
Instrumentation Amplifiers

ECE 211 Circuits I

Lecture #16

Please visit [Bison Academy](#) for corresponding
lecture notes, homework sets, and solutions

Instrumentation Amplifiers

A very common type of op-amp amplifier is an instrumentation amplifier

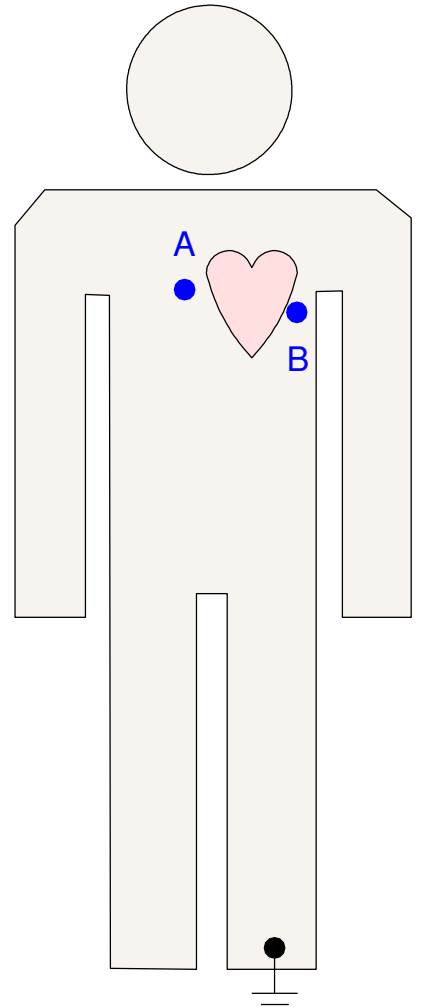
- Also known as a *differential amplifier*

The output is a gain time the difference between two voltages

$$Y = k(V_a - V_b)$$

This lets you...

- Remove 60Hz noise from a signal
 - *Both V_a and V_b are corrupted by 60Hz noise*
- Measure EKG signals (heartbeats)
 - *Amplify only signal created between A and B*
- Measure a fetus's heartbeat
 - *Both V_a and V_b are corrupted by the mother's heartbeat*
-



This Lecture

This lecture will cover three types of instrumentation amplifiers

- Single op-amp design
- Two op-amp design
- Three op-amp design

Using an instrumentation amplifier to measure temperature

- 0V @ 0C
- 5V @ 50C

Single Op-Amp Design

Covered previously

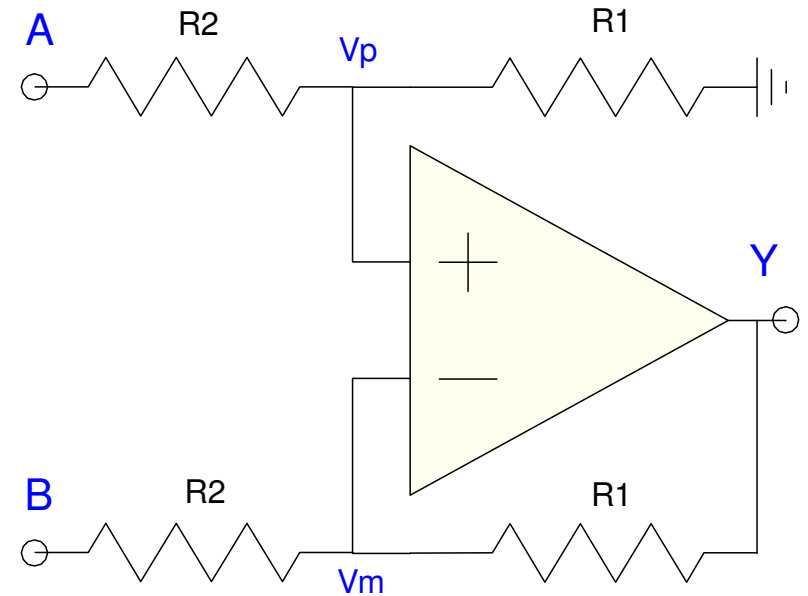
$$Y = \left(\frac{R_1}{R_2} \right) (A - B)$$

