
Phasors & Voltage Nodes

ECE 211 Circuits I

Lecture #25

Please visit [Bison Academy](#) for corresponding
lecture notes, homework sets, and solutions

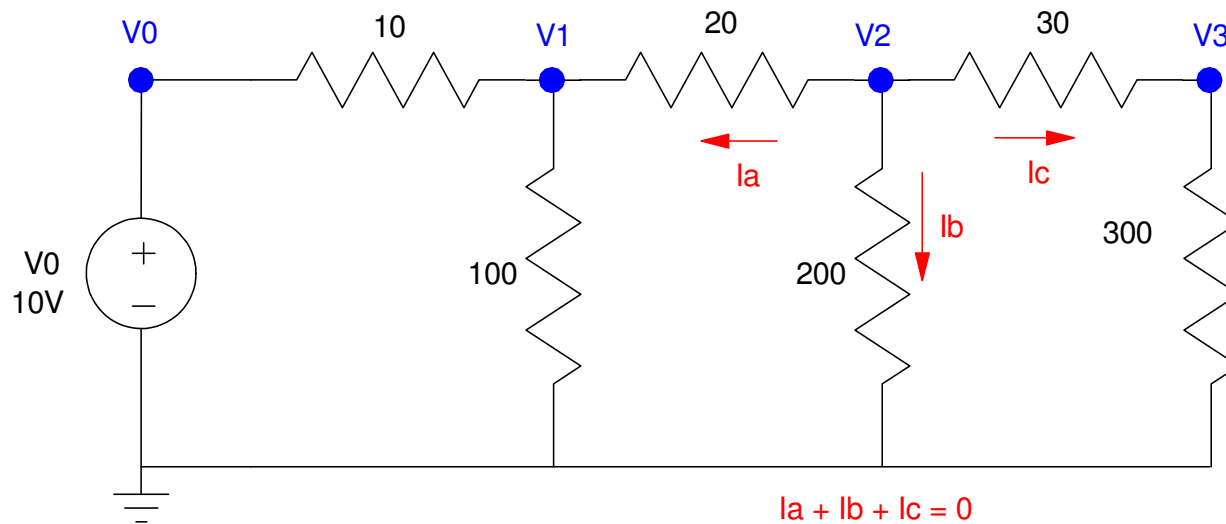
Voltage Nodes:

One tool that's useful in solving a given circuit is *voltage nodes*

- Given a circuit with N unknown voltages
- Write N equations for N unknowns

Each voltage node equation uses conservation of current

- The net current flowing from a node must be zero



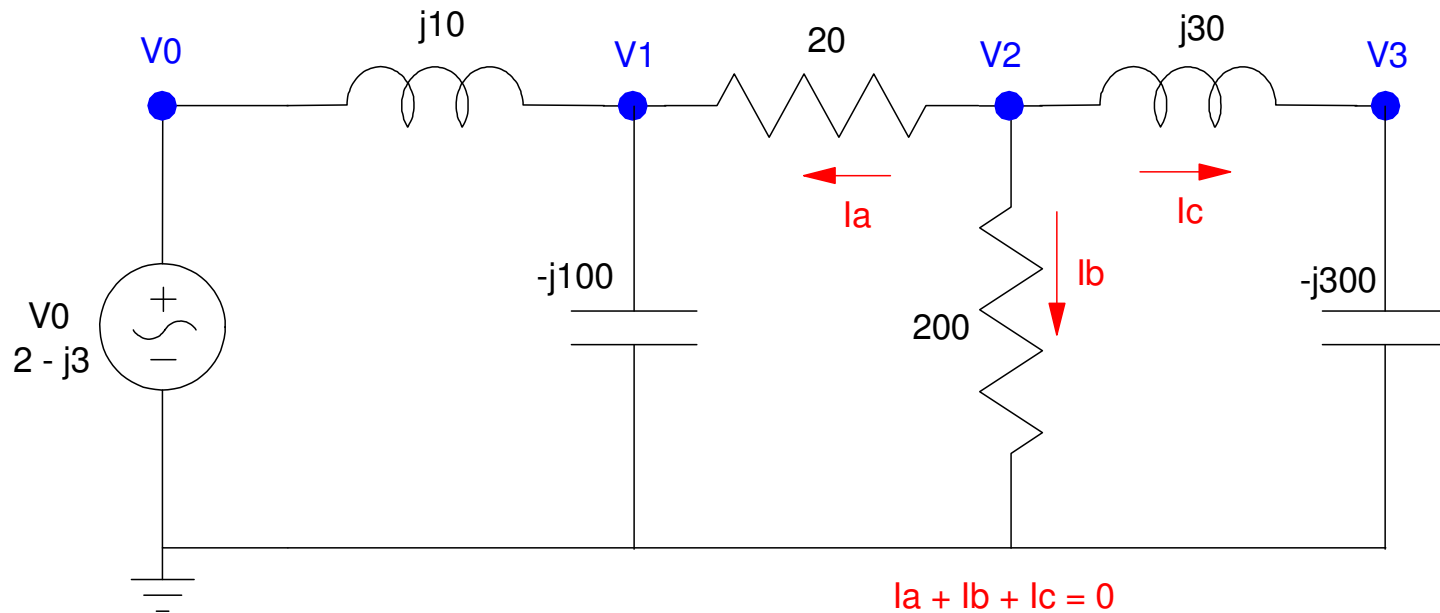
Voltage Nodes with Phasors

Voltage Nodes works for DC signals

- Node equations use real numbers

Voltage Nodes works for AC signals

- You now need to use phasors and complex numbers



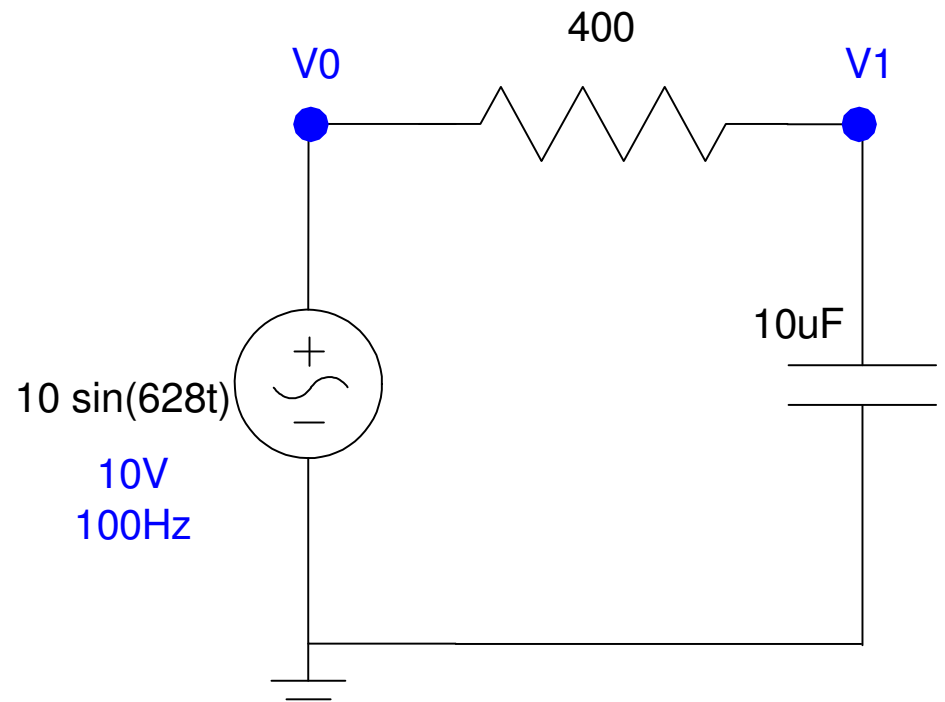
Phasor Voltages & Impedances

Recall: The phasor representation of voltages and impedances are:

