
Active Filters

ECE 211: Circuits I

Lecture #29

Please visit [Bison Academy](#) for corresponding
lecture notes, homework sets, and solutions

Background:

Up to now, we've been analyzing RLC circuits

- With constant inputs, and
- With a sinusoidal input

If you have an RLC circuit with a specific input this works

- Phasor analysis

What if the frequency of the input varies?

Active Filters

A filter is any circuit whose behaviour changes with frequency

- Any circuit with capacitors and/or inductors
- An active filter is an RLC circuit which uses an op-amp

Op-amp circuits are usually used for filters

- They allow the gain to be > 1
- They allow you to avoid using inductors

Element	Phasor Impedance
R	R
L	$j\omega L$
C	$1 / j\omega C$

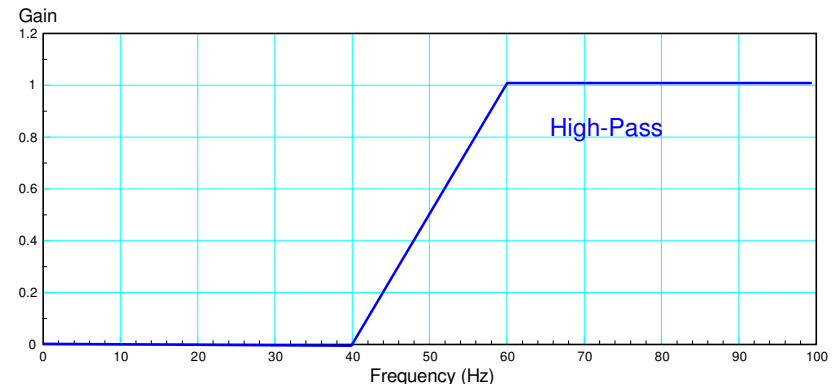
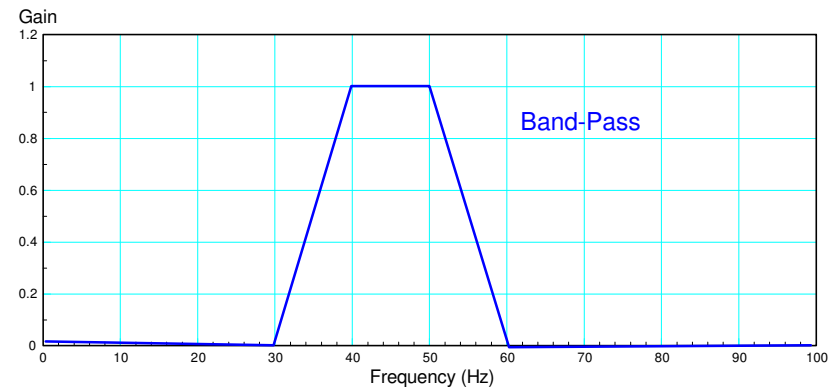
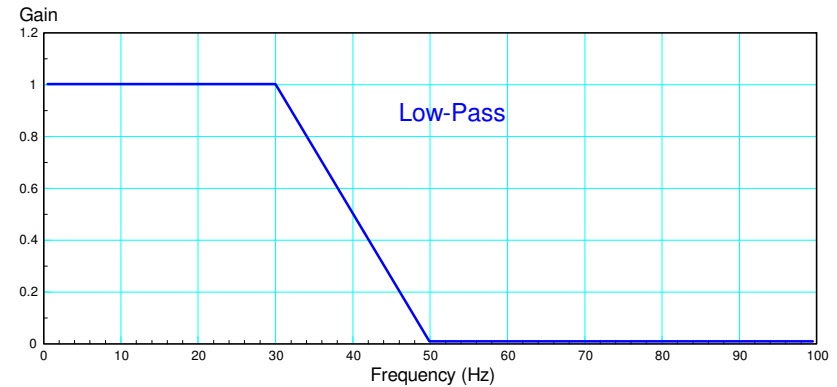
Code Plots

If the frequency of the input is unknown or varies, the gain can be plotted vs. frequency

- The result is called a *Bode Plot*

Bode plots describe how a filter behaves

- Low-Pass:
 - Pass low frequencies
 - Reject high frequencies
- Band-Pass:
 - Pass freq over a range
 - Reject others
- High-Pass:
 - Pass high frequencies
 - Reject low frequencies



This Lecture

In this lecture, we'll look at five active filters

- Bass-boost
- 1st-order low-pass filter
- 2nd-order low-pass filter
- Bandpass filter

Circuits II and Electronics covers filter design in more detail.

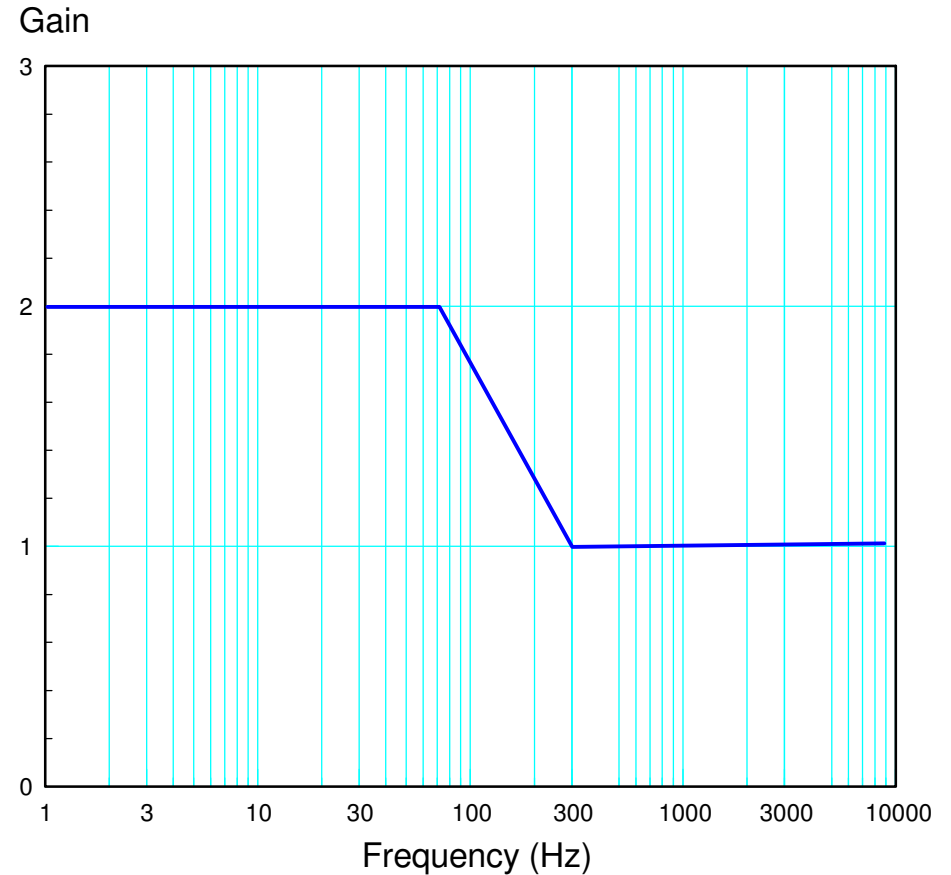
Bass-Boost Filter

Design a filter where

- Gain = 2
 - $f \ll 100\text{Hz}$
- Gain = 1
 - $f \gg 100\text{Hz}$

Purpose:

- Amplify the bass of a song



Bass-Boost

- Calculations

The gain will be

$$gain = -\left(\frac{Z_1}{Z_2}\right)$$

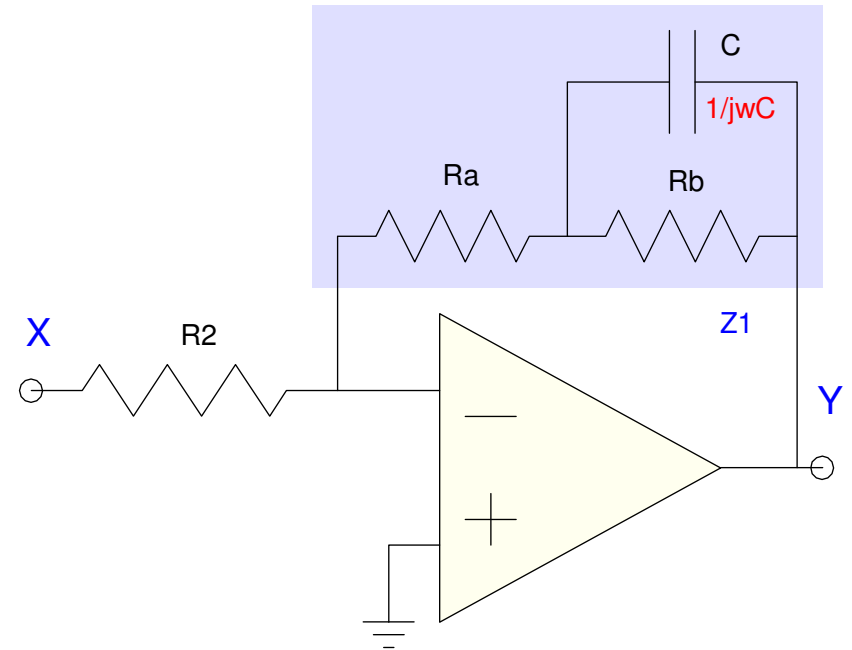
Z_1 is the impedance of R_a , R_b , and C