Advantages

- Simple design
- Only one op-amp

Disadvantages

- Need to change two resistors to adjust gain
- Common-mode gain if R's are different
- Loading on A and B



Common-Mode Gain

Assume R1's are slightly different

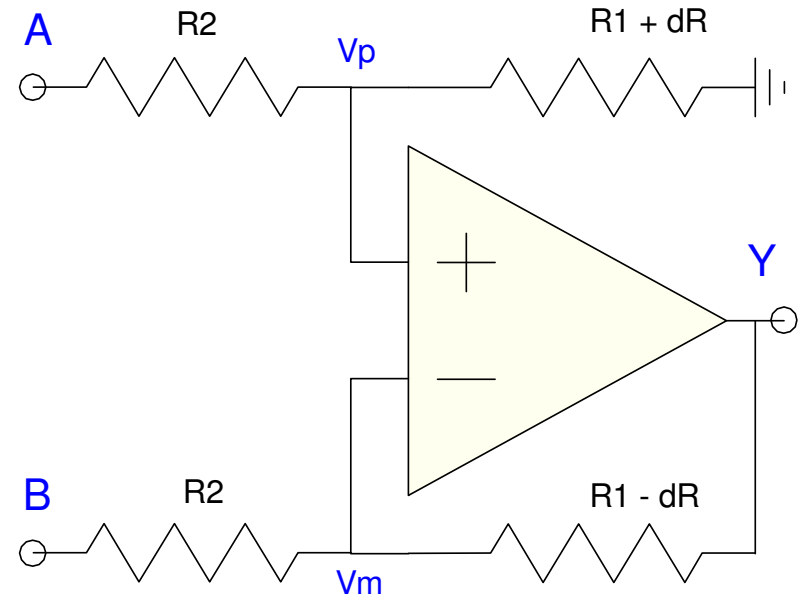
The overall gain is

$$Y = \left(\frac{R_1 + \delta R}{R_2} \right) A - \left(\frac{R_1 - \delta R}{R_2} \right) B$$

Doing some algebra

$$Y = \left(\frac{R_1}{R_2} \right) (A - B) + \left(\frac{\delta R}{R_2} \right) (A + B)$$

