
Superposition with Phasors

ECE 211 Circuits I Lecture #31

Please visit [Bison Academy](#) for corresponding
lecture notes, homework sets, and solutions

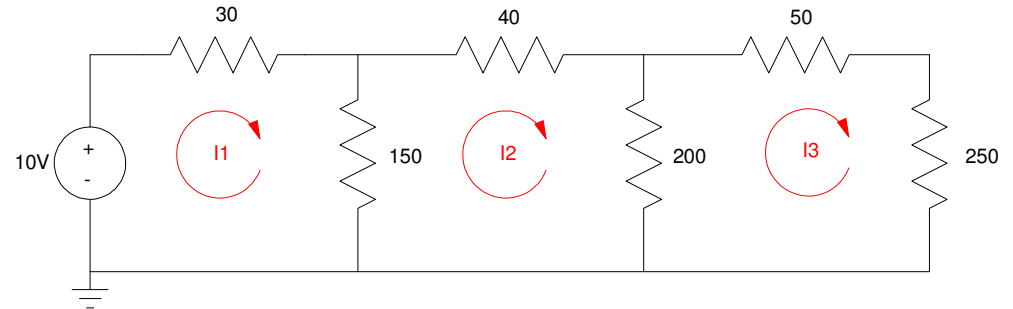
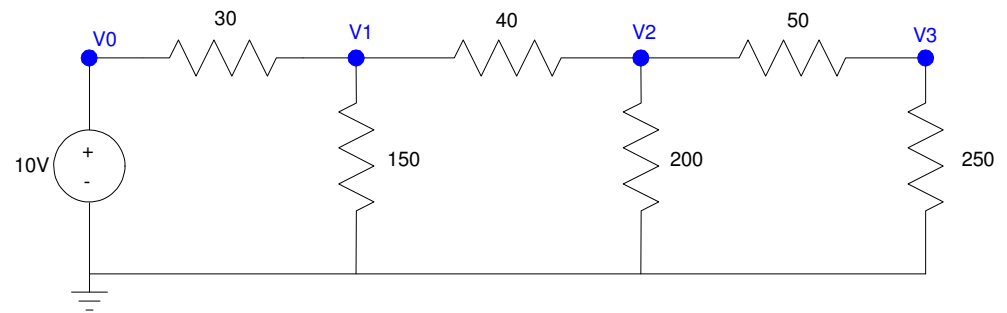
Recap

At the start of this course, we looked at circuits with constant inputs

- DC voltage and/or current sources

The resulting voltages and currents can be found using

- Real numbers,
- Voltage Nodes,
- Current Loops, and
- Thevenin equivalents



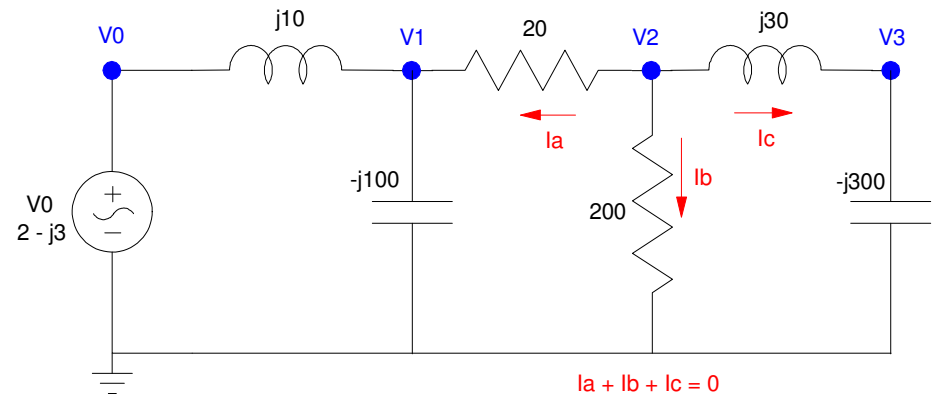
Recap (cont'd)

We then looked at circuits with sinusoidal inputs

- Voltage sources and/or current sources

The resulting voltages and currents can be found using

- Complex numbers (phasors),
- Voltage Nodes,
- Current Loops, and
- Thevenin equivalents



Problem:

How do you analyze a circuit with both?

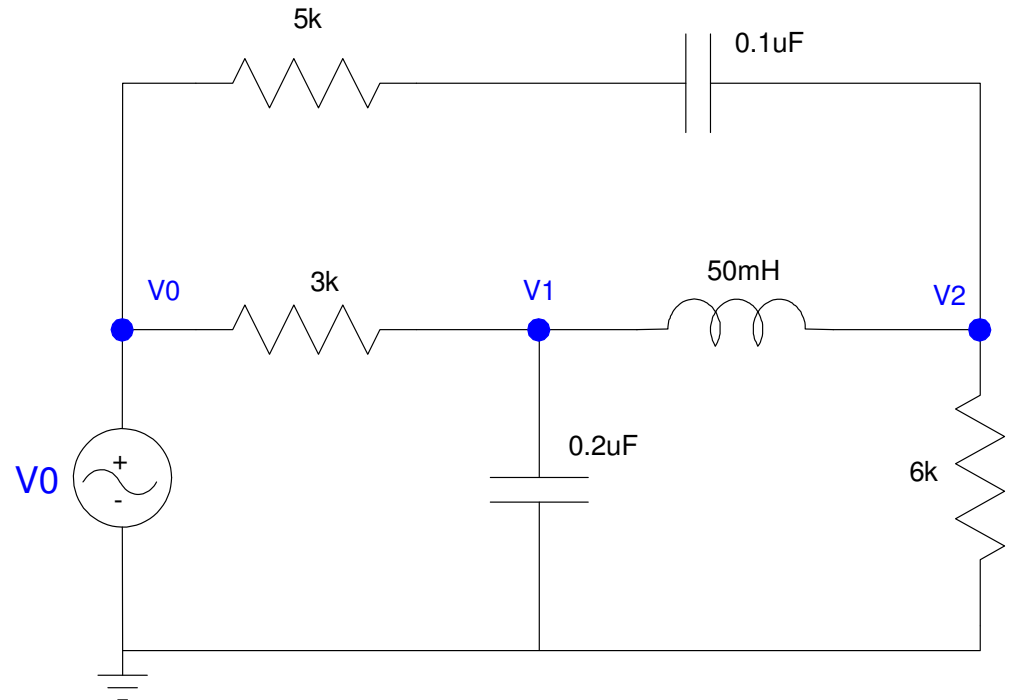
- DC source and
- AC source?