$$Z_1 = R_a + R_b \parallel \frac{1}{j\omega C}$$

$$Z_1 = R_a + \left(\frac{R_b}{1+j\omega R_b C}\right)$$

The overall gain is then

$$G = -\left(\frac{R_a + R_b + j\omega C R_a R_b}{R_2(1+j\omega R_b C)}\right)$$

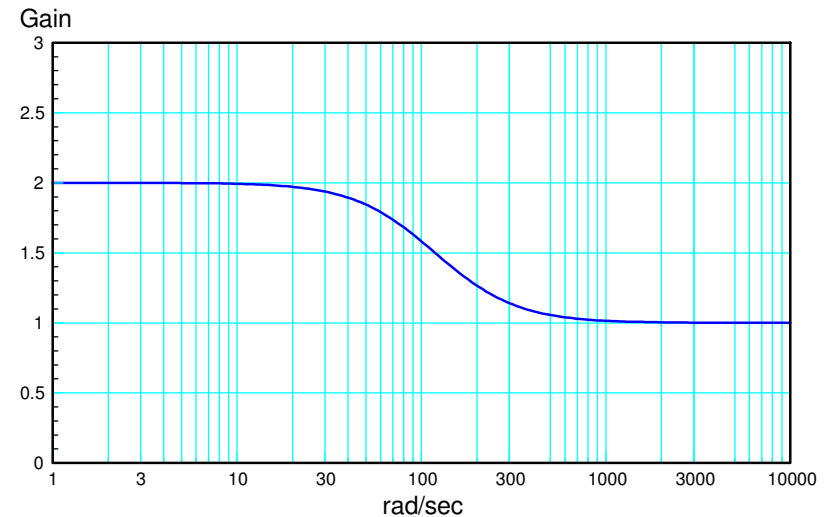
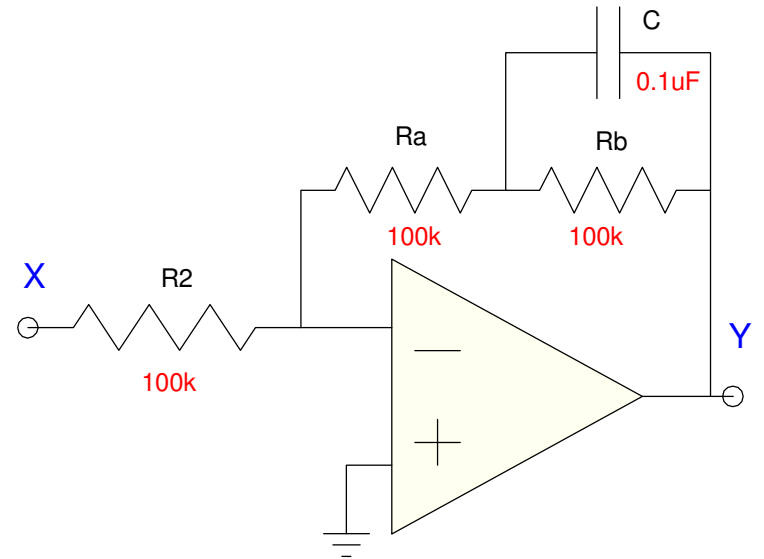


Bass-Boost: Analysis

$$G = -\left(\frac{R_a + R_b + j\omega C R_a R_b}{R_2(1 + j\omega R_b C)}\right)$$

Plot the gain vs. frequency

```
w = logspace(0, 4, 200)';  
Ra = 1e5;  
Rb = 1e5;  
R2 = 1e5;  
C = 0.1e-6;  
num = Ra + Rb + j*w*C*Ra*Rb;  
den = R2*(1 + j*w*C*Rb);  
G = -num ./ den;  
semilogx(w, abs(G))  
xlabel('rad/sec')  
ylabel('Gain')
```

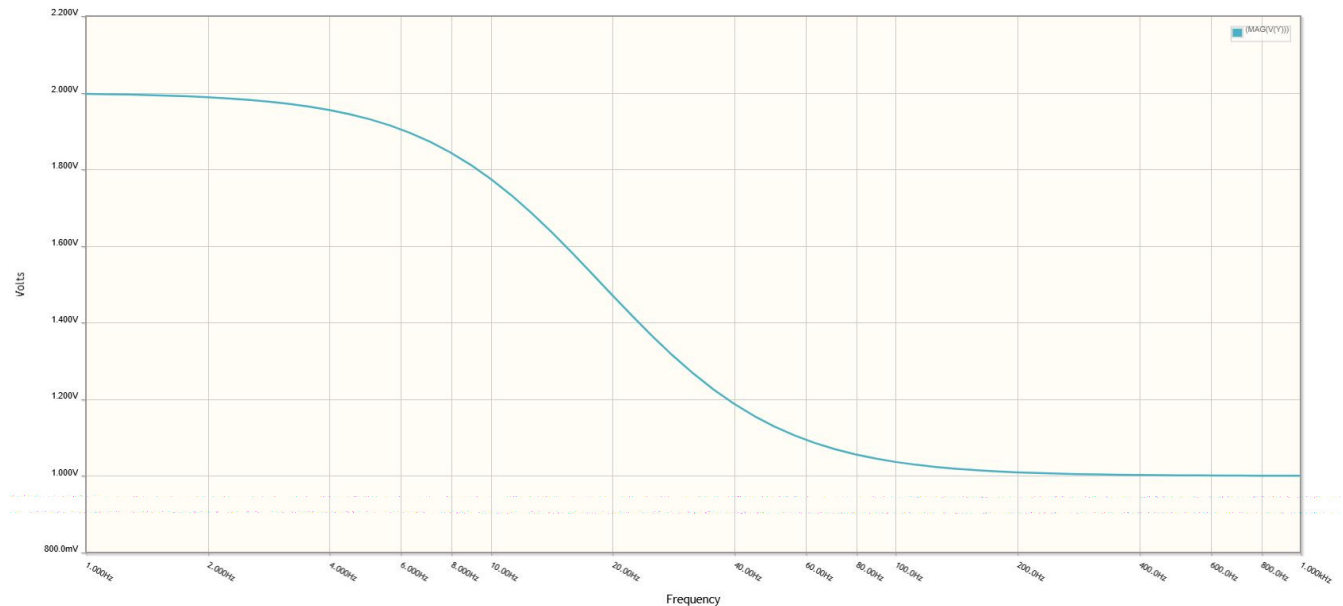
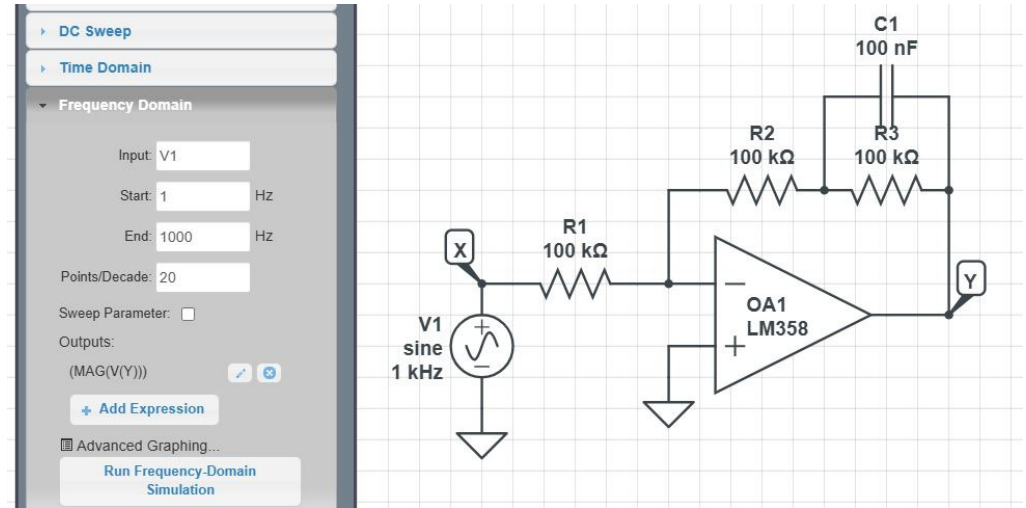


CircuitLab Check

Frequency Domain

- Set the input to V1
- Sweep from 1Hz to 1000Hz
- 20 points per decade

Matches calculations



Bass-Boost: Design

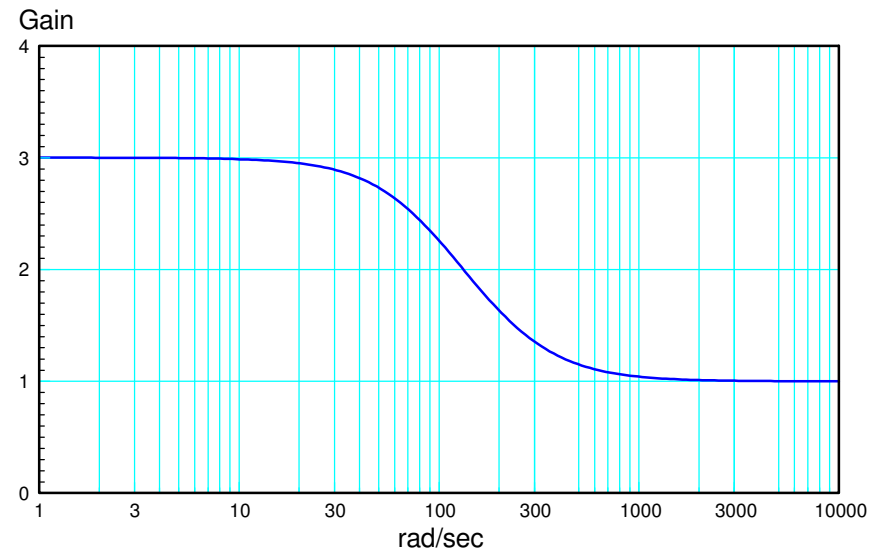
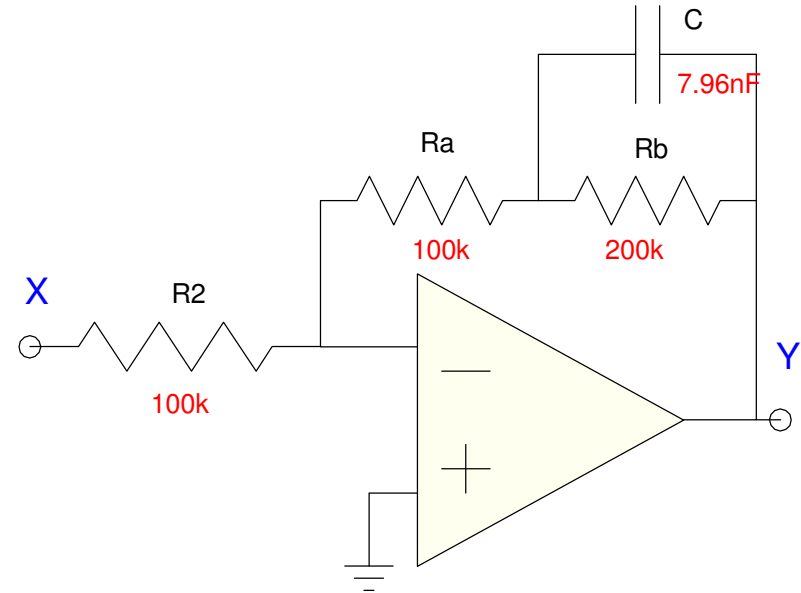
Design a circuit with