The common-mode (A+B) gets through to the output



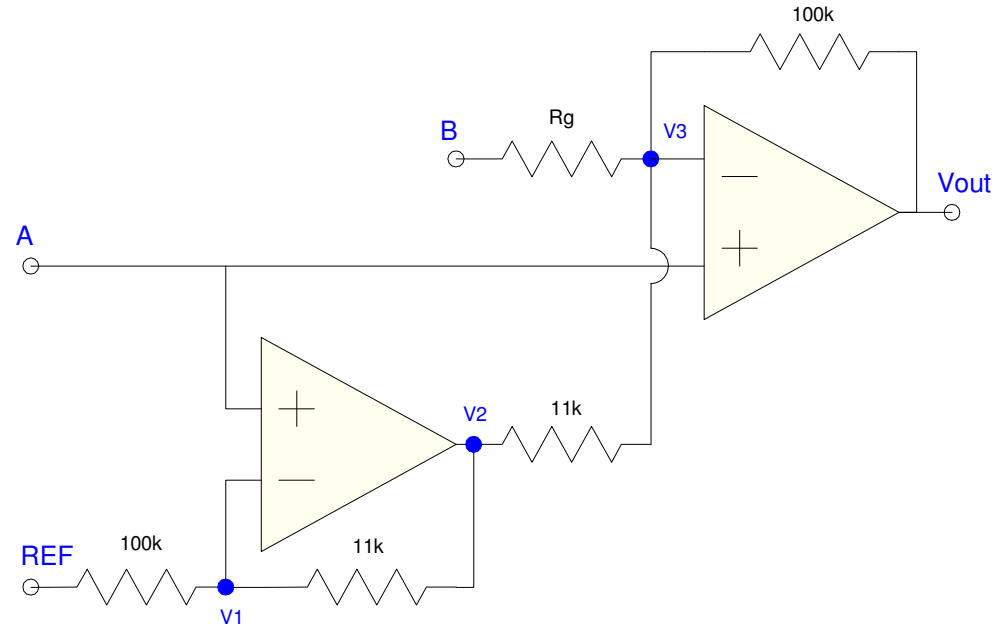
Two Op-Amp Design

- AMP04 op-amp (\$19 ea)
- \$0.12 if built using LM358 op-amps

$$V_{out} = \left(\frac{100k\Omega}{R_g} \right) (A - B) + V_{ref}$$

Advantage

- Single resistor (R_g) adjusts gain
- 100dB Common Mode Rejection Ratio
- $4G\Omega$ Input Impedance
 - *Buffers at inputs not shown*
- Output can be offset by REF
 - *Allows you to offset the output by 2.5V*



Calculations

$$V_p = V_m$$

$$V_1 = A$$

$$V_3 = A$$

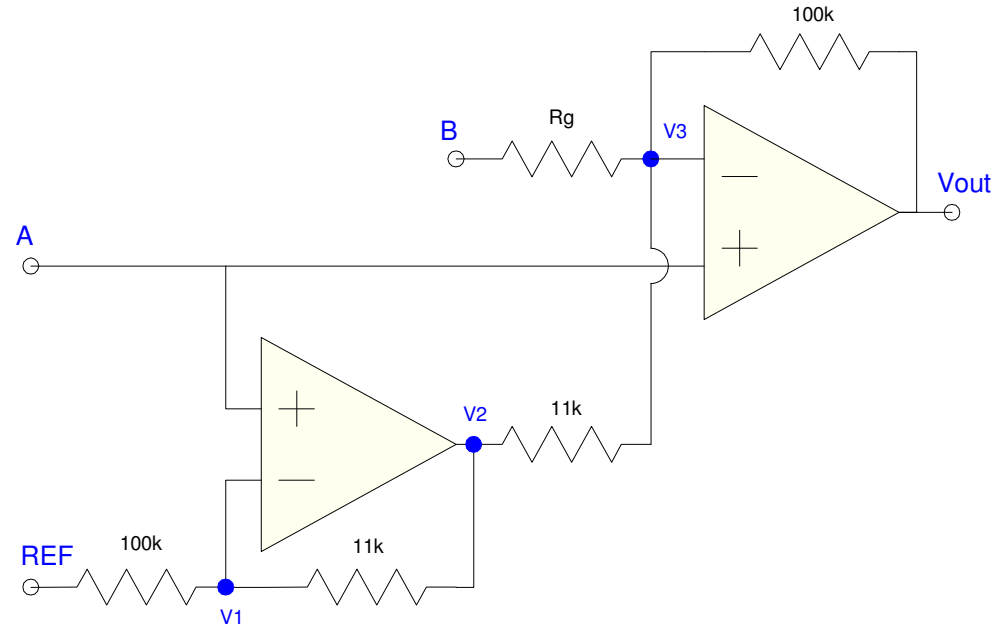
Voltage Node Equations

$$\left(\frac{V_1 - V_{ref}}{100k} \right) + \left(\frac{V_1 - V_2}{11k} \right) = 0$$

$$\left(\frac{V_3 - V_2}{11k} \right) + \left(\frac{V_3 - B}{R_g} \right) + \left(\frac{V_3 - V_{out}}{100k} \right) = 0$$

lots of algebra...