	VI relationship	Phasor Notation
Voltage	$v(t) = a \cos(\omega t) + b \sin(\omega t)$	$V = a - jb$
Current	$i(t) = a \cos(\omega t) + b \sin(\omega t)$	$I = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L \frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C \frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

Example: RC Circuit

Determine the node voltages for the following circuit:



Step 1: Replace the elements with their phasor values (shown in red)

Voltage Source

$$a \cos(\omega t) + b \sin(\omega t) \rightarrow a - jb$$

$$10 \sin(628t) \rightarrow 0 - j10$$

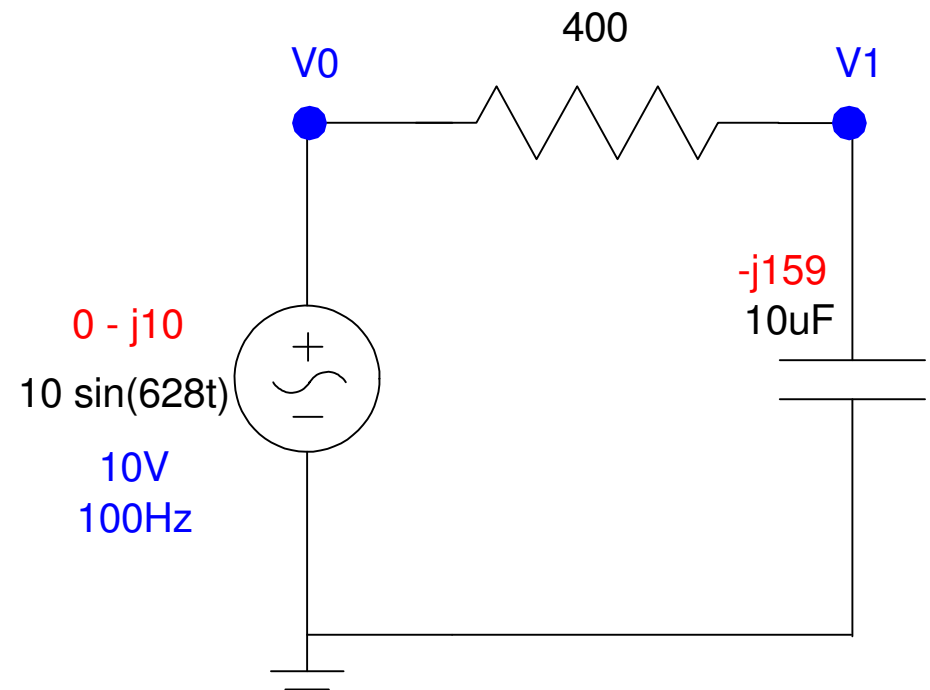
Resistors

$$R \rightarrow R$$

Capacitors

$$C \rightarrow \frac{1}{j\omega C}$$

$$10\mu F \rightarrow -j159.24\Omega$$



Step 2: Write N equations for N unknowns

Node V0

$$V_0 = -j10$$

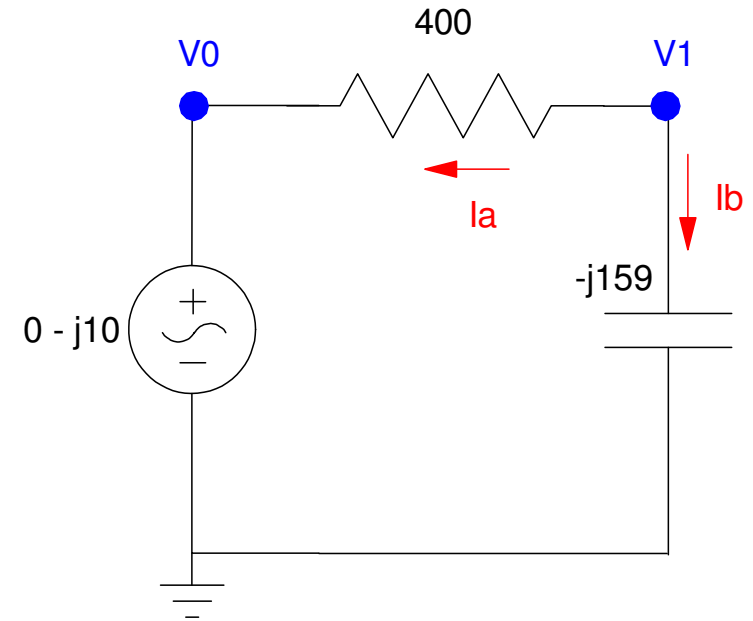
Node V1

$$\left(\frac{V_1 - V_0}{400}\right) + \left(\frac{V_1}{-j159}\right) = 0$$

Solve

$$V_0 = -j10$$

$$\left(\frac{-1}{400}\right)V_0 + \left(\frac{1}{100} + \frac{1}{-j159}\right)V_1 = 0$$



Using Matlab

In matrix form

$$\begin{bmatrix} 1 & 0 \\ \left(\frac{-1}{400}\right) & \left(\frac{1}{400} + \frac{1}{-j159}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \end{bmatrix} = \begin{bmatrix} -j10 \\ 0 \end{bmatrix}$$