- DC gain = 3
- High-freq gain = 1
- Corner = 100Hz

Solution:

- $R_2 = 100k$
- $R_a = 100k$ (hf gain = 1)
- $R_b = 200k$ (DC gain = 3)
- $C = 7.76nF$

$$\omega R_b C = 1 \quad @ \quad 100Hz$$

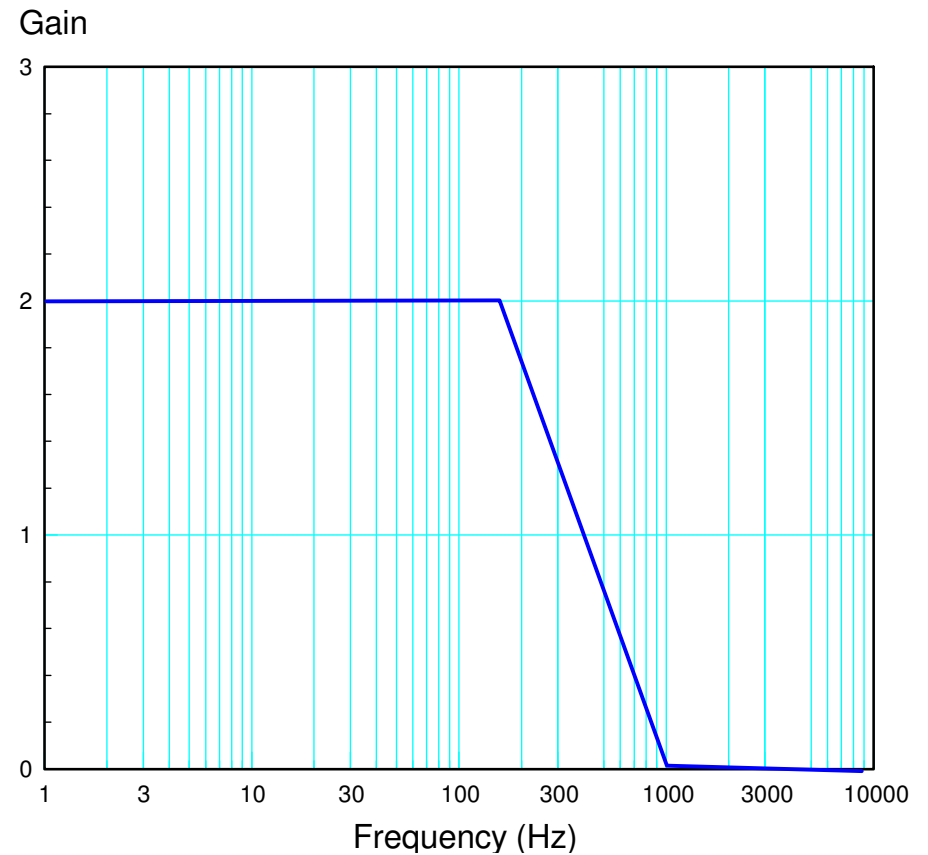


1st-Order Low-Pass Filter

- Pass low frequencies
- Reject high frequencies
- Corner = 250Hz
- DC gain = 2.00

Purpose

- Subwoofer crossover
- Remove noise from a signal



1st-Order LPF: Calculations

$$V_1 = V_p = V_m = Y$$

Non-inverting amplifier

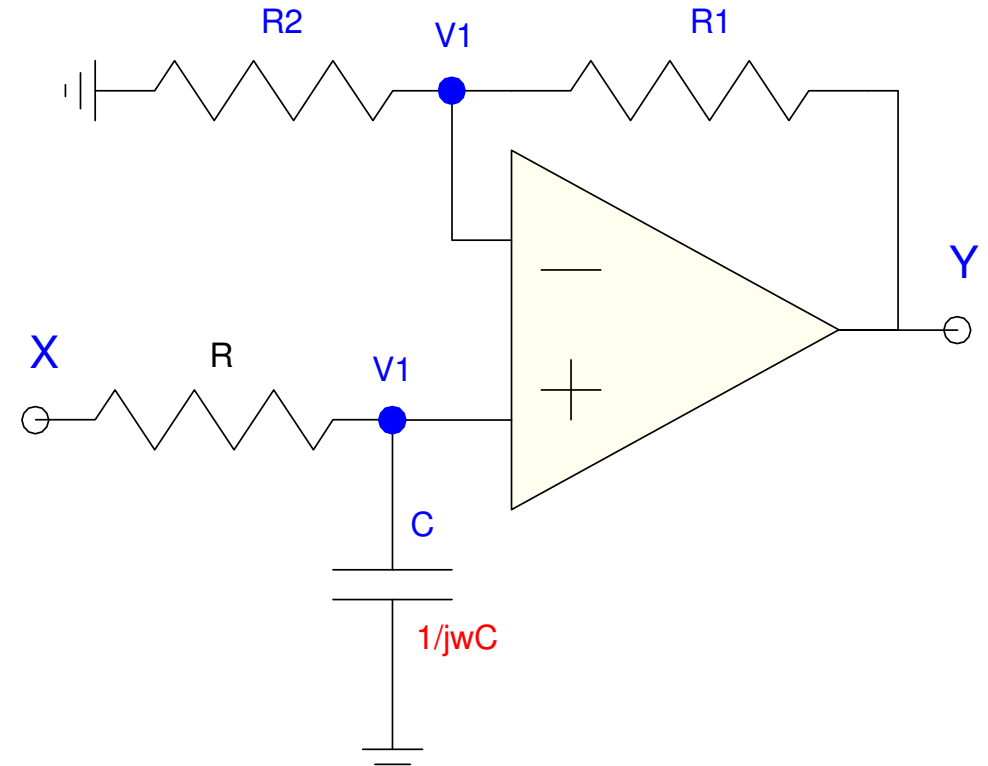
$$Y = \left(1 + \frac{R_1}{R_2}\right) V_1 = k V_1$$

Voltage Nodes

$$\left(\frac{V_1 - X}{R}\right) + \left(\frac{V_1}{1/j\omega C}\right) = 0$$

$$V_1 = \left(\frac{1}{j\omega RC + 1}\right) X$$

$$Y = \left(\frac{k}{j\omega RC + 1}\right) X$$



Analysis

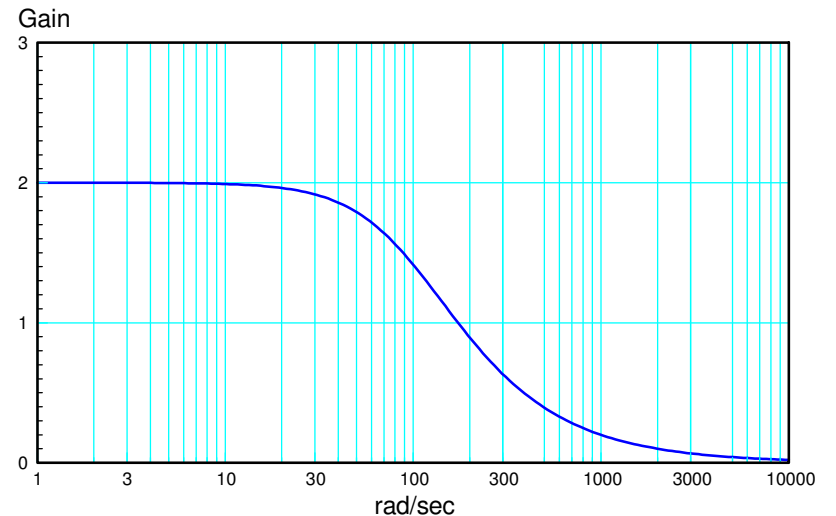
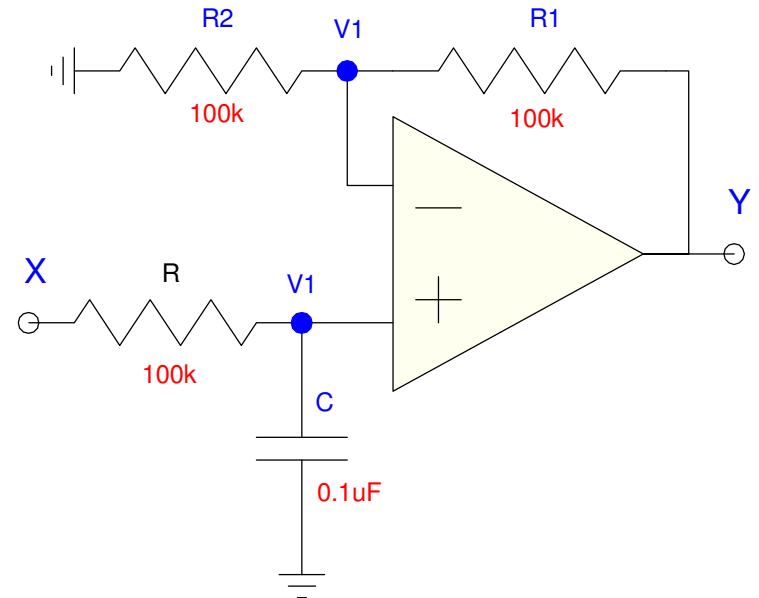
Plot the gain vs. frequency

$$G = \left(\frac{k}{j\omega RC + 1} \right)$$

$$k = 1 + \frac{R_1}{R_2}$$

In Matlab

```
w = logspace(0, 4, 200)';  
R1 = 1e5;  
R2 = 1e5;  
R = 1e5;  
C = 0.1e-6;  
k = 1 + R1/R2;  
G = k ./ (1 + j*w*R*C);  
semilogx(w, abs(G))  
xlabel('rad/sec')  
ylabel('Gain')
```