How do you analyze a circuit with

- Multiple AC sources,
- Each at a different frequency?

The answer is superposition

- Topic for this lecture



a) $V_0 = 2 + 3 \cos(200t)$

b) $V_0 = 3 \cos(200t) + 2 \sin(300t)$

Linear Systems

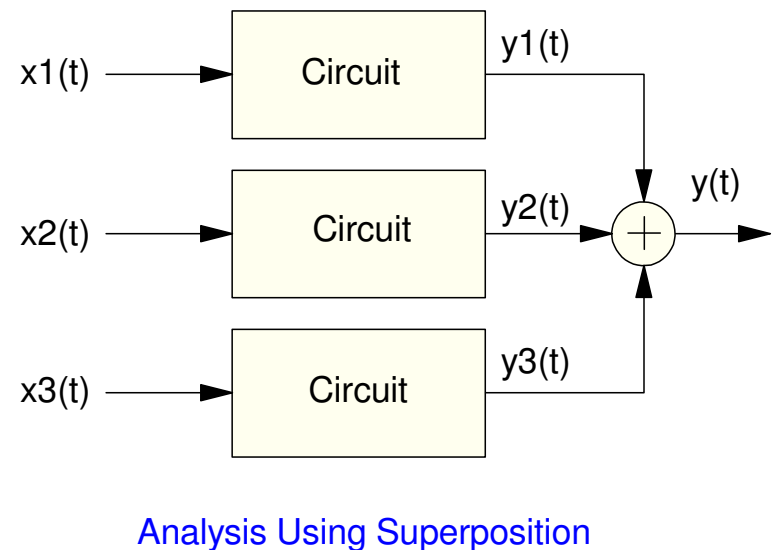
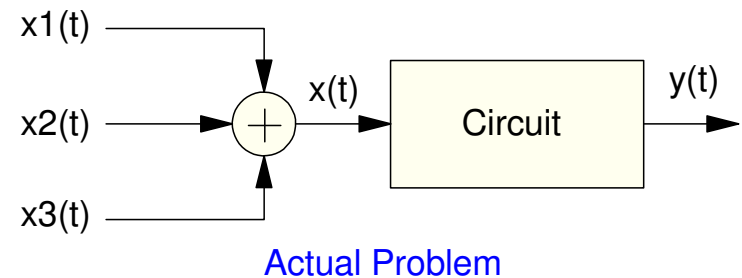
Op-Amp circuits with RLC components are linear circuits.

Linear means

$$f(x_1 + x_2 + x_3) = f(x_1) + f(x_2) + f(x_3)$$

In English, this means that

- If the input has N terms,
- You can treat this as N separate problems
- Solve each problem separately
- Add them together to get the total output



DC + AC

Example: RLC Filter

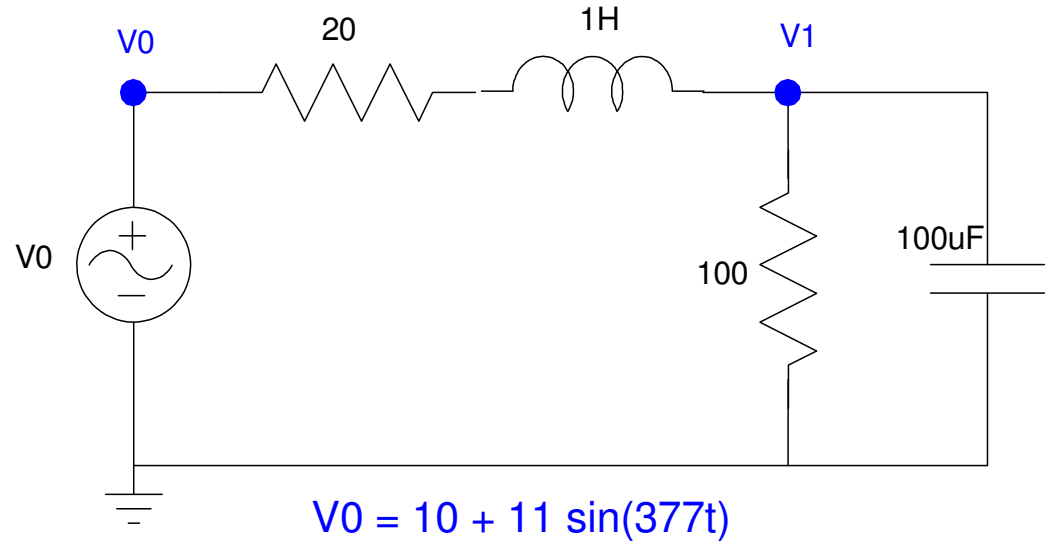
- Determining $V_1(t)$

Why?