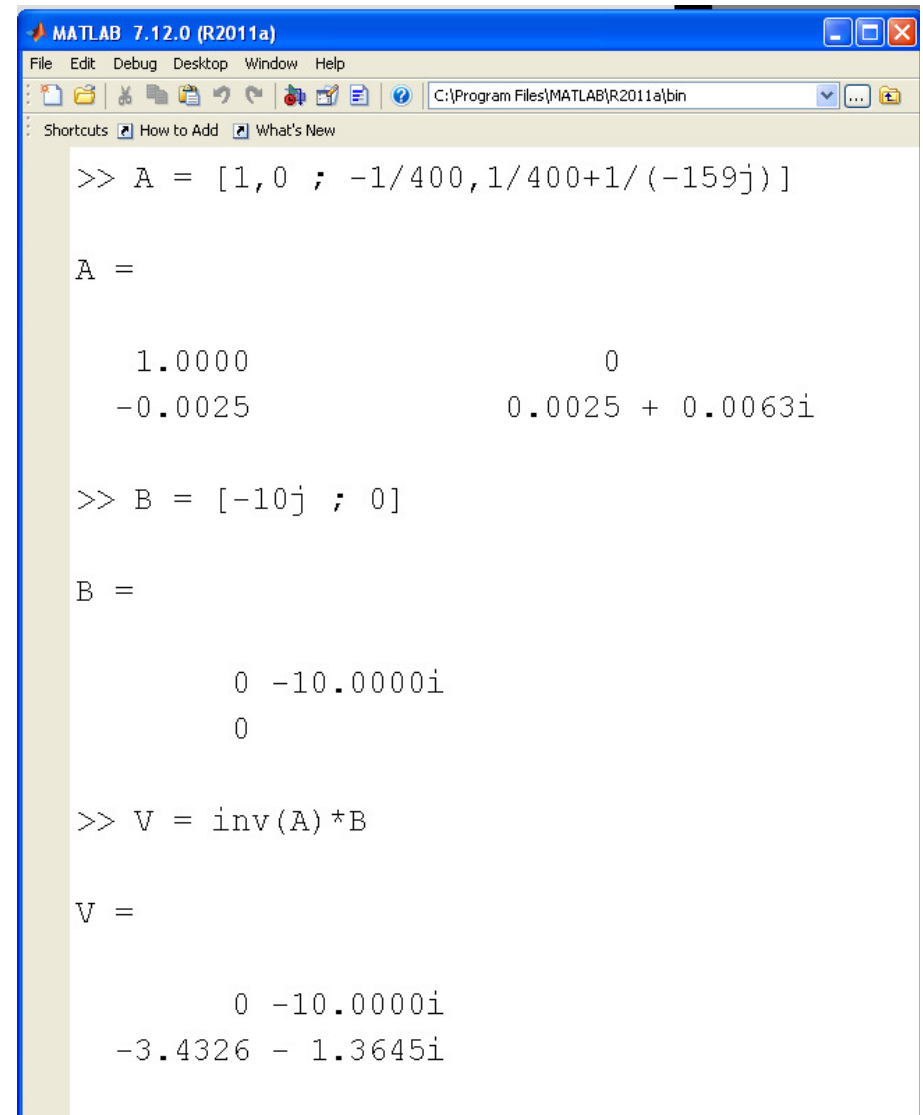
Result

$$V_0 = -j10$$

$$V_1 = -3.4326 - j1.3646$$

Translating

$$v_1(t) = -3.43 \cos(628t) + 1.36 \sin(628t)$$



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MATLAB 7.12.0 (R2011a)
File Edit Debug Desktop Window Help
C:\Program Files\MATLAB\R2011a\bin
Shortcuts How to Add What's New

>> A = [1,0 ; -1/400,1/400+1/(-159j)]

A =

    1.0000         0
   -0.0025    0.0025 + 0.0063i

>> B = [-10j ; 0]