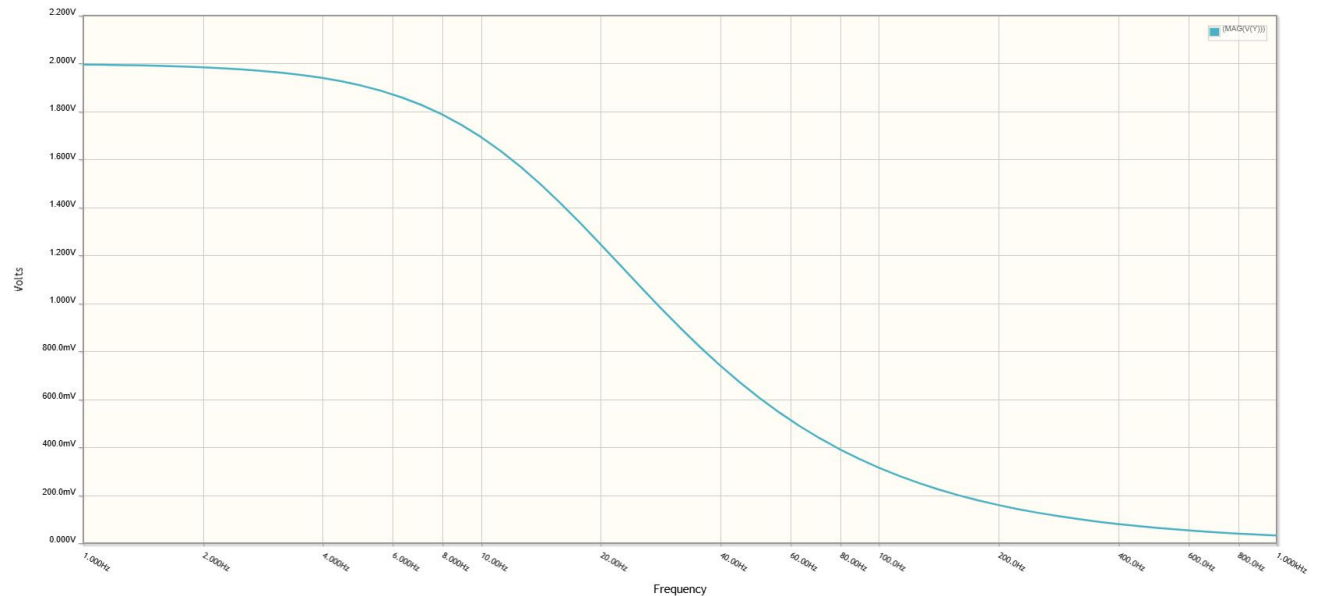
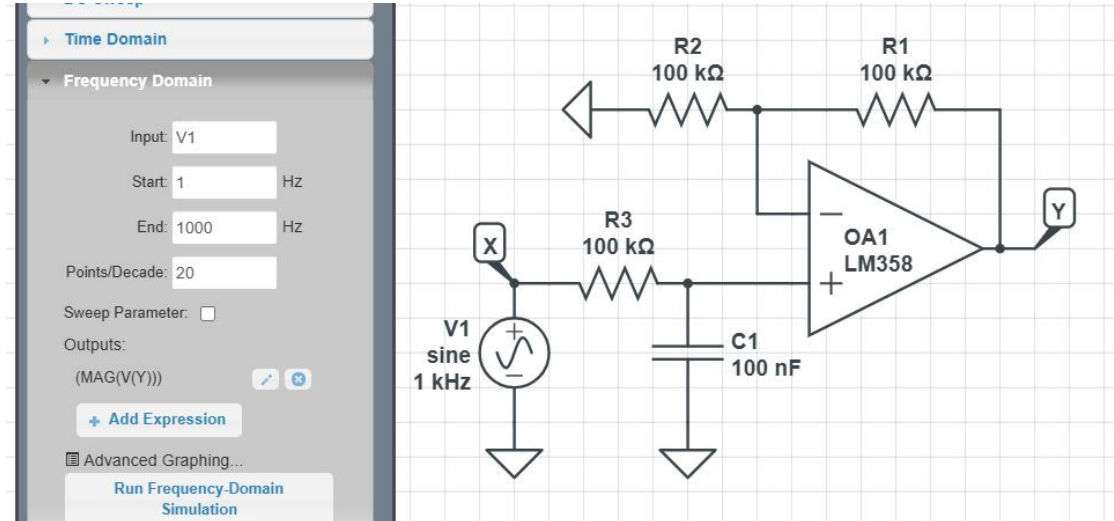


CircuitLab Check

Frequency Domain

- Set the input to V1
- Sweep from 1Hz to 1000Hz
- 20 points per decade

Matches calculations

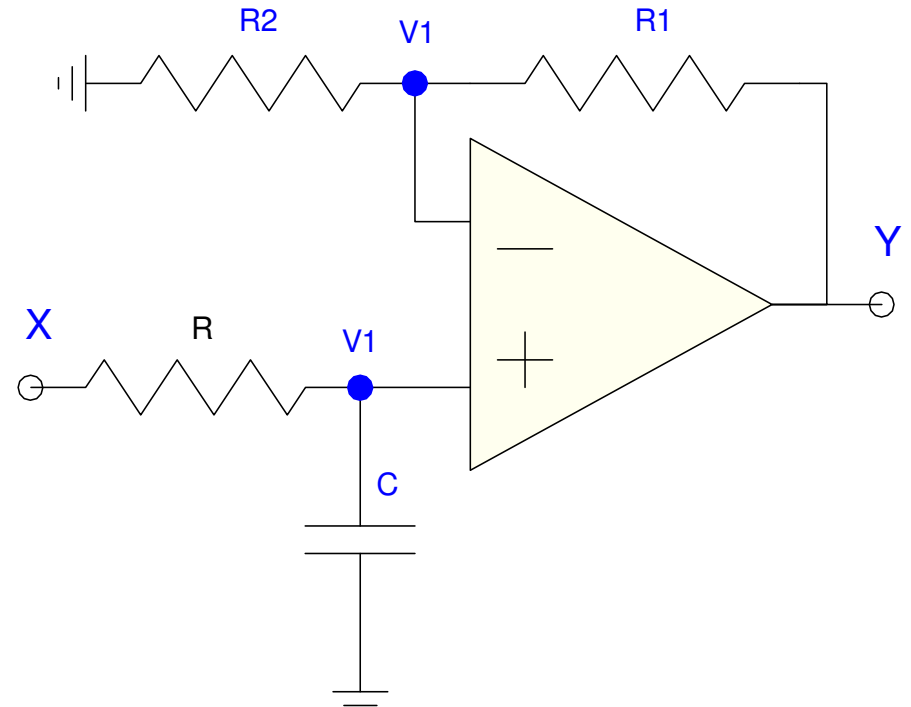


Design

$$G = \left(\frac{k}{j\omega RC + 1} \right)$$

Design a filter

- DC gain = 3
- Corner = 100Hz (628 rad/sec)