- Similar to a AC to DC converter
- Covered in Electronics

V_1 represents

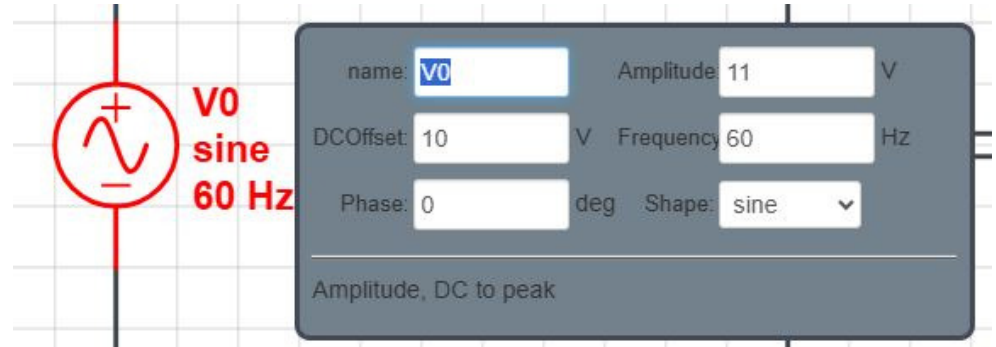
- The DC voltage generated
- With some ripple (AC)



CircuitLab

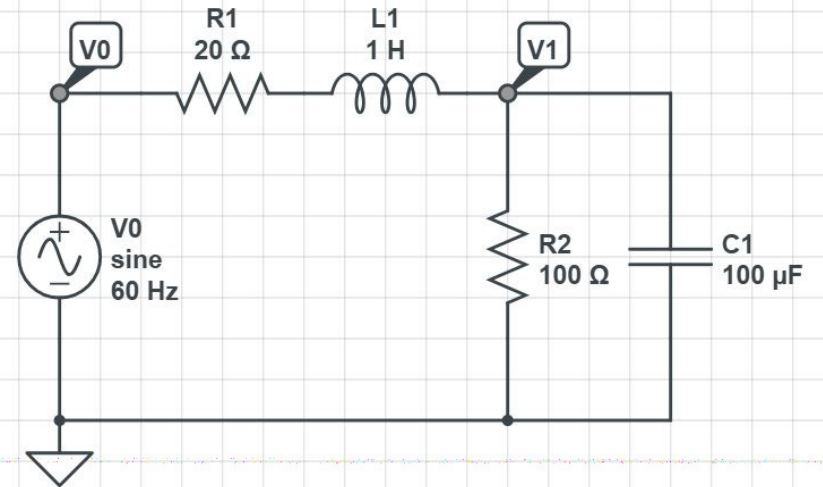
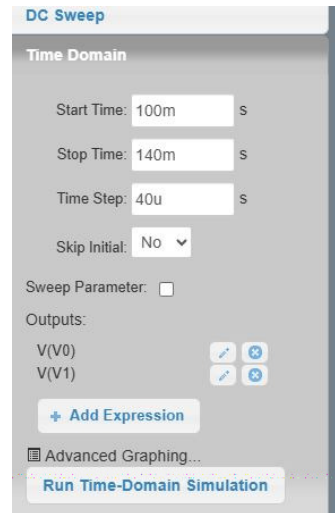
Set V0 to

- 10V Offset (DC)
- 11V Amplitude (AC)
- 60Hz (377 rad/sec)



Time-Domain Simulation

- Start at 100ms
 - Let the transient die out
- Run for 40ms
 - 3 cycles @ 60Hz
- Step size 40us
 - 1000x smaller than run time
 - 1000 points in the plot



CircuitLab Result

V1(t) (orange line)