B =

         0 -10.0000i
         0

>> V = inv(A)*B

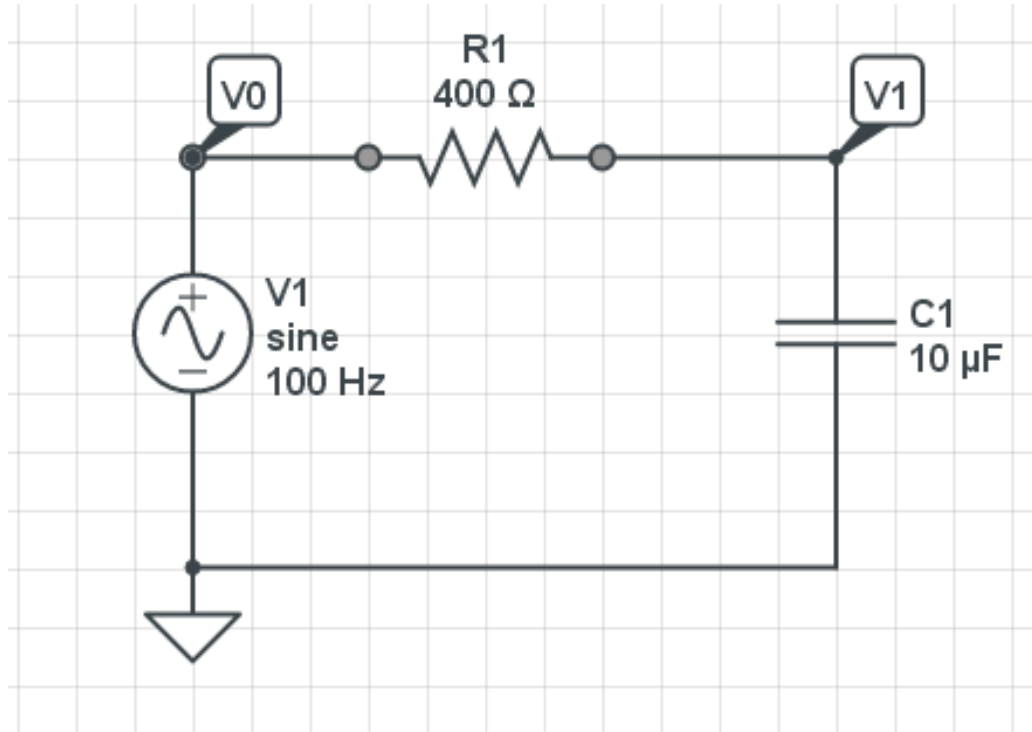
V =

         0 -10.0000i
   -3.4326 - 1.3645i
```

Verify with CircuitLab

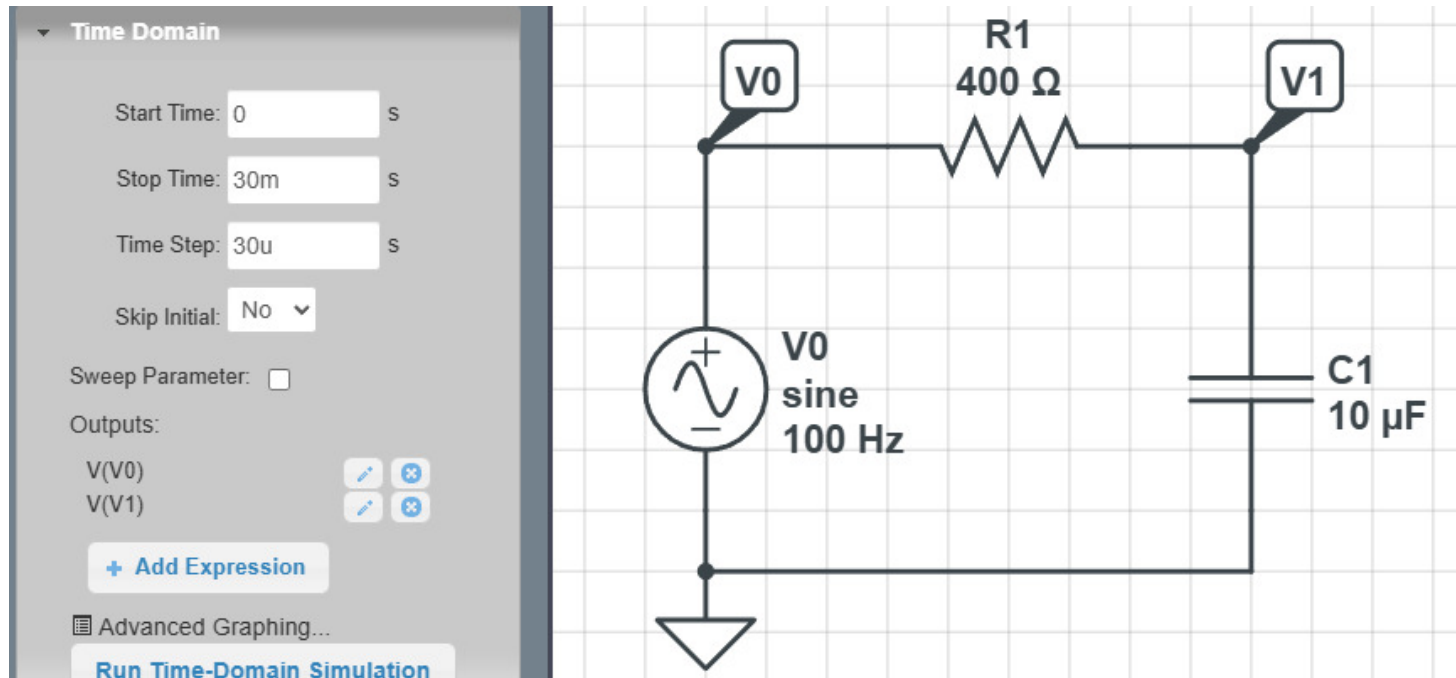
Make the input a sine wave with

- no DC offset
- 10V amplitude
- 100Hz (628 rad/sec)



Run a transient response for

- 30ms (3 cycles)
- 10us step size



CircuitLab Result

- Polar form is easier with CircuitLab

V0 (blue waveform)

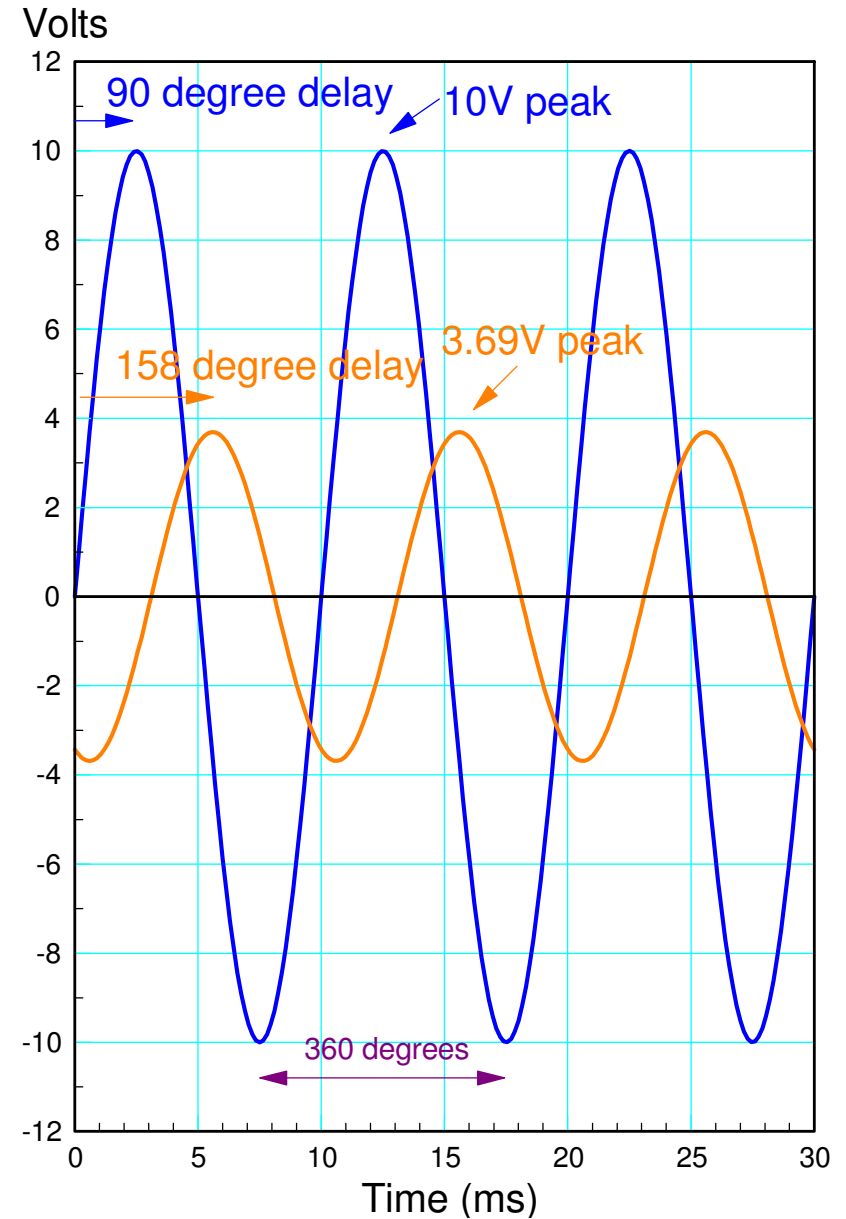
$$\begin{aligned} V_0 &= 0 - j10 \\ &= 10 \angle -90^\circ \end{aligned}$$

- Peak = 10V
- Delay is 90 degrees

V1 (orange waveform)

$$\begin{aligned} V_1 &= -3.43 - j1.36 \\ &= 3.69 \angle -158^\circ \end{aligned}$$

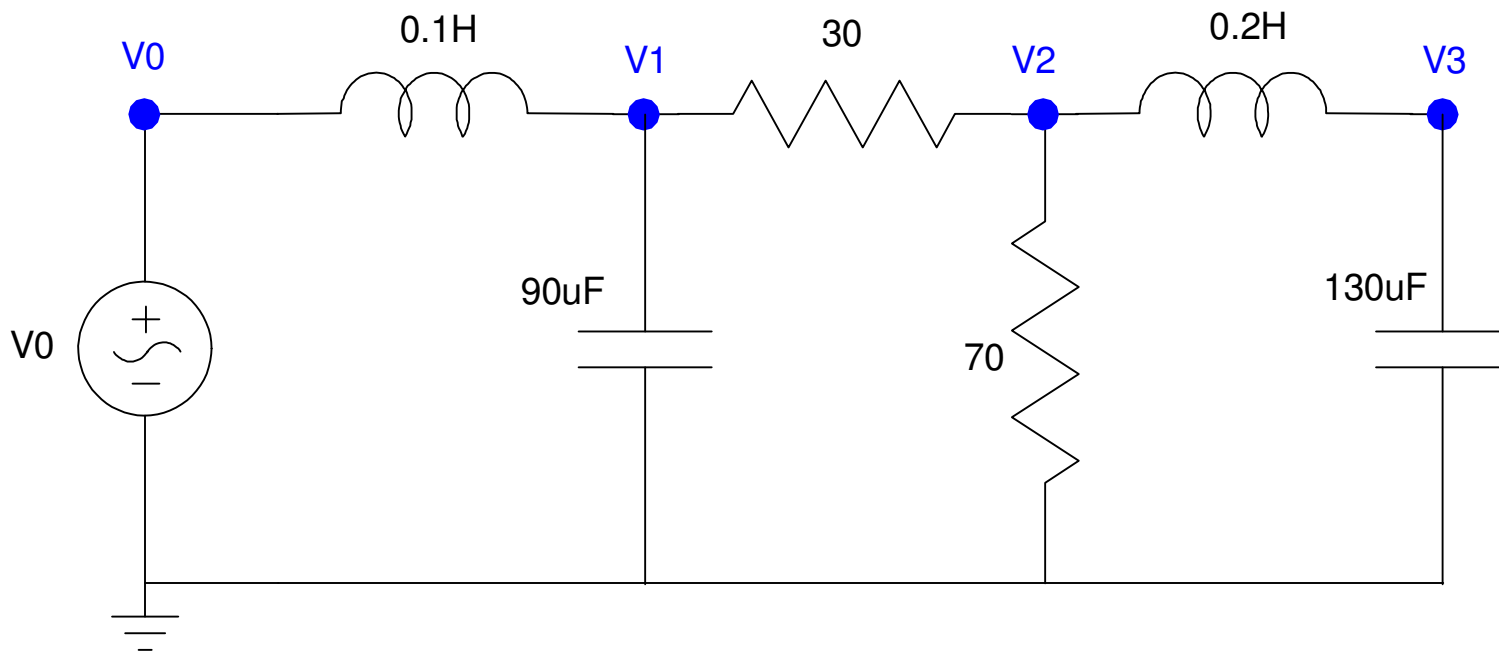
- Peak is 3.69V
- Delay is 158 degrees



Example 2: 3-Stage RC Circuit

Find the voltages for the following circuit when the input is

$$V_0 = 7 \cos(100t) + 3 \sin(100t)$$



Step 1: Express in Phasor form

- (shown in red)

$$V_0 = 7 \cos(100t) + 3 \sin(100t)$$

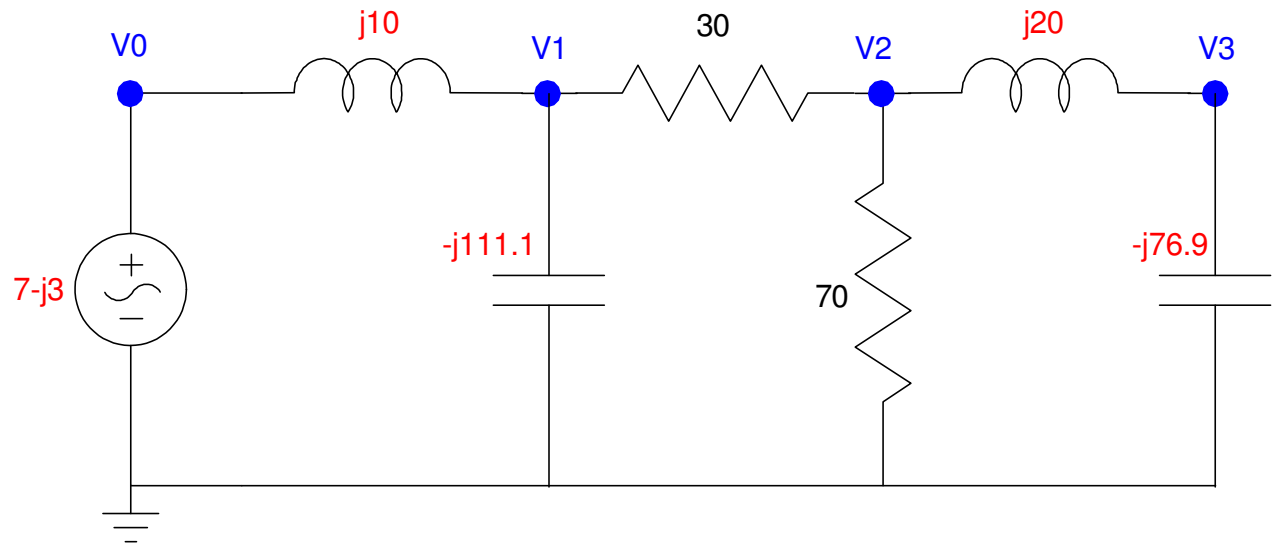
$$= 7 - j3$$

$$0.1H \rightarrow j\omega L = j10$$

$$0.2H \rightarrow j\omega L = j20$$

$$90\mu F \rightarrow \frac{1}{j\omega C} = -j111.1$$

$$130\mu F \rightarrow \frac{1}{j\omega C} = -j76.9$$



Step 2: Write N equations for N unknowns

Voltage source

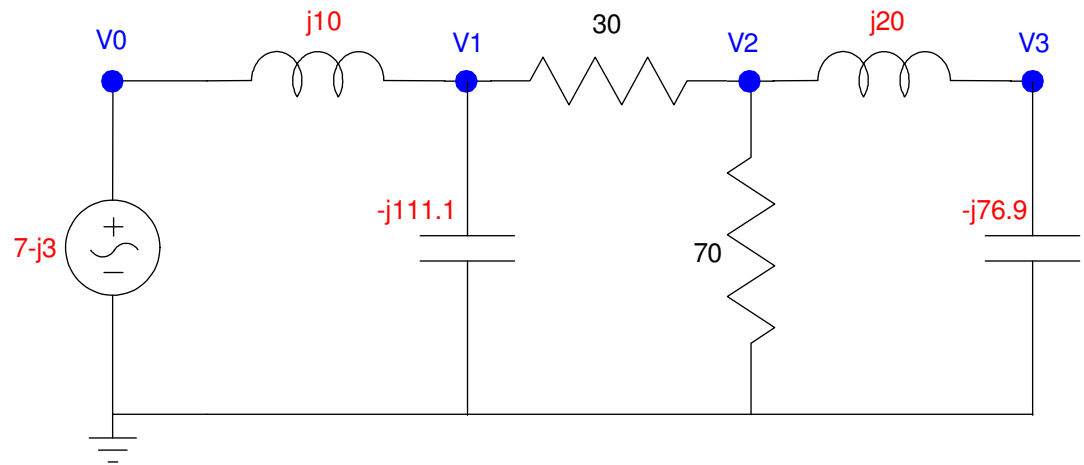
$$V_0 = 7 - j3$$

