
- This increase is a smooth function which is essentially exponential in nature.

Diode VI Characteristics:

The current through a diode is exponential with respect to voltage:

$$I_d = I_o(e^{V_d/\eta V_T} - 1)$$

$$V_d = \eta V_T \cdot \ln(I_d/I_o + 1)$$

where

η is a constant, approximately 1 for germanium, 2 for silicon

$V_T = 0.026V$ at 300K (the volt equivalent of temperature)

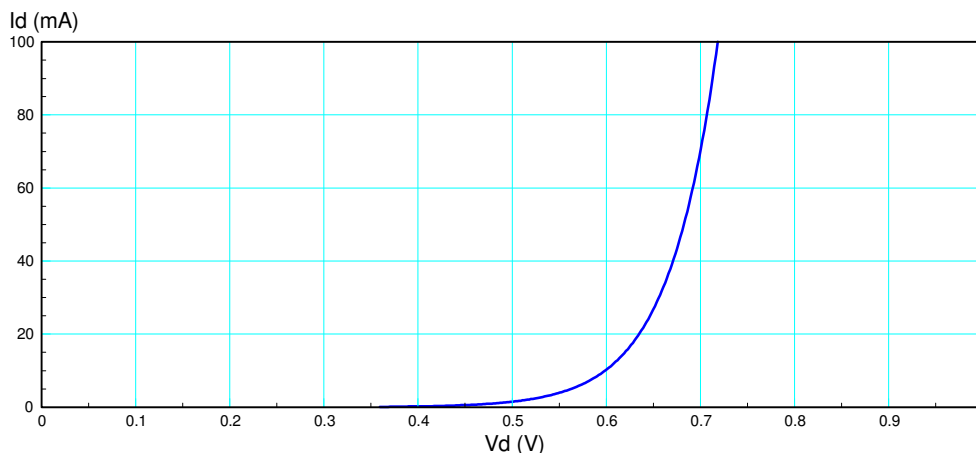
I_o is the reverse saturation current

For silicon diodes, the diode equations are typically:

$$I_d = 10^{-7} \left(\exp\left(\frac{V_d}{0.052}\right) - 1 \right)$$

$$V_d = 0.052 \cdot \ln\left(\frac{I_d}{10^{-7}} + 1\right)$$

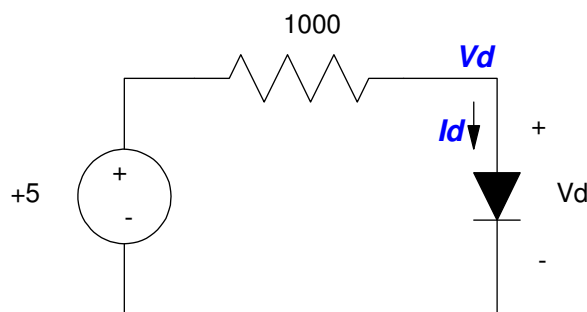
with the following VI characteristics:



Typical VI characteristic of a Silicon diode

This allows you to write the equations which describe circuits with diodes using either voltage nodes or current loop techniques.

Example 1: Write N equations to solve for N unknowns for the following circuit:



Circuit with a Single Diode

Voltage Nodes: You need to write 2 equations to solve for 2 unknowns:

- One equation for the diode,
- A second equation for the voltage node.

Diode Equation:

$$I_d = 10^{-7} \left(\exp \left(\frac{V_d}{0.052} \right) - 1 \right)$$

Node Equations:

$$\left(\frac{V_d - 5}{1000} \right) + I_d = 0$$

Current Loops: You can also write the current loop equation. Again, you have 2 equations for 2 unknowns:

- One for the diode
- The second for the one loop:

Diode Equation:

$$V_d = 0.052 \cdot \ln \left(\frac{I_d}{10^{-7}} + 1 \right)$$

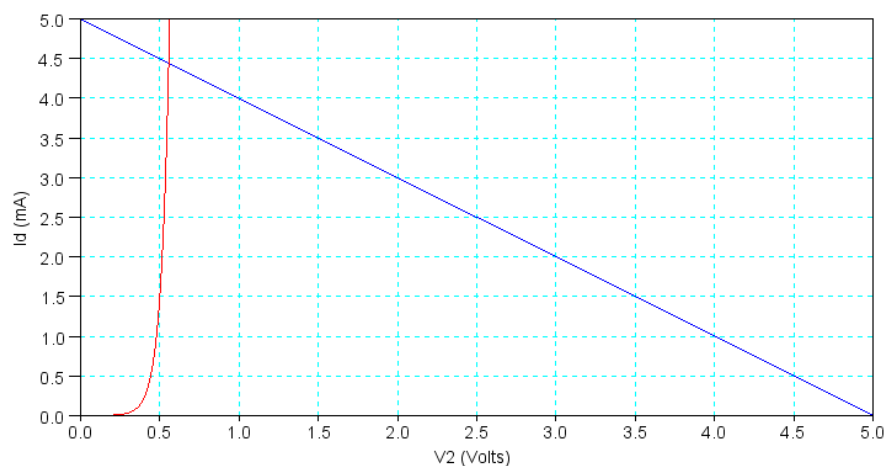
Loop Equation:

$$-5 + 1000 \cdot I_d + V_d = 0$$

Since you have only 2 unknowns, you can solve graphically. In Matlab:

```
I = [0:0.001:1]' * 0.005;
V1 = 0.052*log( 1e7 * I + 1);
V2 = 5 - 1000*I;

plot (V1,I*1000,V2,I*1000);
xlabel('V2 (Volts)')
ylabel('Id (mA)');
```