$$V_{out} = \left(\frac{100k}{R_g} \right) (A - B) + V_{ref}$$



Calculations

$$V_{out} = aA + bB + rR$$

Let $A=1$, $B=R=0$

$$V_3 = V_1 = 1$$

$$V_2 = 1 + \left(\frac{11k}{100k} \right)$$

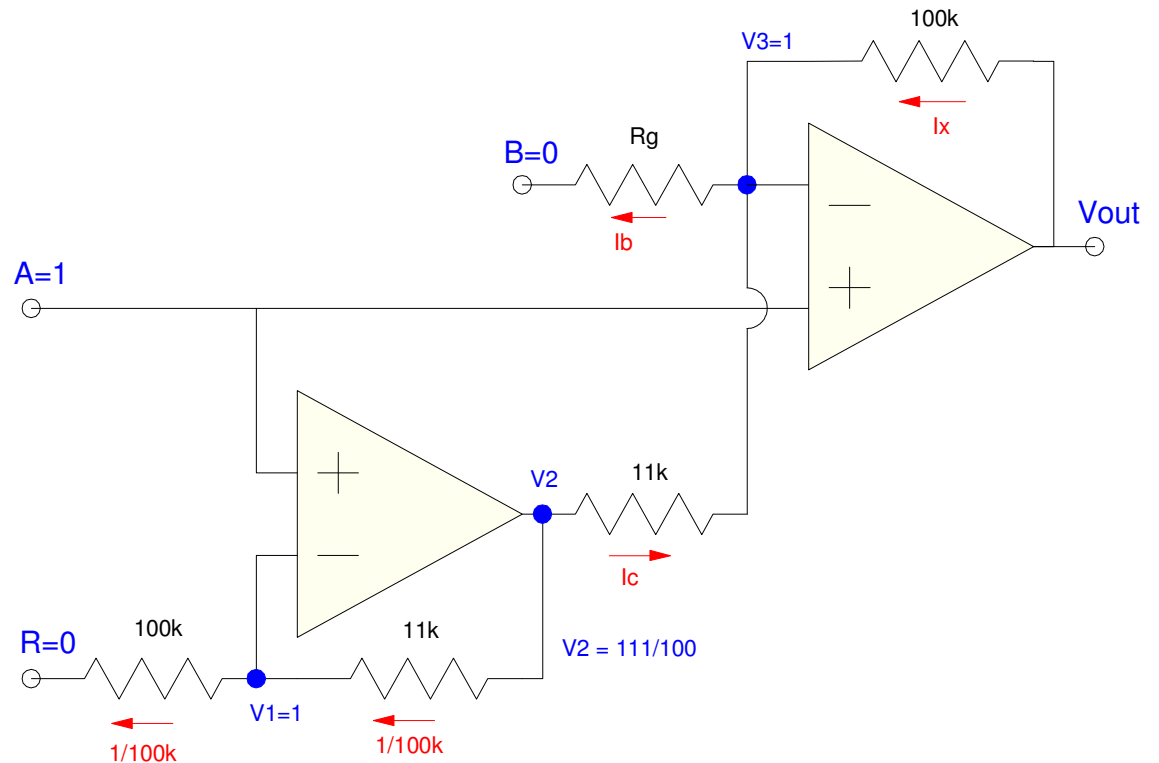
$$I_b = \left(\frac{1}{R_g} \right)$$

$$I_c = \left(\frac{1 + \frac{11k}{100k} - 1}{11k} \right) = \frac{1}{100k}$$

$$I_x = I_b - I_c$$

$$V_{out} = 1 + 100k \cdot I_x$$