Node equations

$$\left(\frac{V_1 - V_0}{j10} \right) + \left(\frac{V_1}{-j111.1} \right) + \left(\frac{V_1 - V_2}{30} \right) = 0$$

$$\left(\frac{V_2 - V_1}{30} \right) + \left(\frac{V_2}{70} \right) + \left(\frac{V_2 - V_3}{j20} \right) = 0$$

$$\left(\frac{V_3 - V_2}{j20} \right) + \left(\frac{V_3}{-j76.9} \right) = 0$$



Step 3: Solve

Group terms

$$V_0 = 7 - j3$$

$$\left(\frac{V_1 - V_0}{j10}\right) + \left(\frac{V_1}{-j111.1}\right) + \left(\frac{V_1 - V_2}{30}\right) = 0$$

$$\left(\frac{V_2 - V_1}{30}\right) + \left(\frac{V_2}{70}\right) + \left(\frac{V_2 - V_3}{j20}\right) = 0$$

$$\left(\frac{V_3 - V_2}{j20}\right) + \left(\frac{V_3}{-j76.9}\right) = 0$$

becomes

$$V_0 = 7 - j3$$

$$-\left(\frac{1}{j10}\right)V_0 + \left(\frac{1}{j10} + \frac{1}{-j111.1} + \frac{1}{30}\right)V_1 - \left(\frac{1}{30}\right)V_2 = 0$$

$$-\left(\frac{1}{30}\right)V_1 + \left(\frac{1}{30} + \frac{1}{70} + \frac{1}{j20}\right)V_2 - \left(\frac{1}{j20}\right)V_3 = 0$$

$$-\left(\frac{1}{j20}\right)V_2 + \left(\frac{1}{j20} + \frac{1}{-j76.9}\right)V_3 = 0$$

Place in matrix form

$$V_0 = 7 - j3$$

$$-\left(\frac{1}{j10}\right) V_0 + \left(\frac{1}{j10} + \frac{1}{-j111.1} + \frac{1}{30}\right) V_1 - \left(\frac{1}{30}\right) V_2 = 0$$

$$-\left(\frac{1}{30}\right) V_1 + \left(\frac{1}{30} + \frac{1}{70} + \frac{1}{j20}\right) V_2 - \left(\frac{1}{j20}\right) V_3 = 0$$