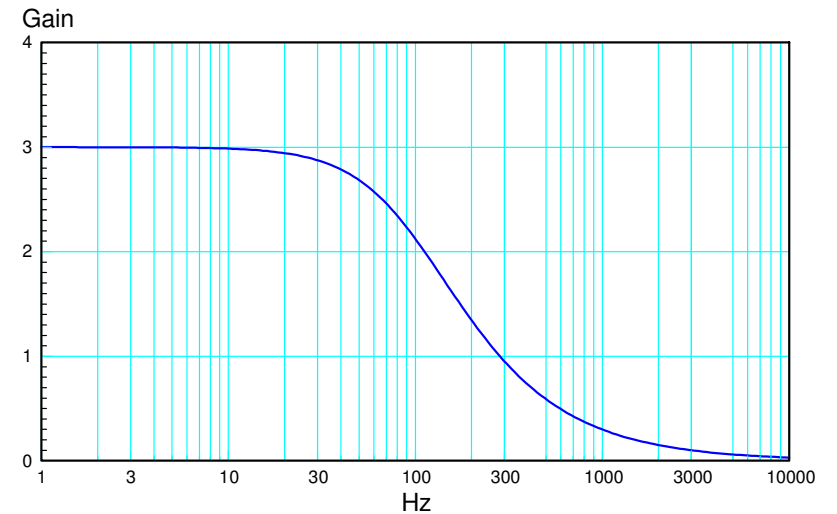
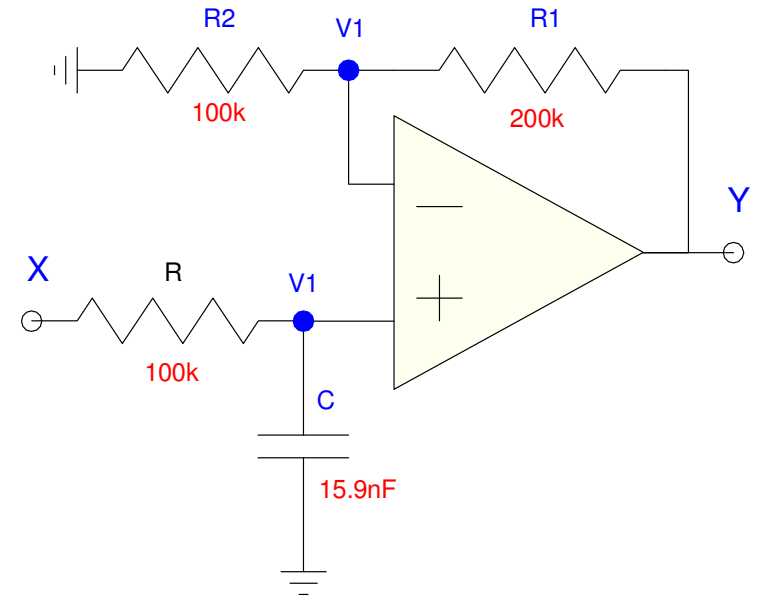


Design

Solution:

- $R2 = 100k$ (arbitrary)
- $R1 = 200k$ (gain = 3)
- $R = 100k$ (arbitrary)
- $C = 15.9nF$

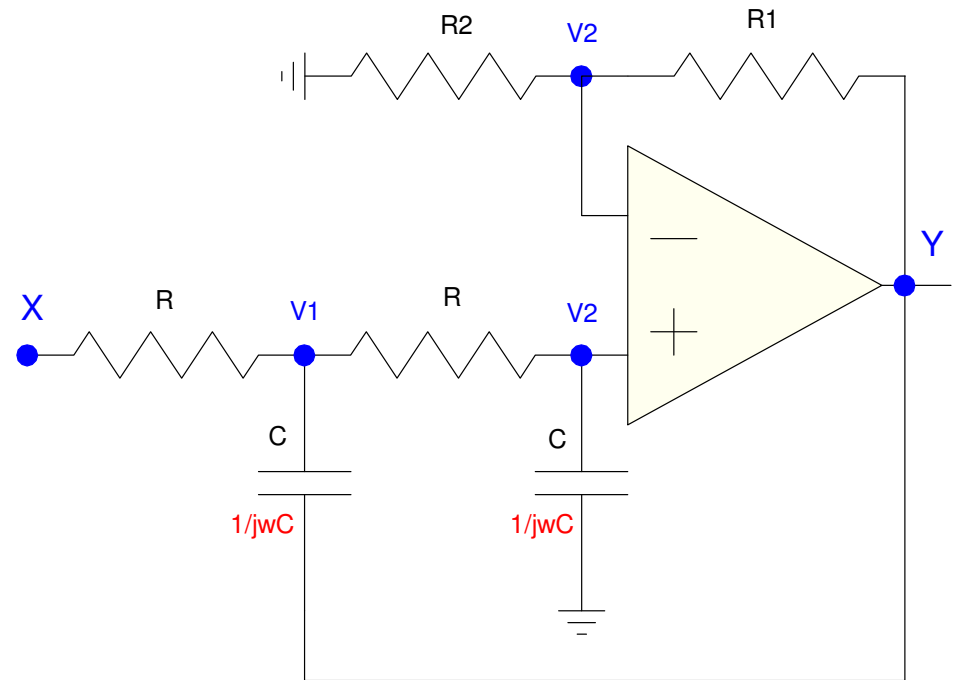
$$\omega RC = 1 \text{ @ } 100Hz$$



2nd-Order LPF

Not an obvious circuit

- Write the node equations



Voltage Node Equations

Non-Inverting Amplifier

$$Y = \left(1 + \frac{R_1}{R_2}\right) V_2 = kV_2$$

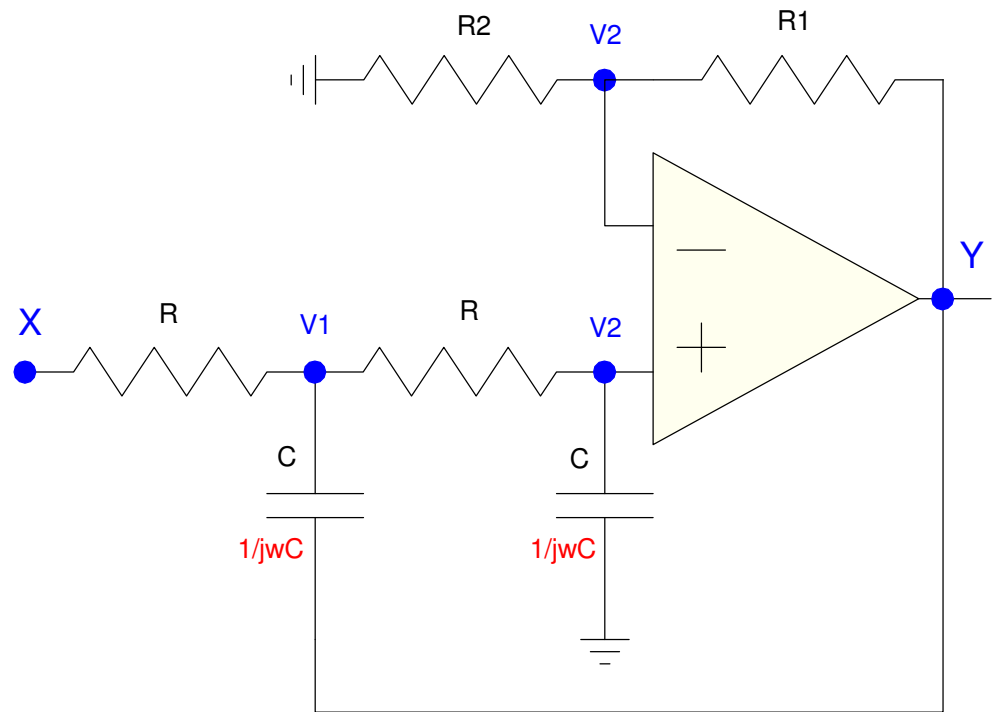
Voltage nodes

$$\left(\frac{V_1 - X}{R}\right) + \left(\frac{V_1 - V_2}{R}\right) + \left(\frac{V_1 - Y}{1/j\omega C}\right) = 0$$

$$\left(\frac{V_2 - V_1}{R}\right) + \left(\frac{V_2}{1/j\omega C}\right) = 0$$

Simplify

$$Y = \left(\frac{k}{(j\omega RC)^2 + (3-k)(j\omega RC) + 1}\right) X$$

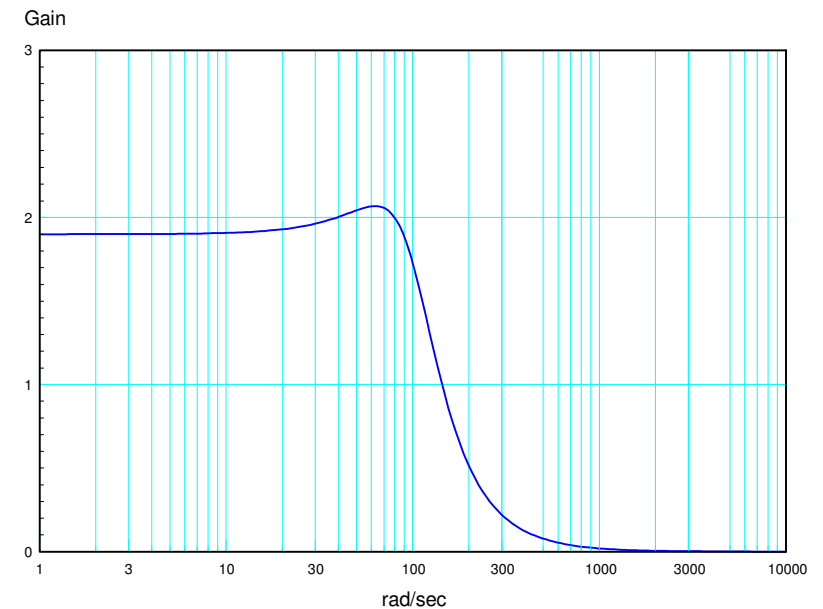
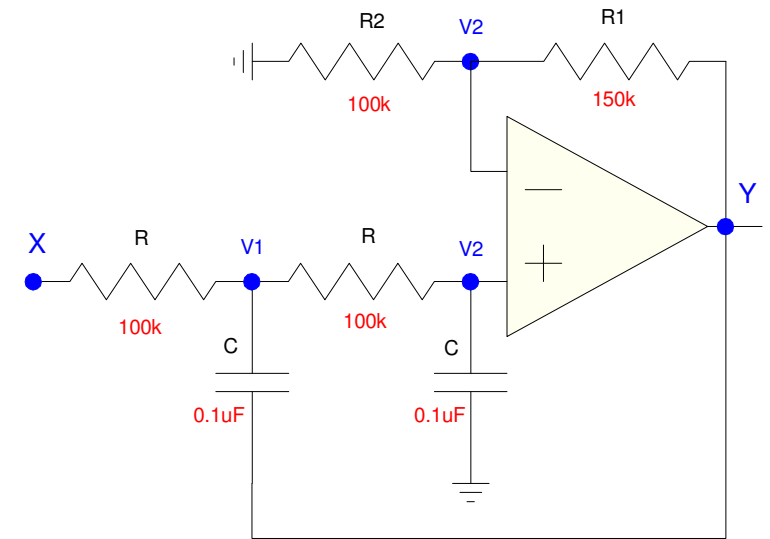