$$V_{out} = \left(\frac{100k}{R_g} \right) = a$$



Calculations

$$V_{out} = aA + bB + rR$$

Let

- $A = 0$
- $B = 1$
- $R = 0$

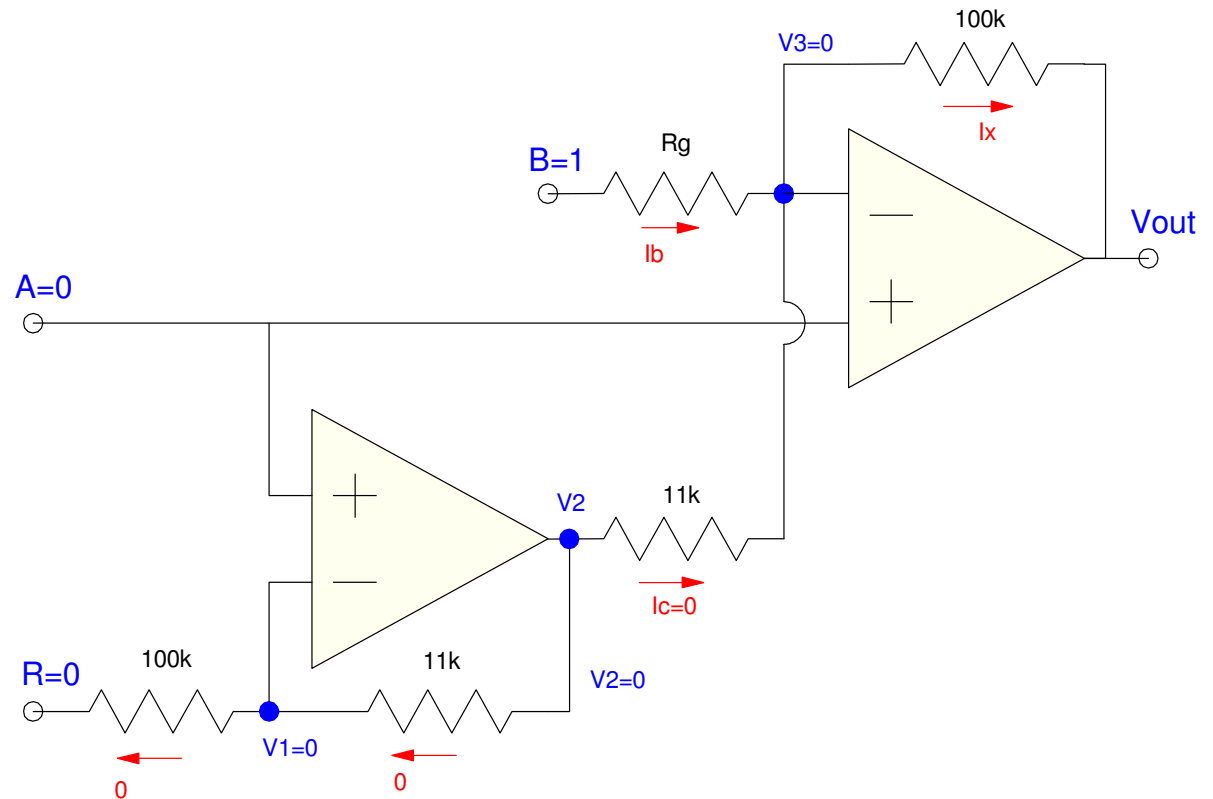
Then

$$I_b = \left(\frac{1}{R_g} \right)$$

$$I_x = I_b$$

$$V_{out} = -100k \cdot I_x$$

$$V_{out} = -\left(\frac{100k}{R_g} \right) = b$$



Calculations

$$V_{out} = aA + bB + rR$$

Let

- $A = B = 0$
- $R = 1$

Then