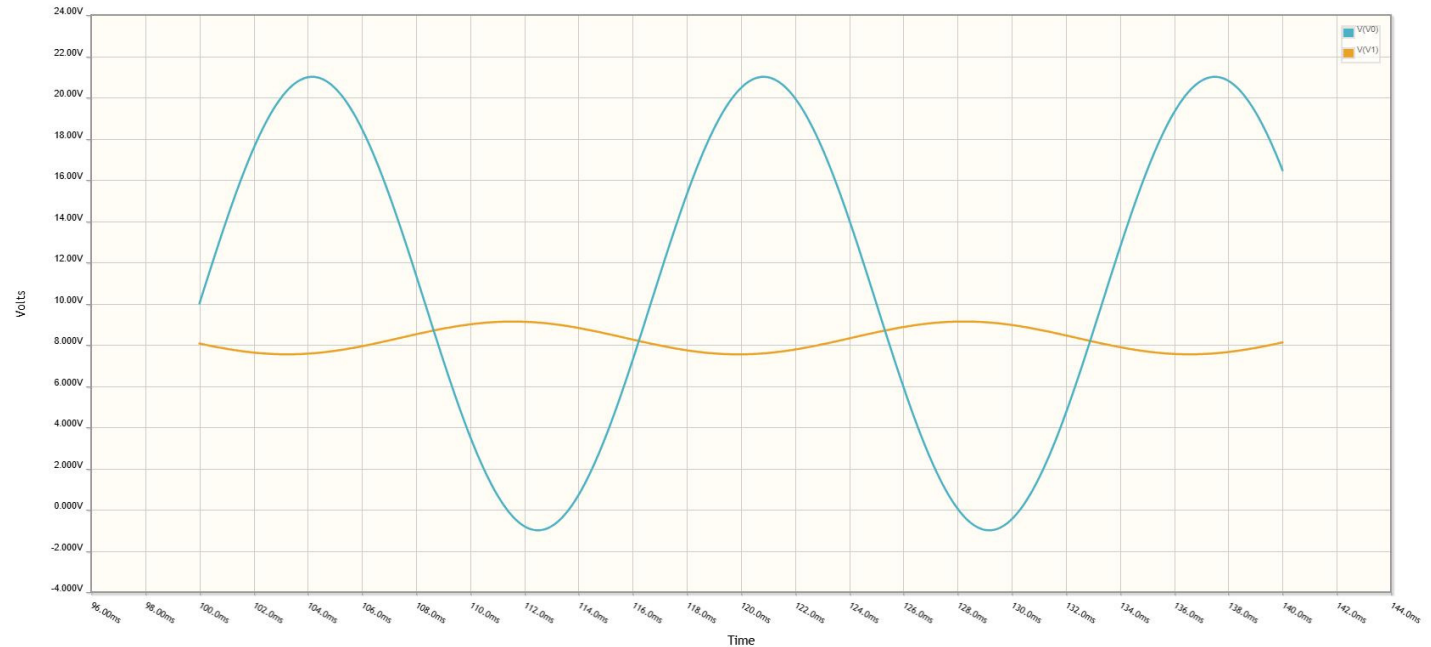
- max = 9.127V
- min = 7.536V

DC term

- 8.332V
- $(\text{max} + \text{min}) / 2$

AC term

- 1.591V_{pp}
- $(\text{max} - \text{min})$
- 795.5mV_p
- $V_{pp} / 2$
- 562.5mV_{rms}
- $V_p / \sqrt{2}$



Analyze at $\omega = 0$ (DC)

$$V_0(t) = 10$$

Phasor analysis at $\omega = 0$

- $L \rightarrow j\omega L = 0$
- $C \rightarrow \frac{1}{j\omega C} = \infty$

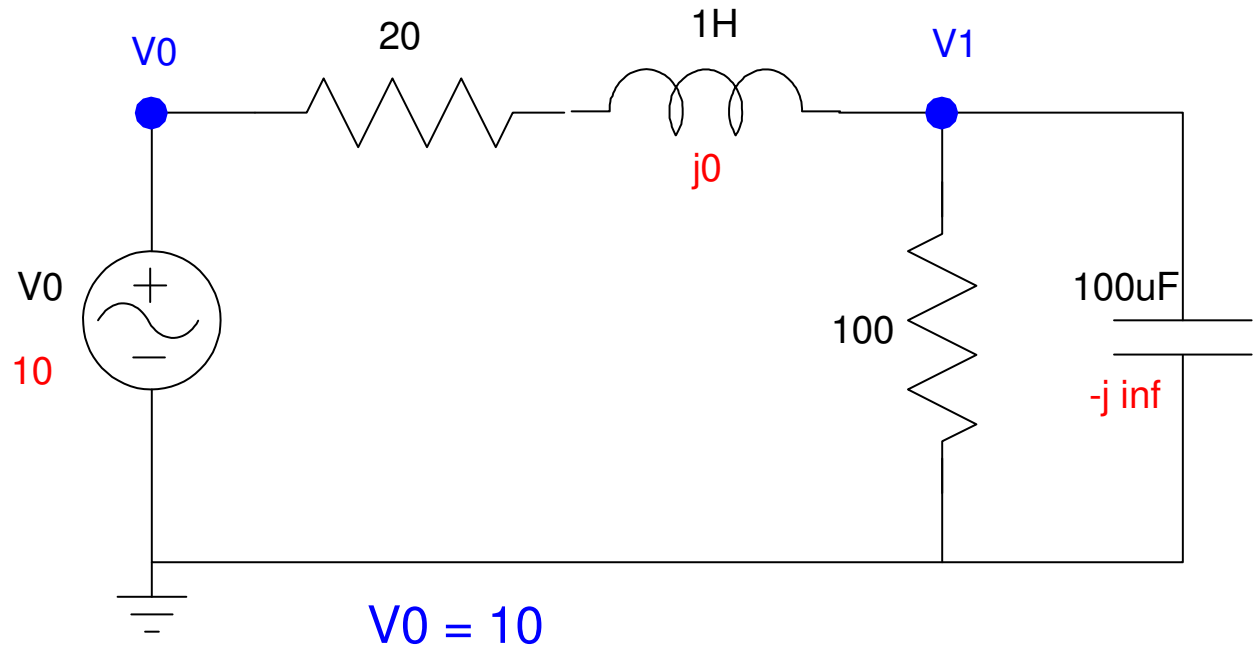
By voltage division

$$V_1 = \left(\frac{100}{100+20} \right) 10$$

$$V_1 = 8.333V$$

Same as CircuitLab

- DC term for V_1



Analyze at $\omega = 377$ rad/sec

$$V_0(t) = 11 \sin(377t)$$

At this frequency

- $\omega = 377$
- $V_0 = 0 - j11$
- $L \rightarrow j\omega L = j377$
- $C \rightarrow \frac{1}{j\omega C} = -j26.5$

Find V_1

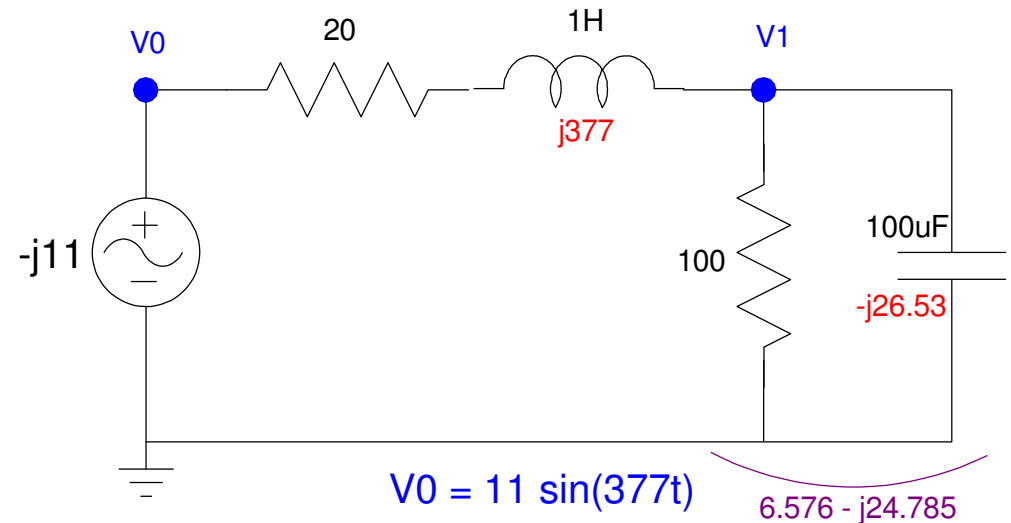
$$(100) \parallel (-j26.54) = 6.576 - j24.785$$