Analysis

$$Y = \left(\frac{k}{(j\omega RC)^2 + (3-k)(j\omega RC) + 1} \right) X$$

In Matlab

```
w = logspace(0, 4, 200)';  
R1 = 90e3;  
R2 = 100e3;  
R = 100e3;  
C = 0.1e-6;  
k = 1 + R1/R2;  
jwRC = j*w*R*C;  
G = k ./ (jwRC.^2 + (3-k)*jwRC + 1);  
semilogx(w, abs(G))  
xlabel('rad/sec')  
ylabel('Gain')
```

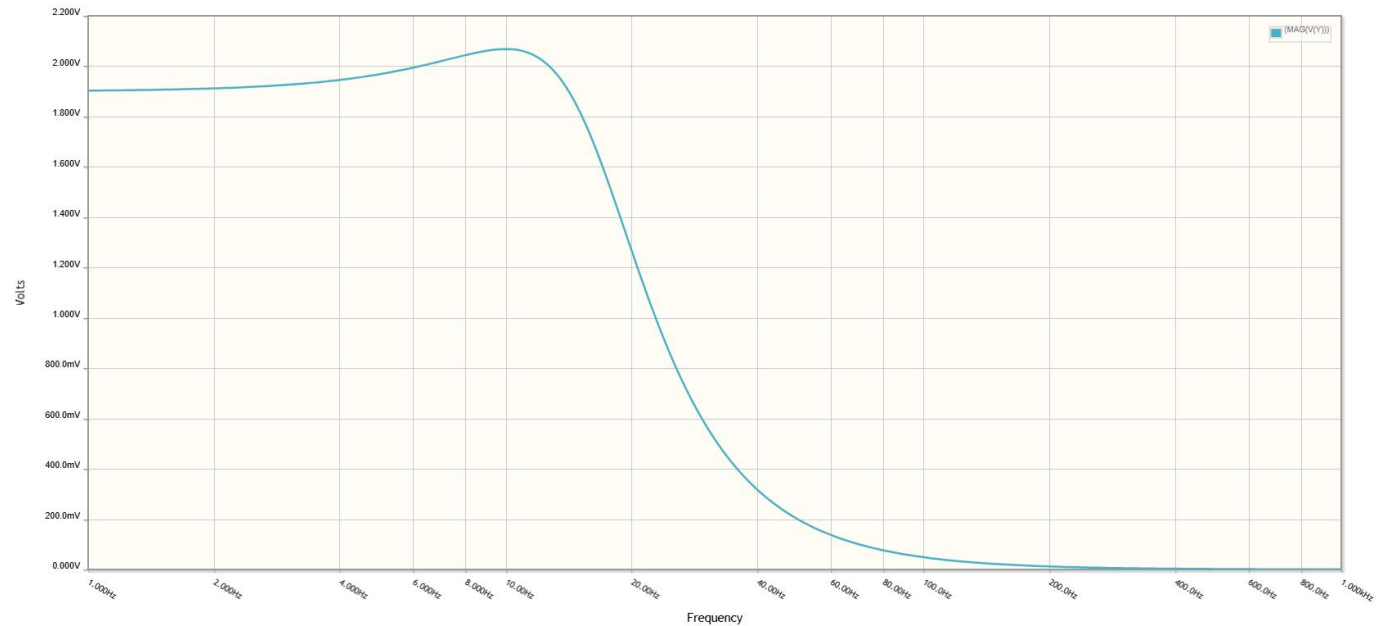
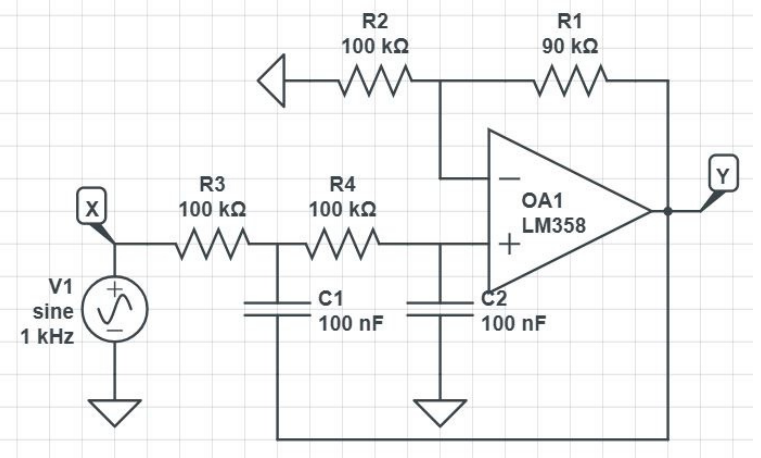
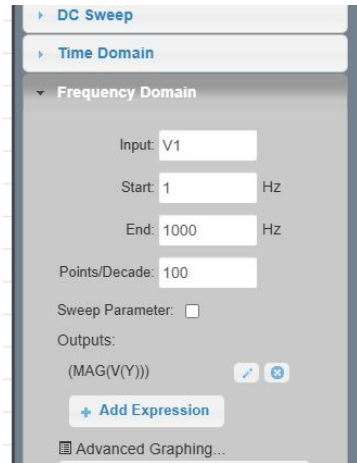


CircuitLab

Frequency Domain

- Set the input to V1
- Sweep from 1Hz to 1000Hz
- 20 points per decade

Matches calculations



Design

- Pass frequencies below 250Hz
- Subwoofer crossover

The corner frequency is $1/RC$

$$\frac{1}{RC} = 500\pi \quad 250Hz$$

$$R = 100k$$

$$C = 6.37nF$$

k sets the angle of the pole

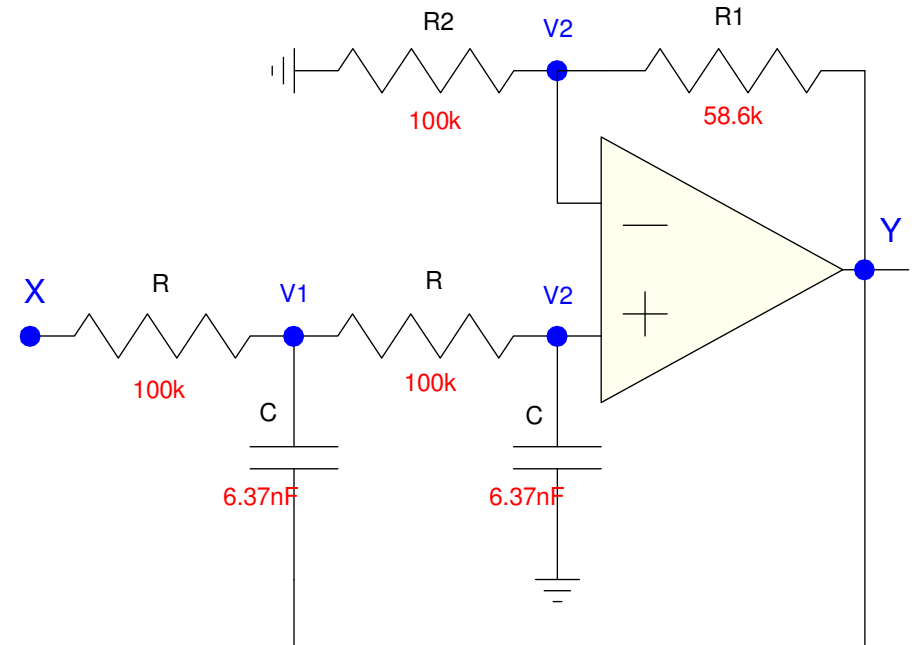
$$3 - k = 2 \cos \theta$$

$$\theta = 45^\circ$$

$$k = 1.586 = 1 + \frac{R_1}{R_2}$$

$$R_2 = 100k$$

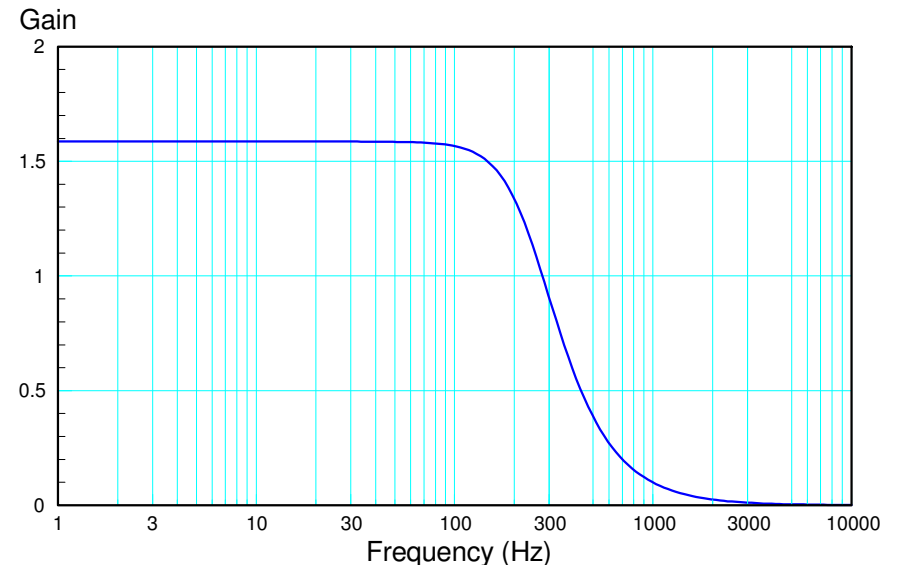
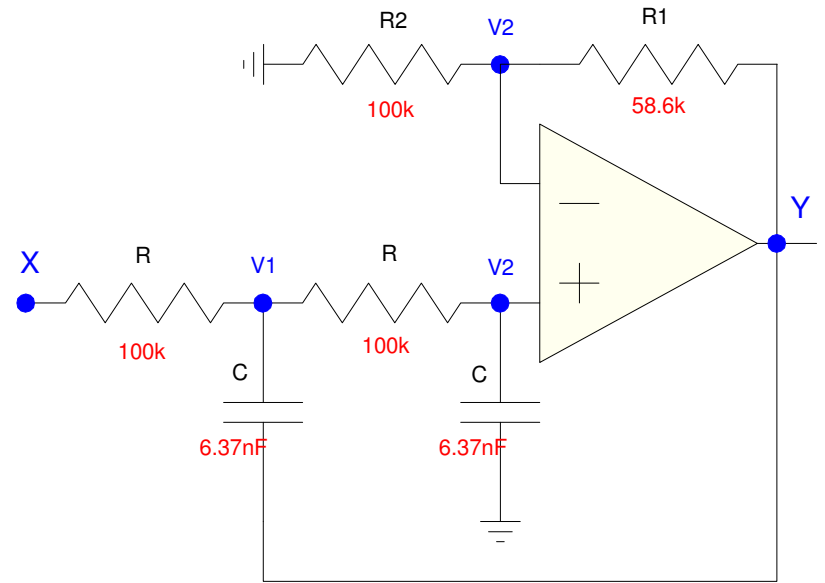
$$R_1 = 58.6k$$