$$-\left(\frac{1}{j20}\right) V_2 + \left(\frac{1}{j20} + \frac{1}{-j76.9}\right) V_3 = 0$$

becomes

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{j10}\right) & \left(\frac{1}{j10} + \frac{1}{-j111.1} + \frac{1}{30}\right) & \left(\frac{-1}{30}\right) & 0 \\ 0 & \left(\frac{-1}{30}\right) & \left(\frac{1}{30} + \frac{1}{70} + \frac{1}{j20}\right) & \left(\frac{-1}{j20}\right) \\ 0 & 0 & \left(\frac{-1}{j20}\right) & \left(\frac{1}{j20} + \frac{1}{-j76.9}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} (7 - j3) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve

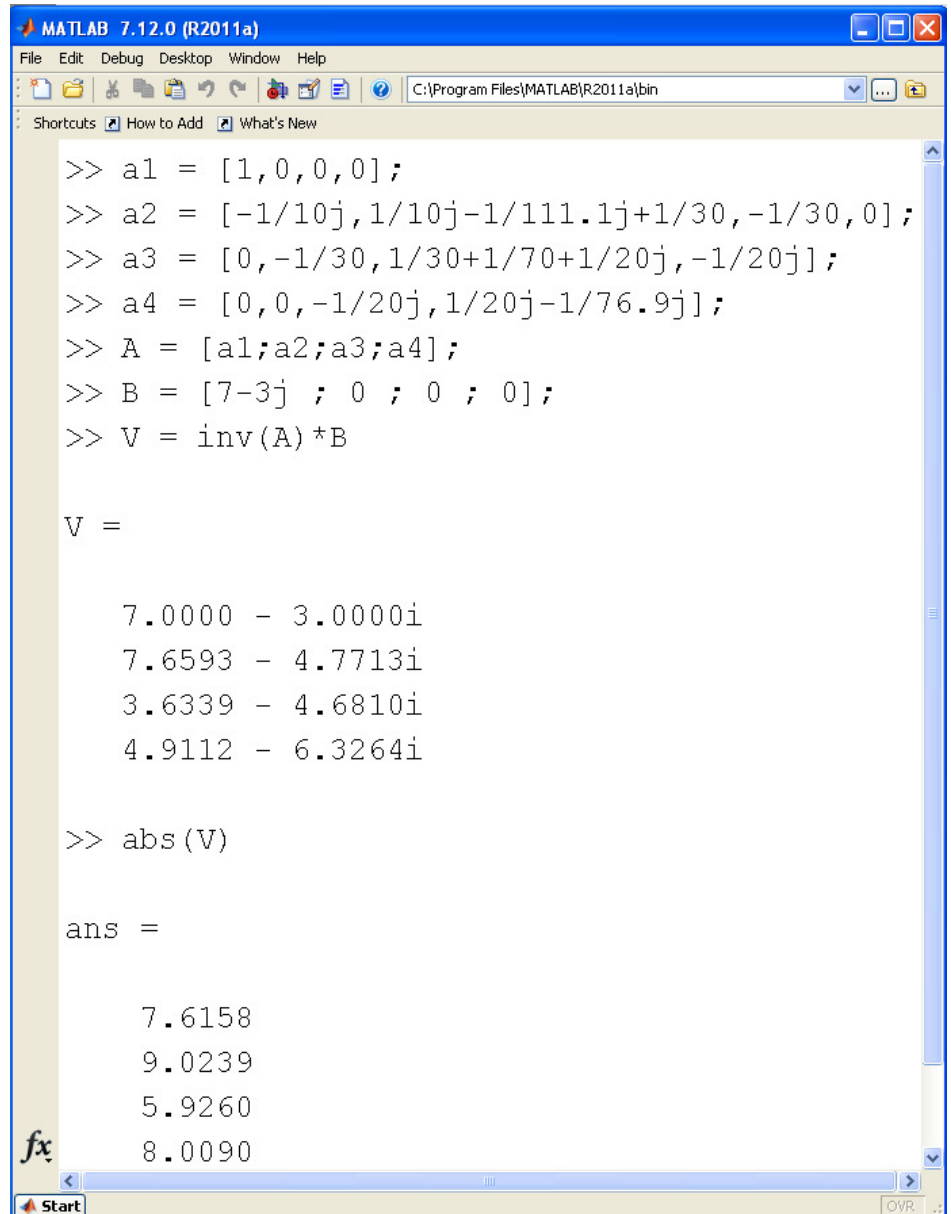
- Matlab is *really* useful here

$$V_0(t) = 7 \cos(100t) + 3 \sin(100t)$$

$$V_1(t) = 7.65\cos(100t)+4.77\sin(100t)$$

$$V_2(t) = 3.63\cos(100t)+4.68\sin(100t)$$

$$V_3(t) = 4.91\cos(100t)+6.32\sin(100t)$$



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MATLAB 7.12.0 (R2011a)
File Edit Debug Desktop Window Help
C:\Program Files\MATLAB\R2011a\bin
Shortcuts How to Add What's New

>> a1 = [1,0,0,0];
>> a2 = [-1/10j,1/10j-1/111.1j+1/30,-1/30,0];
>> a3 = [0,-1/30,1/30+1/70+1/20j,-1/20j];
>> a4 = [0,0,-1/20j,1/20j-1/76.9j];
>> A = [a1;a2;a3;a4];
>> B = [7-3j ; 0 ; 0 ; 0];
>> V = inv(A)*B

V =

    7.0000 - 3.0000i
    7.6593 - 4.7713i
    3.6339 - 4.6810i
    4.9112 - 6.3264i

>> abs(V)

ans =

    7.6158
    9.0239
    5.9260
    8.0090
```