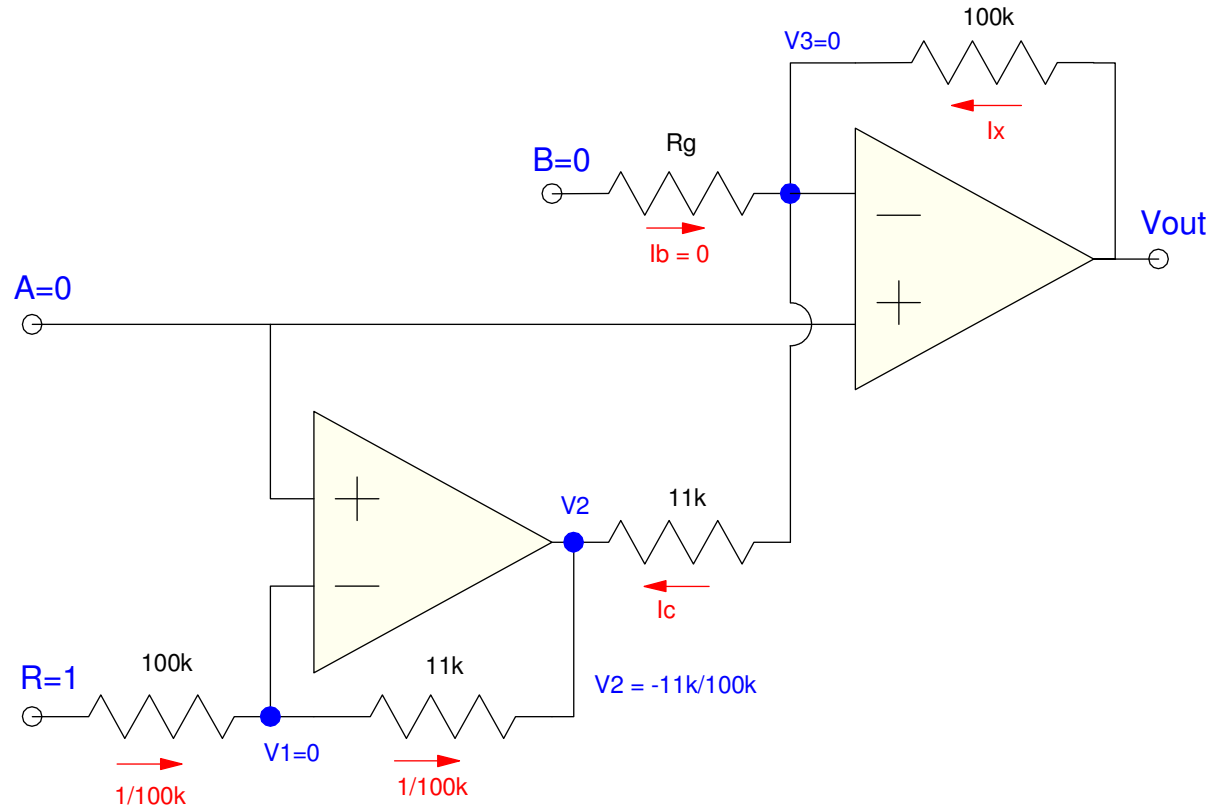
$$V_2 = -\left(\frac{11k}{100k}\right)$$

$$I_c = \left(\frac{0 - V_2}{11k}\right) = \left(\frac{1}{100k}\right)$$

$$I_x = I_c$$

$$V_{out} = 100k \cdot I_x$$

$$V_{out} = 1 = r$$



Example

A CdS light sensor is used to measure changing light levels.

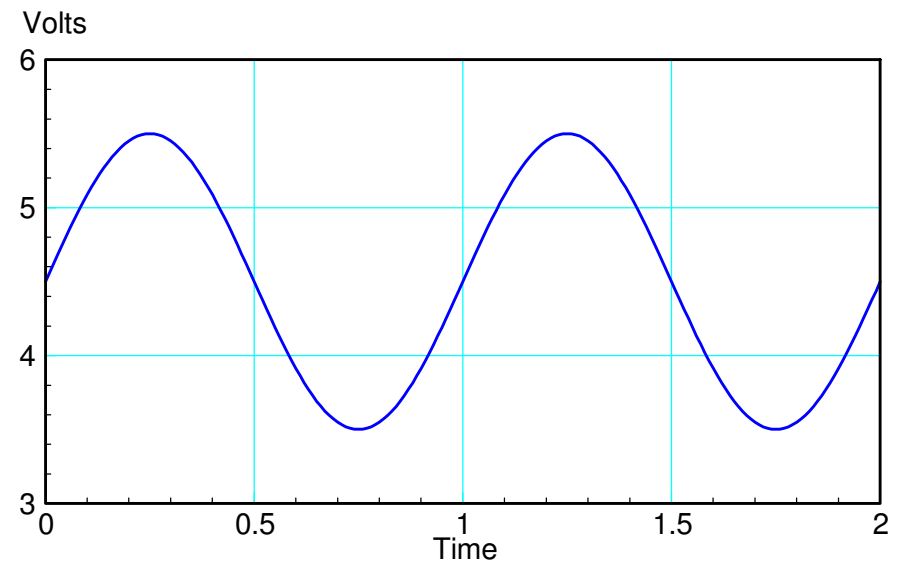
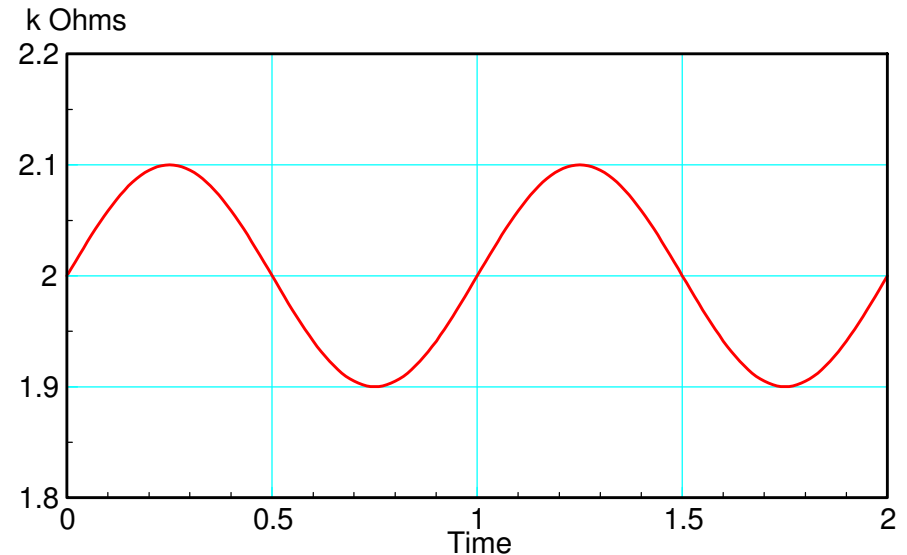
The resistance varies as