$$V_1 = \left(\frac{(6.576 - j24.785)}{(6.576 - j24.785) + (20 + j377)} \right) \cdot (0 - j11)$$

$$V_1 = -0.241 + j0.763$$

$$|V_1| = 0.800V$$

- Matches CircuitLab
- $V_{1p} = 795.5\text{mV}$



Total Answer

DC:

$$V_1 = 8.333$$

$$v_1(t) = 8.333$$

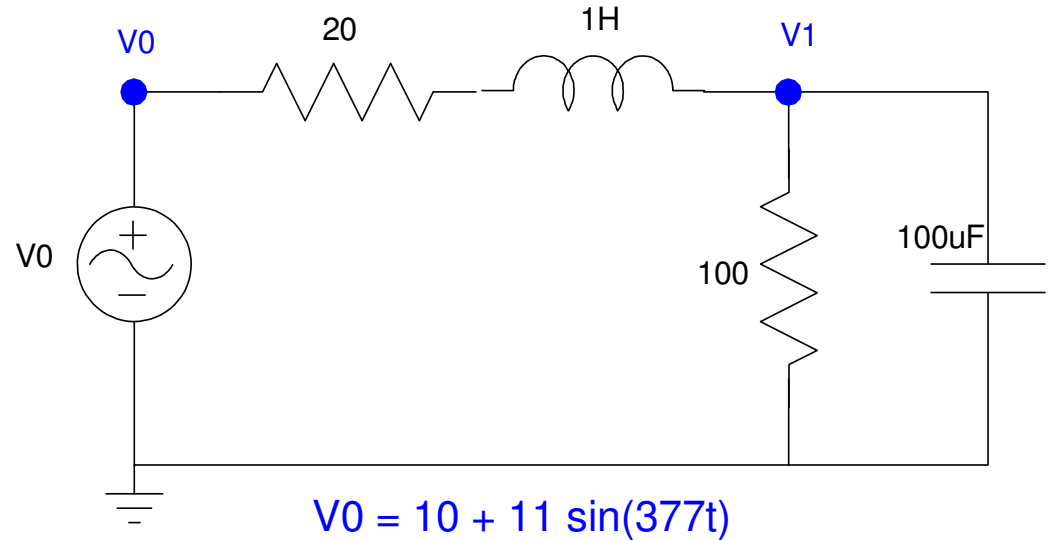
AC:

$$V_1 = -0.241 + j0.763$$

$$v_1(t) = -0.241 \cos(377t) - 0.763 \sin(377t)$$

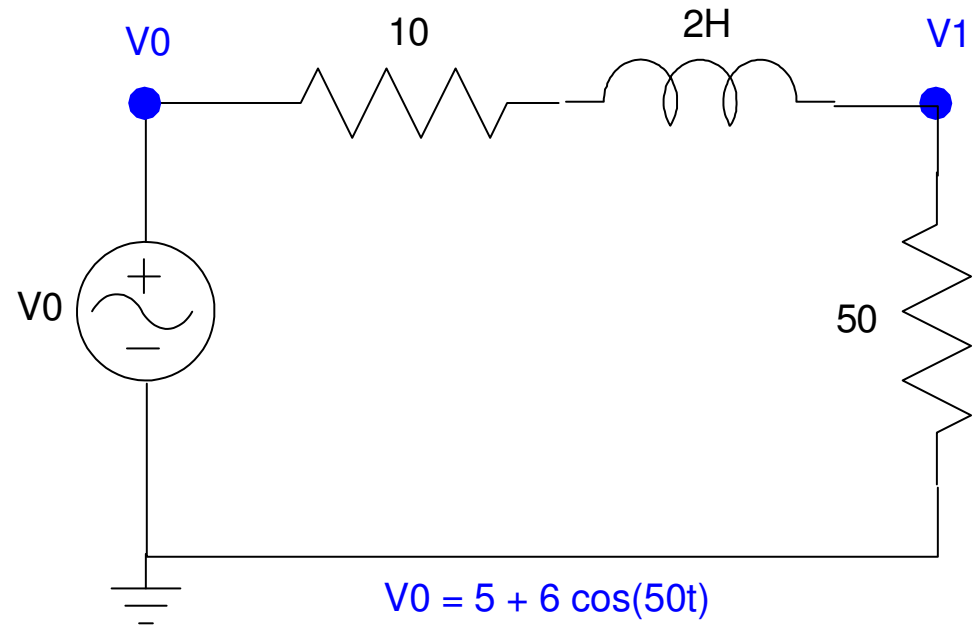
Total = DC + AC

$$v_1(t) = 8.333 - 0.241 \cos(377t) - 0.763 \sin(377t)$$

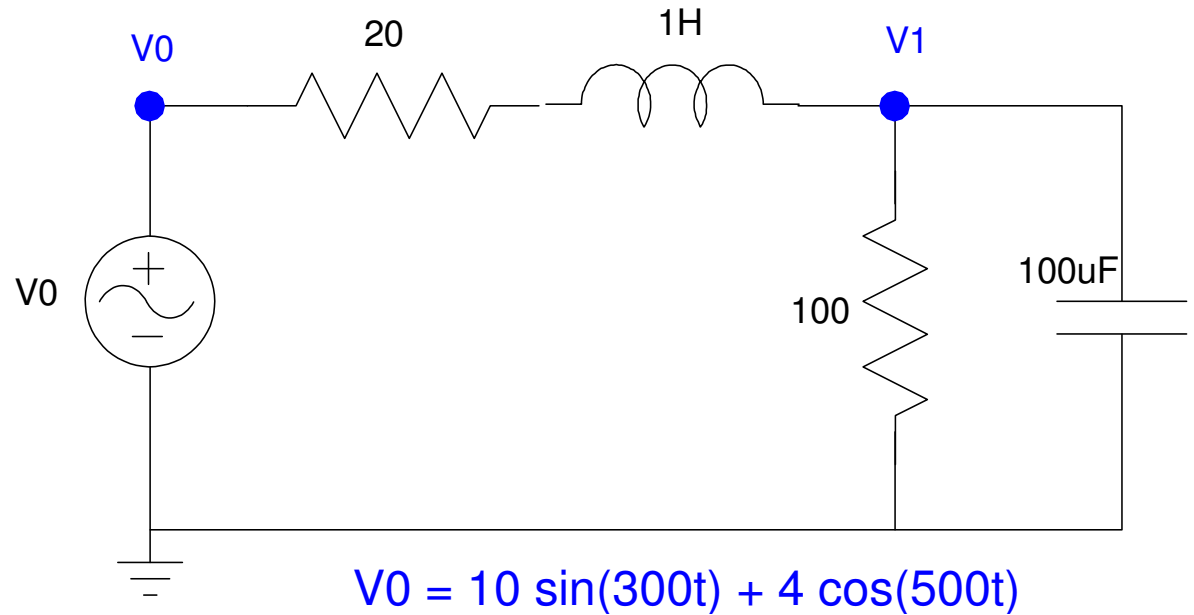


Practice Problem

Find $v_1(t)$



What if $V_1(t)$ has two AC terms?

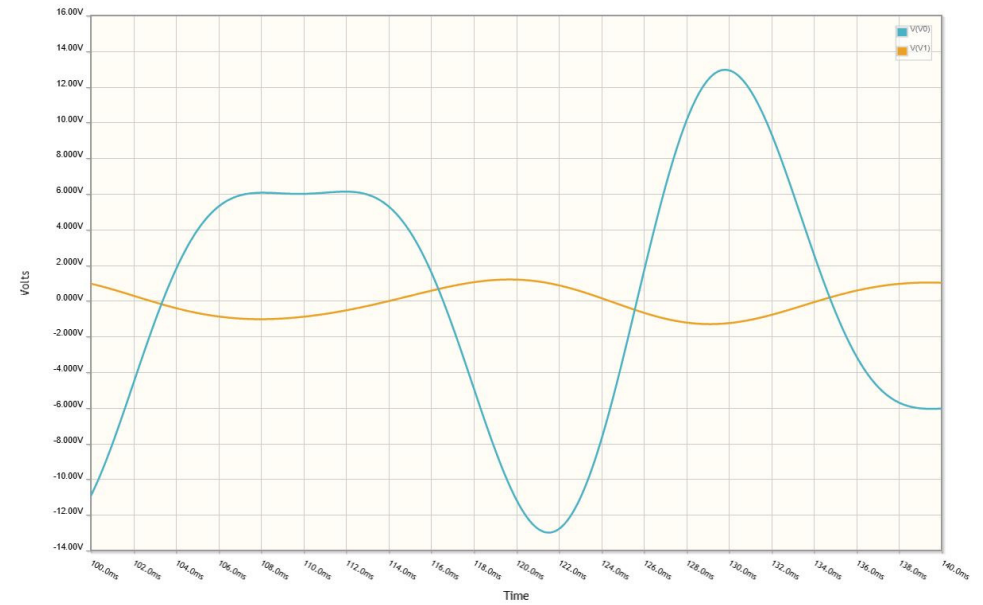
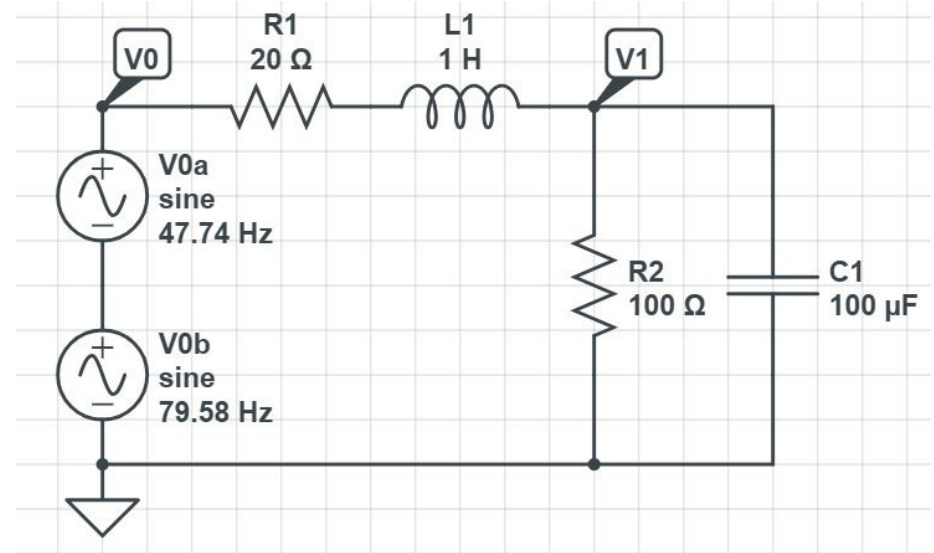