You can solve these using numerical methods. Creating a cost function in Matlab where you guess I and it returns the squared error in the voltages:

Matlab Function:

```
function [ J ] = diodel( Vd )
    Id = 1e-7 * ( exp(Vd/0.052) - 1);
    E = 5 - 1000*Id - Vd;
    J = E ^ 2;
end
```

Calling from Matlab:

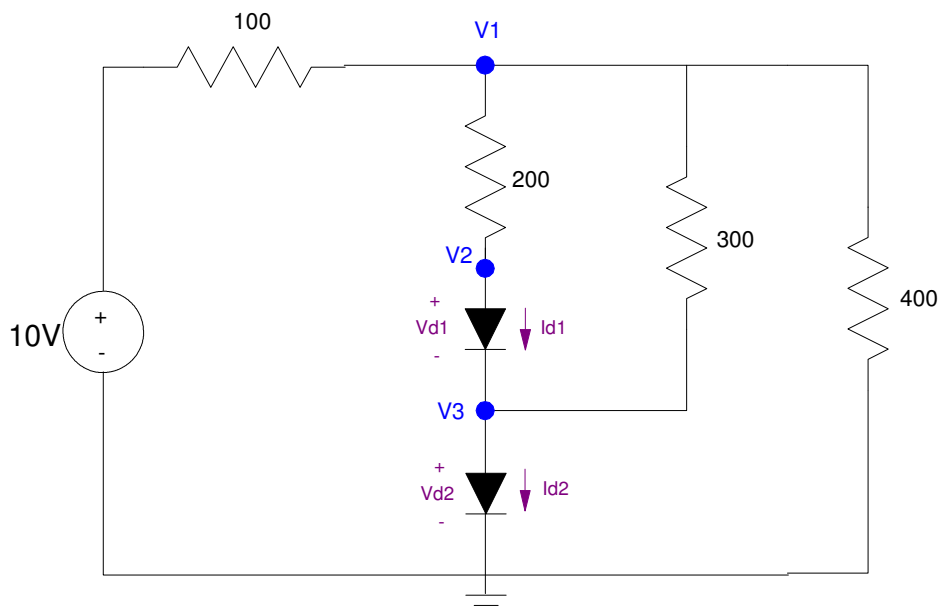
```
>> Vd = fminsearch('diodel',0.7)

Vd =    0.5565

>> Id = (5 - Vd) / 1000

Id =    0.0044
```

The solution is (0.5565V, 4.4mA)

Example 2: Circuit with Two Diodes:

Voltage Nodes for a 2-Diode Circuit. Note that $V_{d1} = (V_2 - V_3)$ and $V_{d2} = V_3$

Voltage Nodes:

With two diodes and three voltage nodes, you need to write 5 equations to solve for 5 unknowns:

Diode Equations:

$$I_{d1} = 10^{-7} \left(\exp \left(\frac{V_2 - V_3}{0.052} \right) - 1 \right)$$

$$I_{d2} = 10^{-7} \left(\exp \left(\frac{V_3 - 0}{0.052} \right) - 1 \right)$$

Node V1:

$$\left(\frac{V_1 - 10}{100} \right) + \left(\frac{V_1 - V_2}{200} \right) + \left(\frac{V_1 - V_3}{300} \right) + \left(\frac{V_1}{400} \right) = 0$$

Node V2:

$$\left(\frac{V_2 - V_1}{200} \right) + I_{d1} = 0$$

Node V3:

$$-I_{d1} + I_{d2} + \left(\frac{V_3 - V_1}{300} \right) = 0$$

To solve, graphical methods no longer work well: there are too many variables to plot. Numerical methods still work though.

First, create a cost function where you guess (V1, V2, V3)

```
function [ J ] = diode2( X )

    V1 = X(1);
    V2 = X(2);
    V3 = X(3);

    Id1 = 1e-7 * ( exp( (V2 - V3) / 0.052 ) - 1 );
    Id2 = 1e-7 * ( exp( (V3 - 0) / 0.052 ) - 1 );

    E1 = (V1 - 10)/100 + (V1 - V2)/200 + (V1 - V3)/300 + V1/400;
    E2 = (V2 - V1)/200 + Id1;
    E3 = -Id1 + Id2 + (V3 - V1)/300;

    J = E1^2 + E2^2 + E3^2;

end
```

Now solve with `fminsearch`:

```
>> V = fminsearch('diode2',[0.7,1.4,2.1])

    5.1884
    1.1950
    0.6346
```

Check: Is the sum squared error almost zero?

```
>> diode2(V)

    1.2387e-012
```

Yup - the solution is

```
V1 = 5.1884
V2 = 1.1950
V3 = 0.6346
```