$$R = 2000 + 100 \cdot \sin(\omega t) \Omega$$

Design a circuit which outputs a

- 2Vpp signal
- Offset by 4.5V

so that you can use a single 9V supply



Solution

Convert R to V using a voltage divider

Assume a 2k resistor

$$V_a = \left(\frac{R}{R+2000} \right) 9V$$

Va has a spread of 0.2251 Vpp

Vout has a spread of $2V_{pp}$

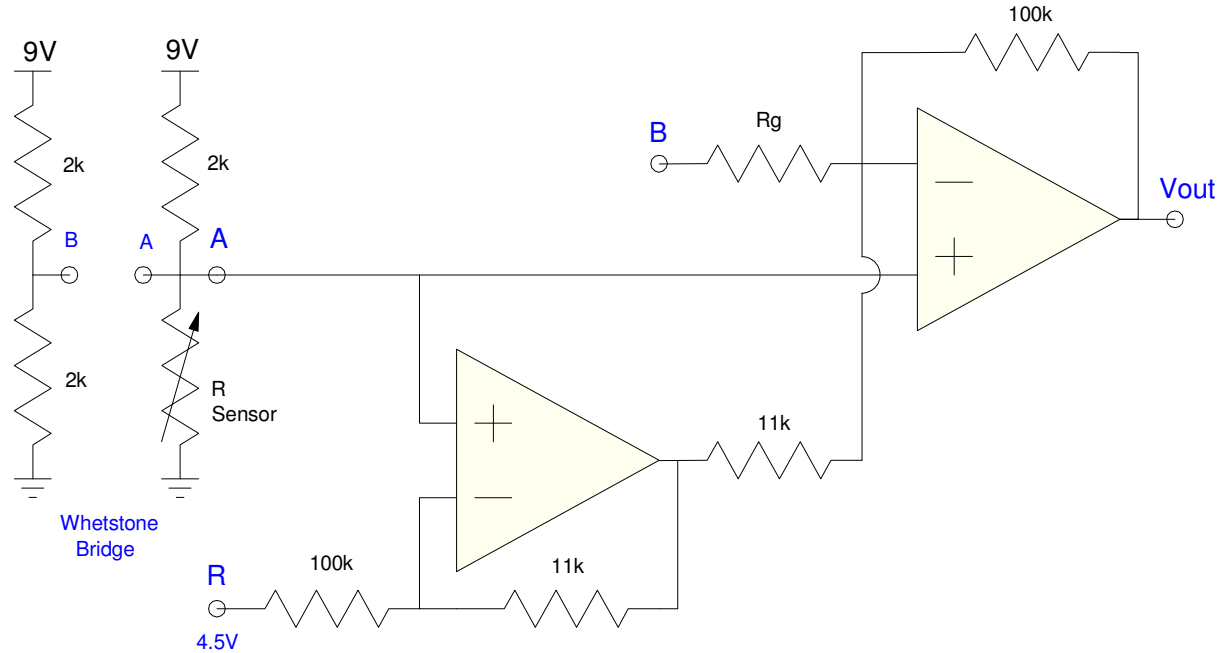
The gain needed is