CircuitLab

Use two inputs in series

Run a transient simulation

- Somewhat erratic signal



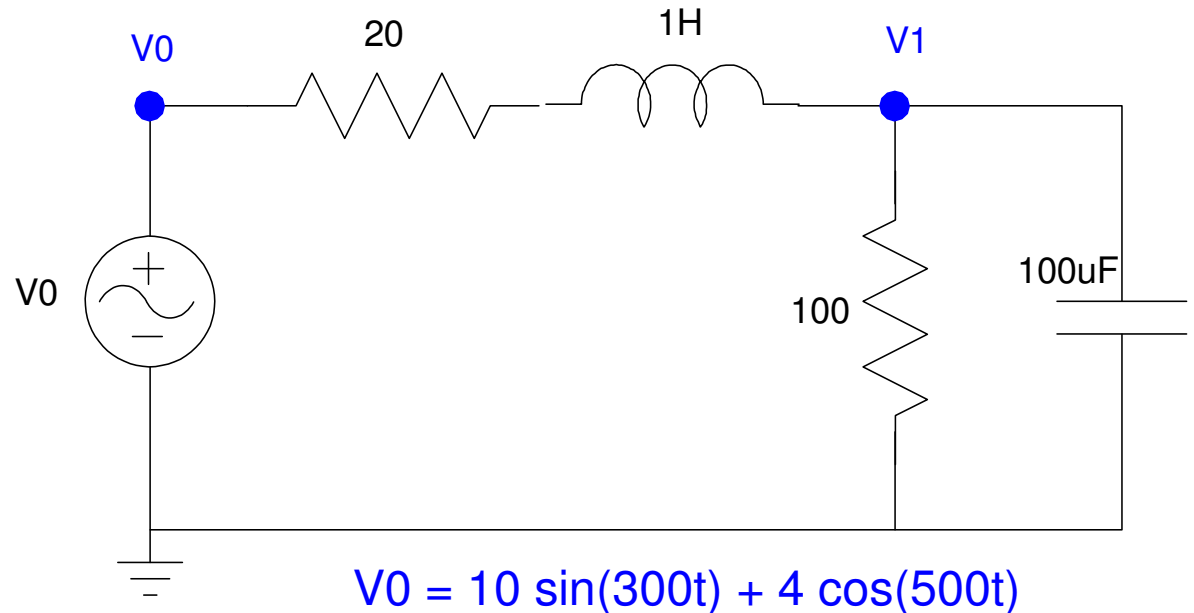
Analysis

Treat as two separate problems

- $V_0 = 10 \sin(300t)$
- $V_0 = 4 \cos(500t)$

Find V_1 for each case

The total answer is the sum of the two



First frequency

$$V_0 = 10 \sin(300t)$$

- $\omega = 300$
- $V_0 = 0 - j10$
- $L \rightarrow j\omega L = j300$
- $C \rightarrow \frac{1}{j\omega C} = -j33.33$

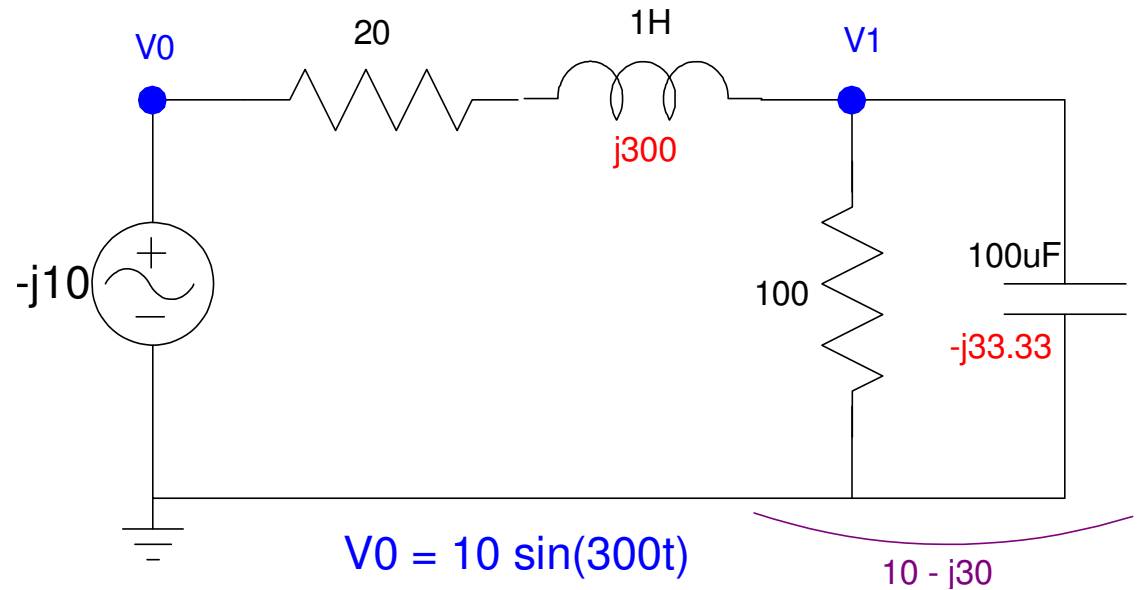
By voltage division

$$V_1 = \left(\frac{(10 - j30)}{(10 - j30) + (20 + j300)} \right) (-j10)$$

$$V_1 = -0.4878 + j1.0568$$

meaning

$$v_1(t) = -0.4878 \cos(300t) - 1.0569 \sin(300t)$$



Second Frequency

$$V_0 = 4 \cos(500t)$$

- $\omega = 500$
- $V_0 = 4 - j0$
- $L \rightarrow j\omega L = j500$
- $C \rightarrow \frac{1}{j\omega C} = -j20$