Design

Check the design in Matlab

```
f = logspace(0,4,200)';  
w = 2*pi*f;  
k = 1.586;  
R = 100e3;  
C = 6.37e-9;  
jwRC = j*w*R*C;  
G = k./(jwRC.^2+(3-k)*jwRC+1);  
semilogx(w,abs(G))  
xlabel('Hz')  
ylabel('Gain')
```

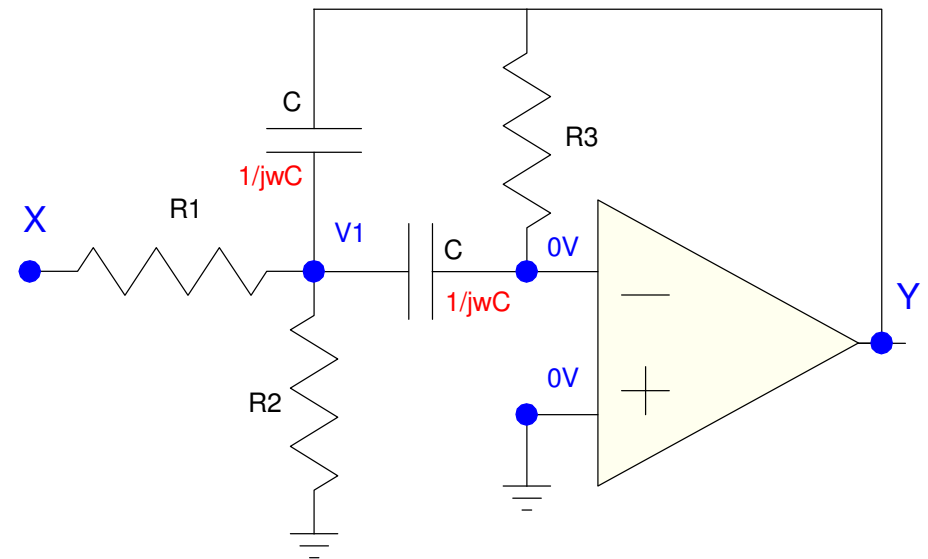


Band-Pass Filter

Also not an obvious topology

- DC gain = 0
- High-frequency gain = 0

Write the node equations



Voltage Nodes

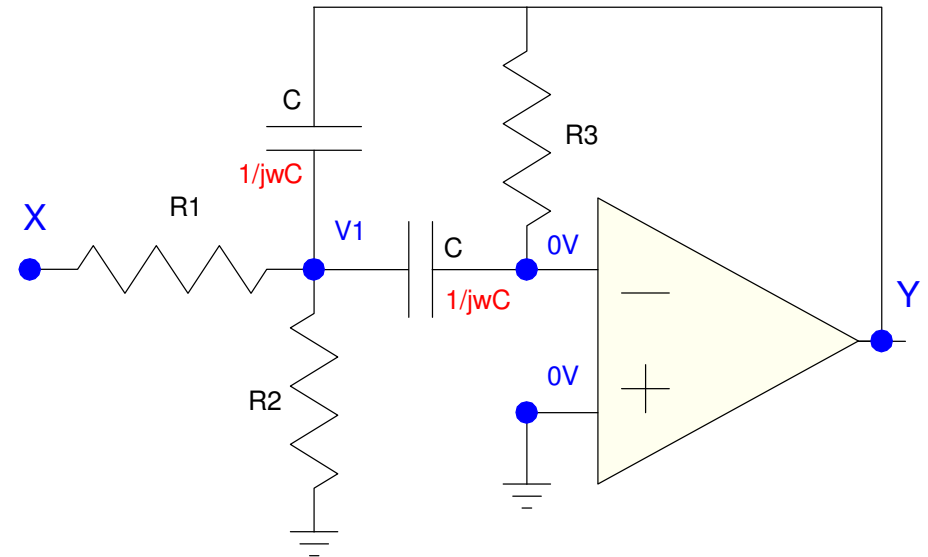
Node equation

$$\left(\frac{V_1 - X}{R_1}\right) + \left(\frac{V_1 - 0}{1/j\omega C}\right) + \left(\frac{V_1 - Y}{1/j\omega C}\right) + \left(\frac{V_1}{R_2}\right) = 0$$

$$\left(\frac{0 - V_1}{1/j\omega C}\right) + \left(\frac{0 - Y}{R_3}\right) = 0$$

Simplify

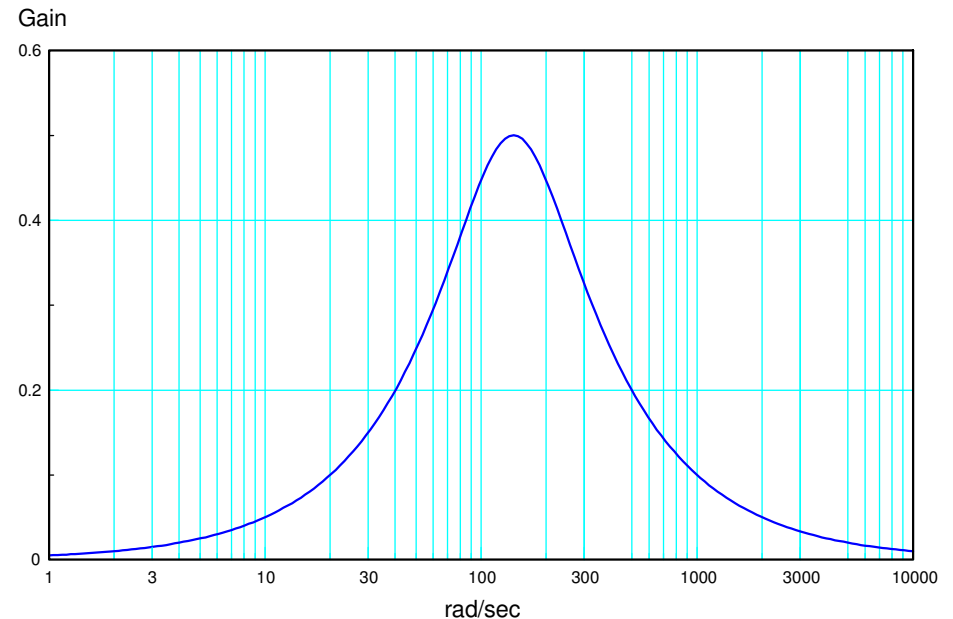
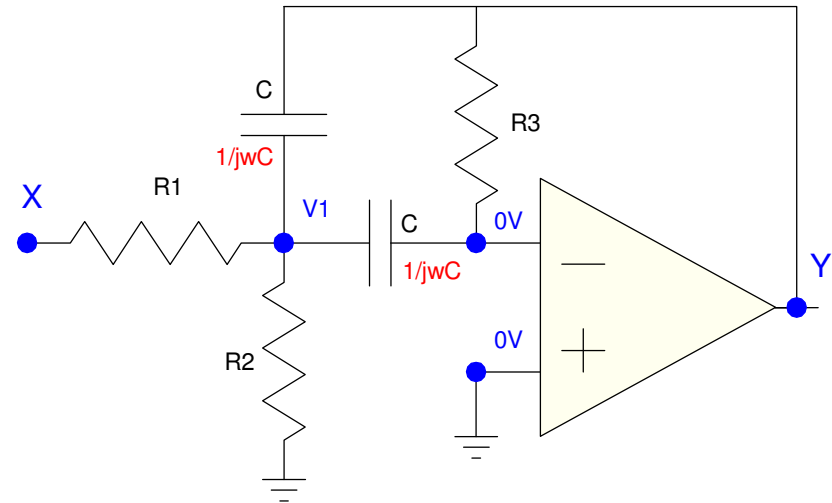
$$Y = \left(\frac{-j\omega \left(\frac{1}{CR_1} \right)}{(j\omega)^2 + \left(\frac{2}{CR_3} \right) (j\omega) + \left(\frac{R_1 + R_2}{R_1 R_2 R_3 C^2} \right)} \right) X$$



Analysis

$$G = \left(\frac{-j\omega \left(\frac{1}{CR_1} \right)}{(j\omega)^2 + \left(\frac{2}{CR_3} \right) (j\omega) + \left(\frac{R_1 + R_2}{R_1 R_2 R_3 C^2} \right)} \right)$$