Check in CircuitLab

V0 is 100 rad/sec

$$f = \frac{\omega}{2\pi} = 15.9155 \text{ Hz}$$

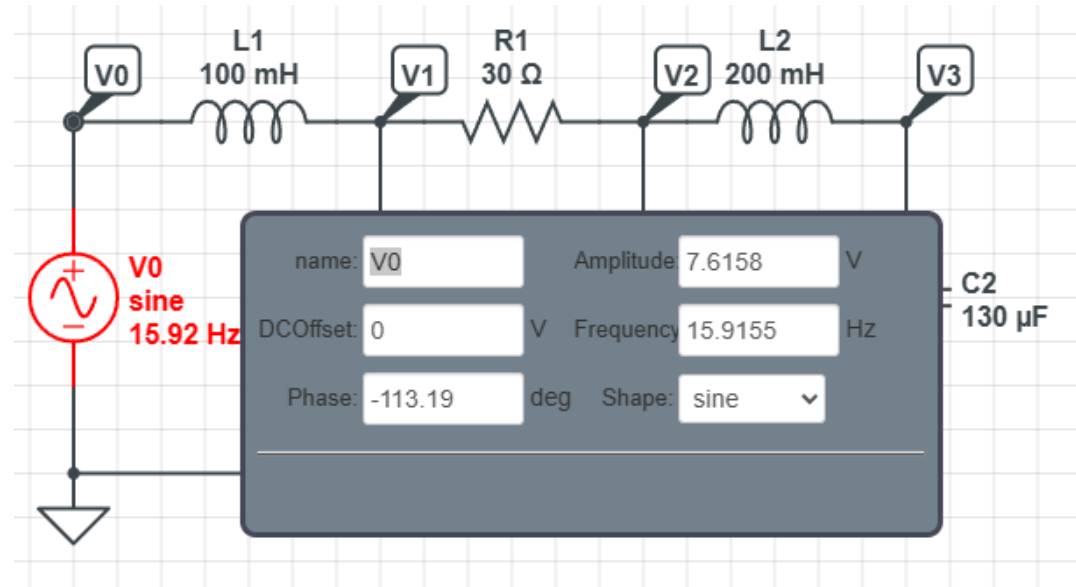
V0(t) in polar form

$$V_0 = 7 - j3$$
$$= 7.6158 \angle -23.1986^\circ$$

Switching to sine waves

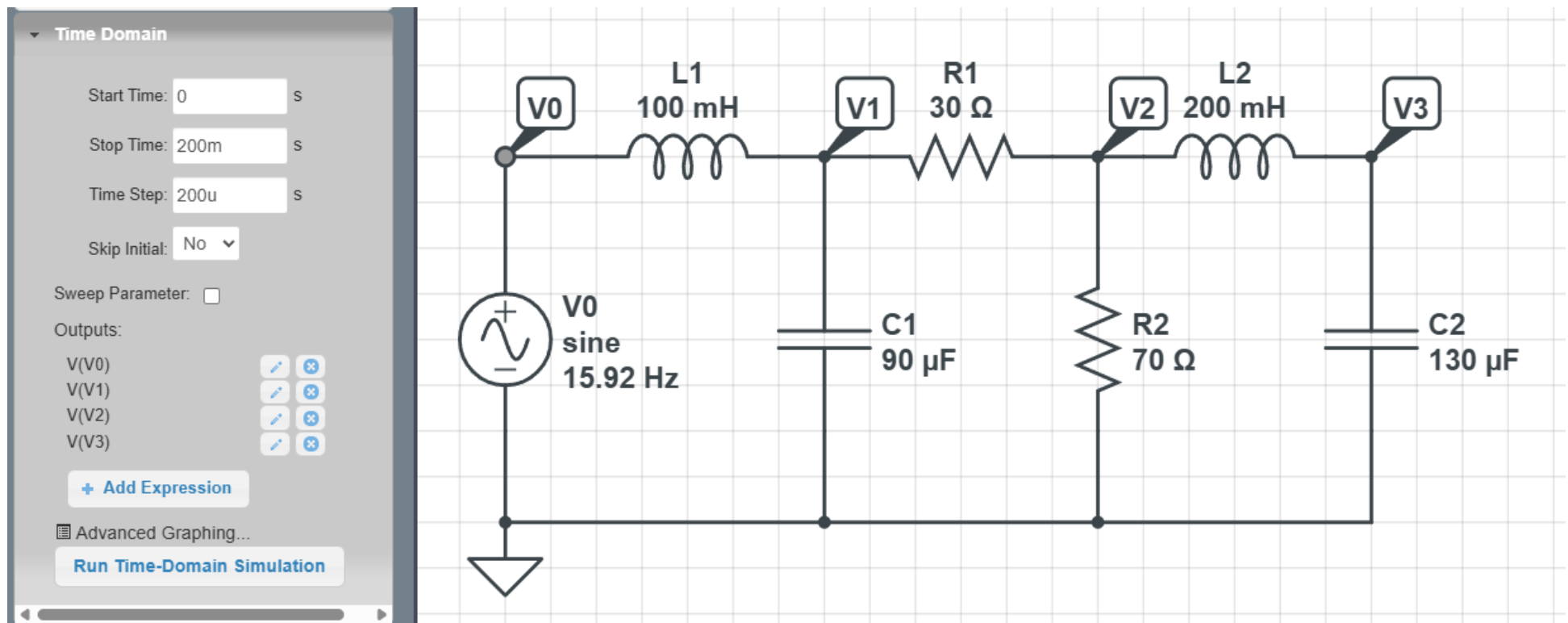
- Cosine is default, but
- CircuitLab uses sine waves