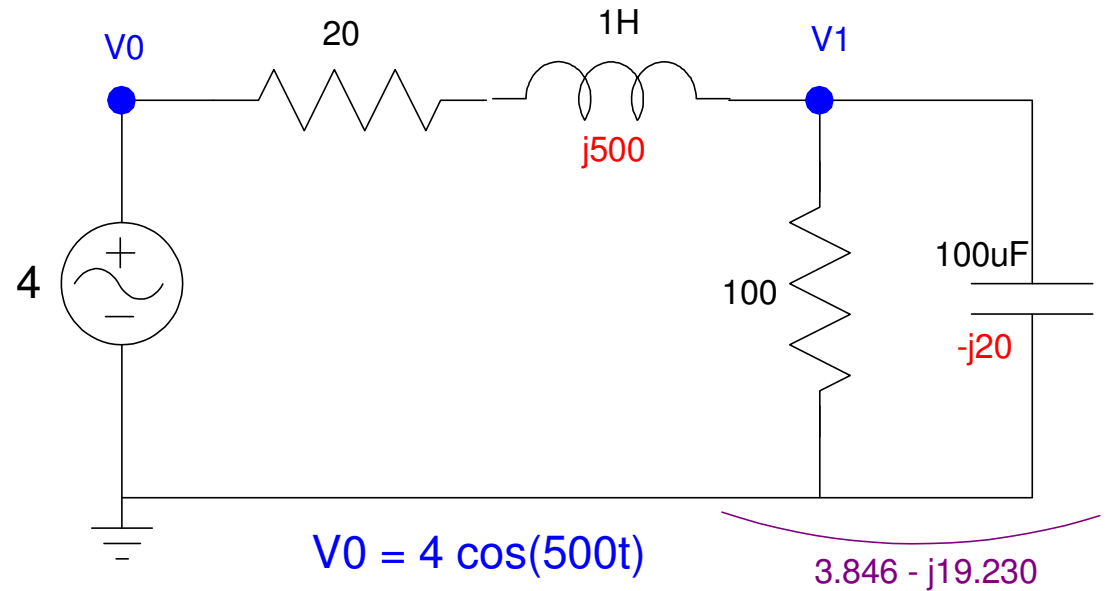
Find V_1

$$V_1 = \left(\frac{(3.846 - j19.23)}{(3.846 - j19.23) + (20 + j300)} \right) (4)$$

$$V_1 = -0.1580 - j0.0398$$

meaning

$$v_1(t) = -0.1580 \cos(500t) + 0.0398 \sin(500t)$$



Total Answer

From superposition, $v_1(t)$ is the sum of these two answers

300 rad/sec

$$v_1(t) = -0.4878 \cos(300t) - 1.0569 \sin(300t)$$

500 rad/sec

$$v_1(t) = -0.1580 \cos(500t) + 0.0398 \sin(500t)$$

Total Answer

$$v_1(t) = -0.4878 \cos(300t) - 1.0569 \sin(300t) - 0.1580 \cos(500t) + 0.0398 \sin(500t)$$

This is the function that CircuitLab plotted for $V_1(t)$

- Orange line
-

Common Mistake:

A common mistake is to try to simplify

At 300 rad/sec

$$V_1 = -0.4878 + j1.0568$$

At 500 rad/sec

$$V_1 = -0.1580 - j0.0398$$

Add the two

$$V_1 = -0.6458 + j1.0170$$

You can't do that

- Each phasor represents a different frequency
- $\cos(300t) + \cos(500t)$ doesn't simplify

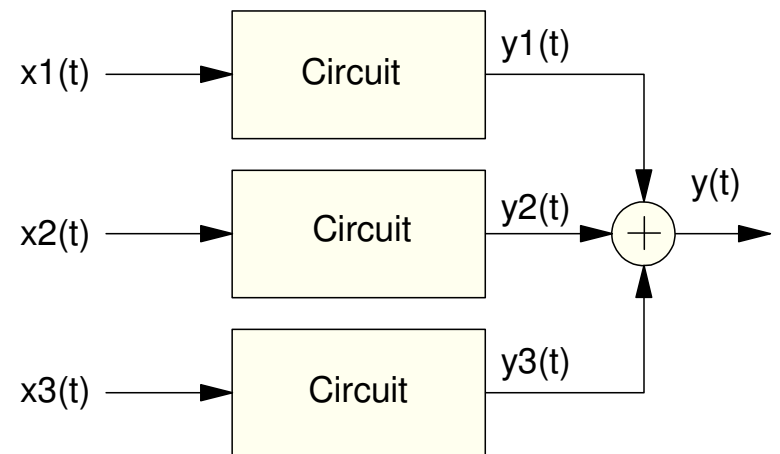
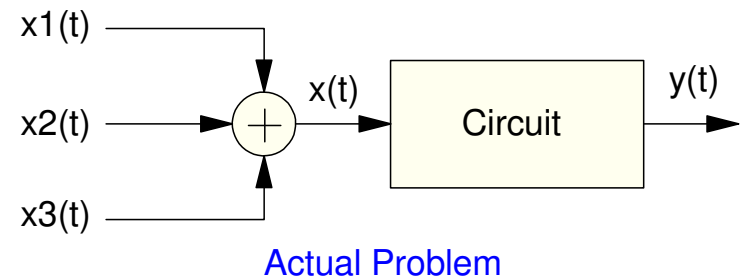
Summary

Superposition uses the property of linearity

$$f(x_1 + x_2 + x_3) = f(x_1) + f(x_2) + f(x_3)$$

If a circuit has an input containing two different frequencies, treat it as two separate problems

- Find the voltages and currents for each frequency separately
- Add the two together to get the total output



Analysis Using Superposition