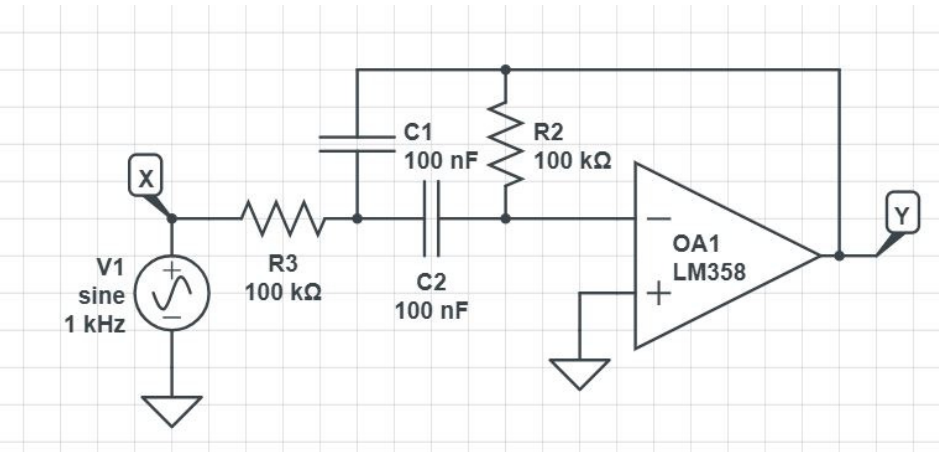
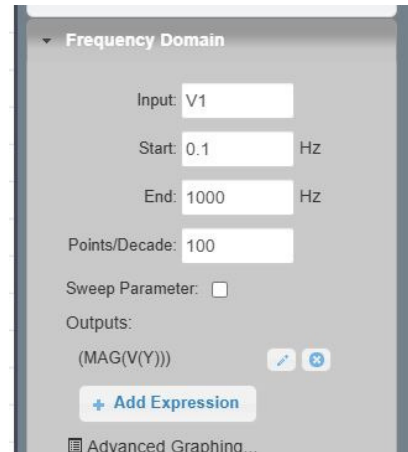
```
w = logspace(0, 4, 200)';
R1 = 100e3;
R2 = 100e3;
R3 = 100e3;
C = 0.1e-6;
num = -(j*w) / (C*R1);
d2 = (j*w).^2;
d1 = (j*w)*2 / (C*R3);
d0 = (R1+R2) / (R1*R2*R3*C*C);
den = d2 + d1 + d0;
G = -num ./ den;
semilogx(w, abs(G))
xlabel('rad/sec')
ylabel('Gain')
```



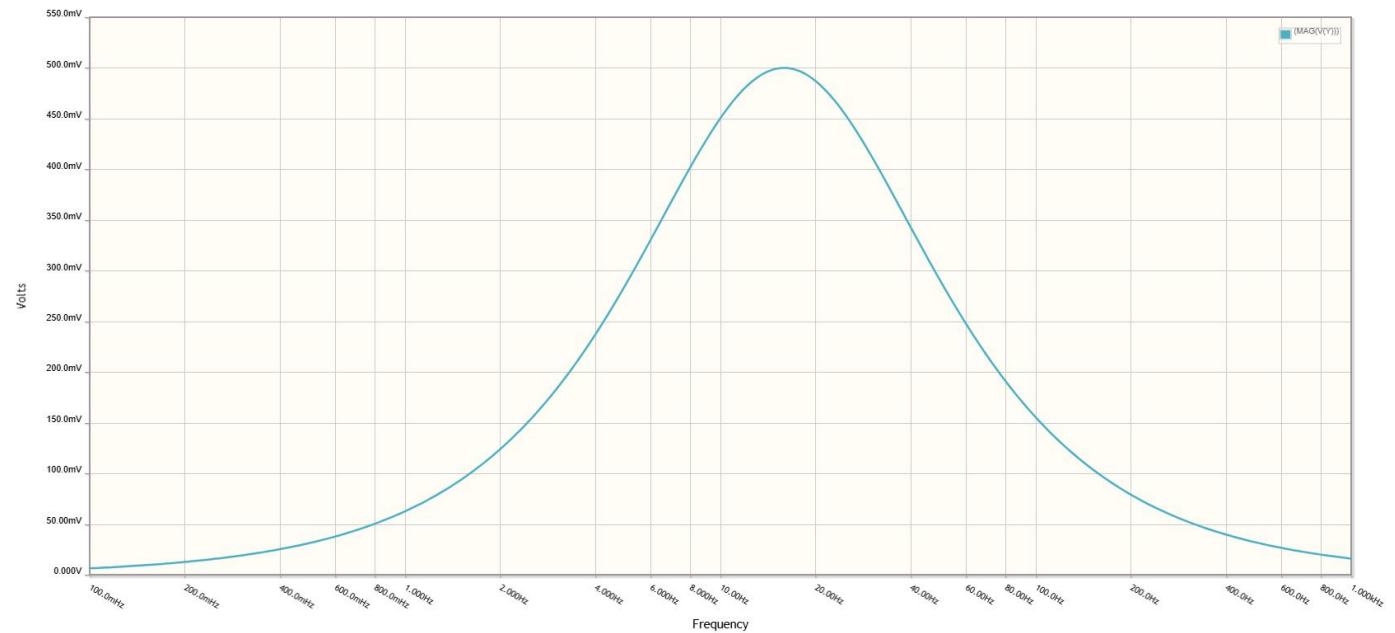
CircuitLab

Frequency Domain

- Set the input to V1
- 0.1Hz to 1000Hz
- 100 points per decade



Matches calculations



Design

Find R and C to implement

$$G = \left(\frac{-20(j\omega)}{(j\omega)^2 + 10(j\omega) + 1000} \right)$$

From before

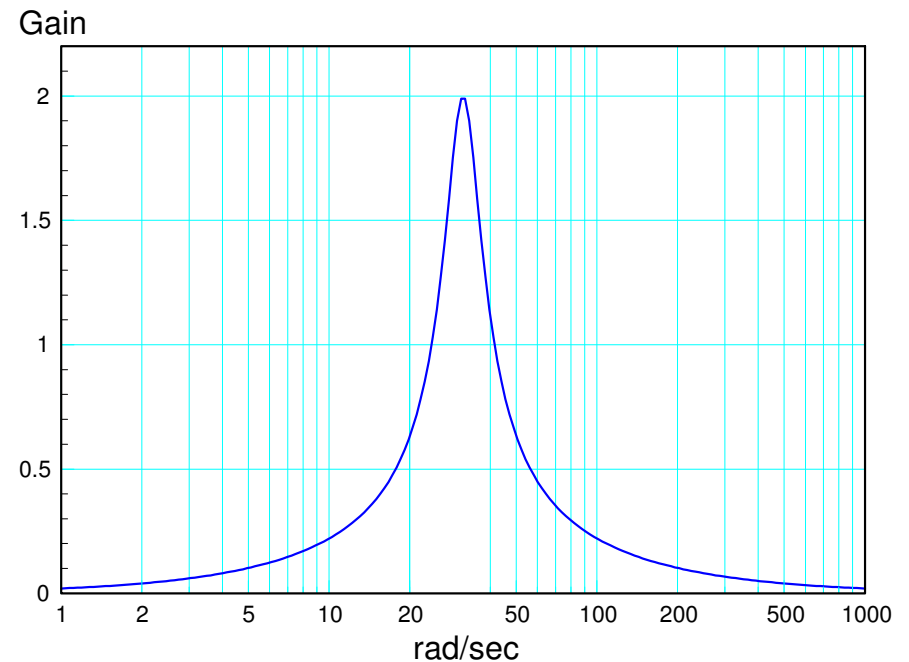
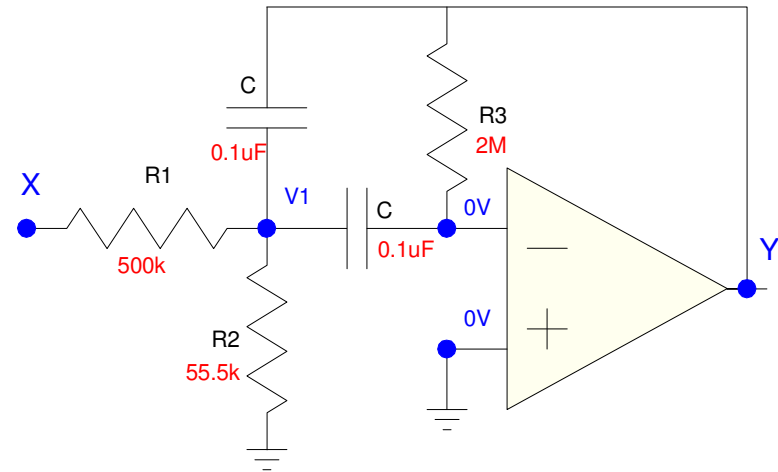
$$G = \left(\frac{-j\omega \left(\frac{1}{CR_1} \right)}{(j\omega)^2 + \left(\frac{2}{CR_3} \right) (j\omega) + \left(\frac{R_1 + R_2}{R_1 R_2 R_3 C^2} \right)} \right)$$

Matching terms

$$\frac{1}{R_1 C} = 20$$

$$\frac{2}{R_3 C} = 10$$

$$\frac{1}{C^2 R_1 R_3} = 1000$$



Summary

Filters are circuits where the gain varies with frequency

- Any circuit with inductors and/or capacitors

Active filters are op-amp circuits where the gain varies with frequency

- Op-amps allow gains > 1
- They also allow you to avoid using inductors

If the operating frequency is unknown, leave it as ω

- You can then use voltage nodes to determine the output as a function of frequency
-