$$= 7.6158 \angle -113.1986^\circ$$



Simulate for 3 cycles

- Let the transients die out
- 15.9Hz
- 3 cycles = 200ms



Results match calculations

$V_{0p} = 7.615V$

- matlab: 7.6158

$V_{1p} = 8.995V$

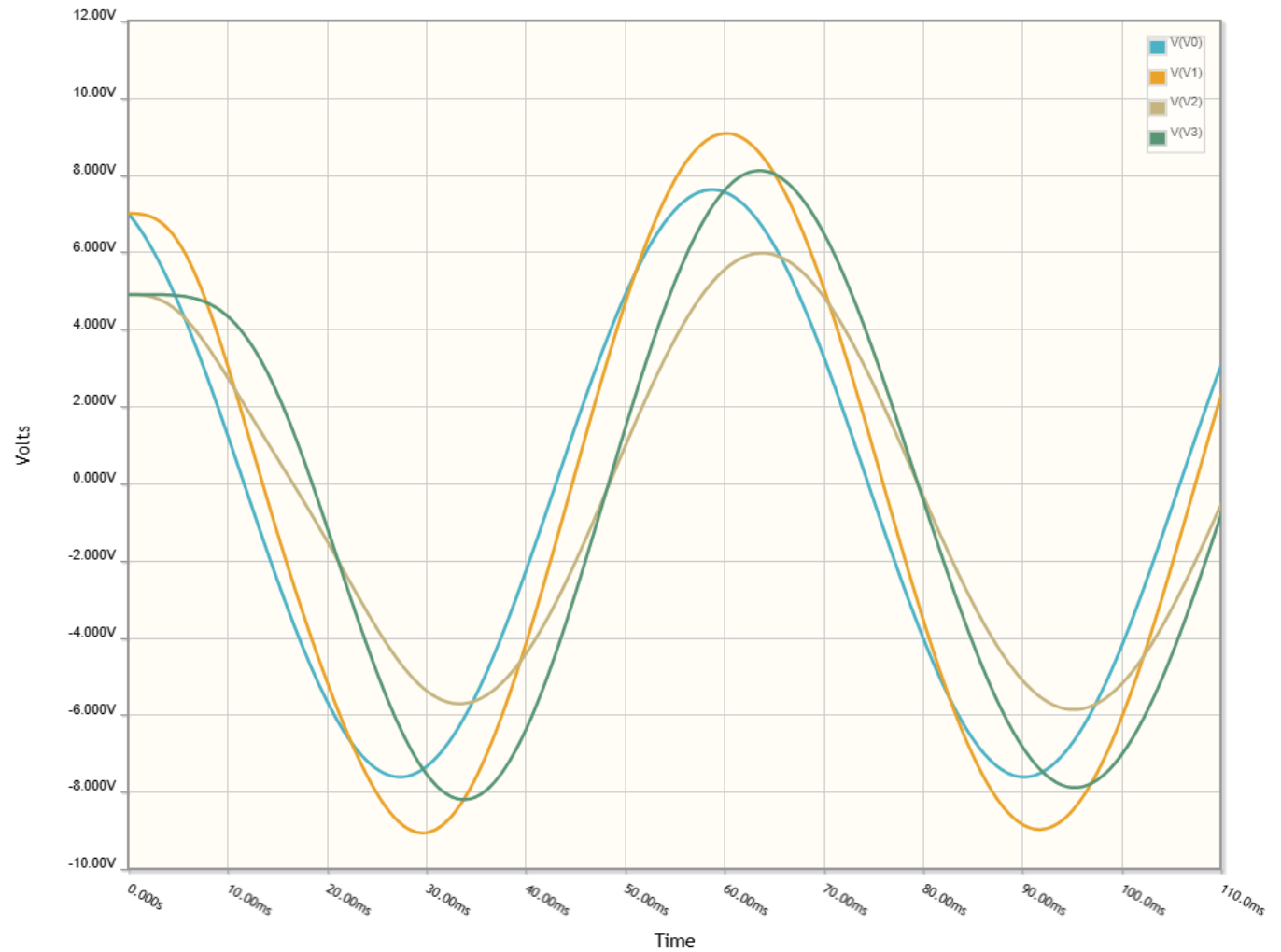
- matlab: 9.0239

$V_{2p} = 5.837V$

- matlab 5.9260

$V_{3p} = 7.936V$

- matlab 8.0090



What happens if you change the frequency?

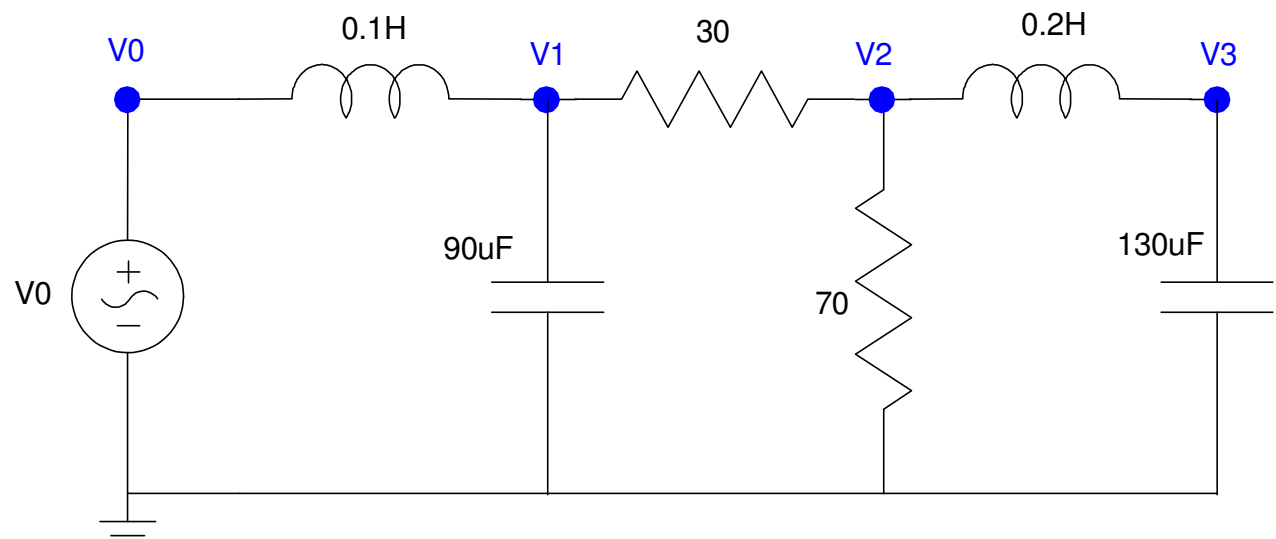
$$V_0 = 7 \cos(200t) + 3 \sin(200t)$$

Completely new problem

- R stays the same
- L changes
- C changes

Start over

- Each frequency gives a different answer



Summary

Voltage nodes works the same

- for DC inputs
- for AC inputs

The only difference is you're now dealing with complex numbers.

Matlab is able to solve N equations for N unknowns

- with real numbers, and
- with complex numbers

This is where Matlab really shines