$$gain = \left(\frac{2V_{pp}}{0.2251V_{pp}} \right) = 8.8833$$

R_g is then

$$gain = \left(\frac{100k}{R_g} \right) \Rightarrow R_g = 11.257k$$

For a 4.5V offset, $\text{Ref} = 4.5\text{V}$

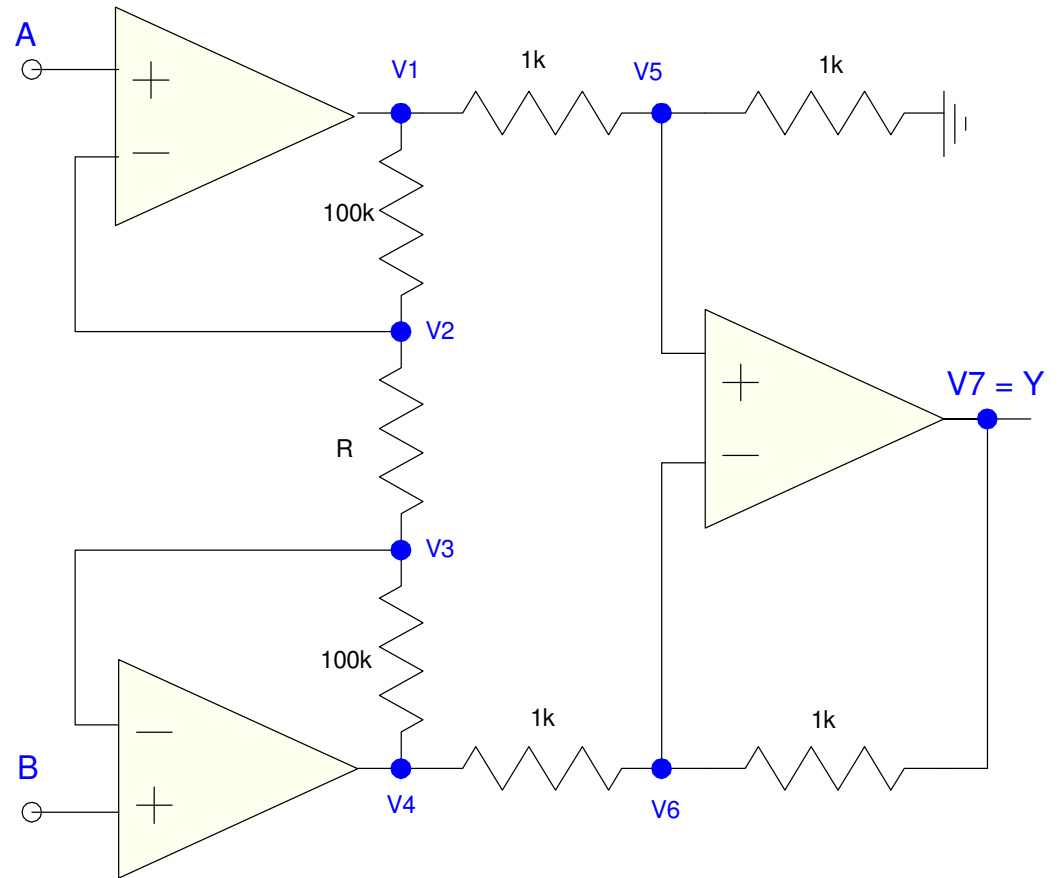


Three Op-Amp Design

$$Y = \left(1 + \frac{200k}{R}\right)(A - B)$$

Advantages

- Single resistor adjusts gain
- High input impedance



Calculations

$$V_p = V_m$$

$$V_2 = A$$

$$V_3 = B$$

$$V_5 = V_6$$

Node equations

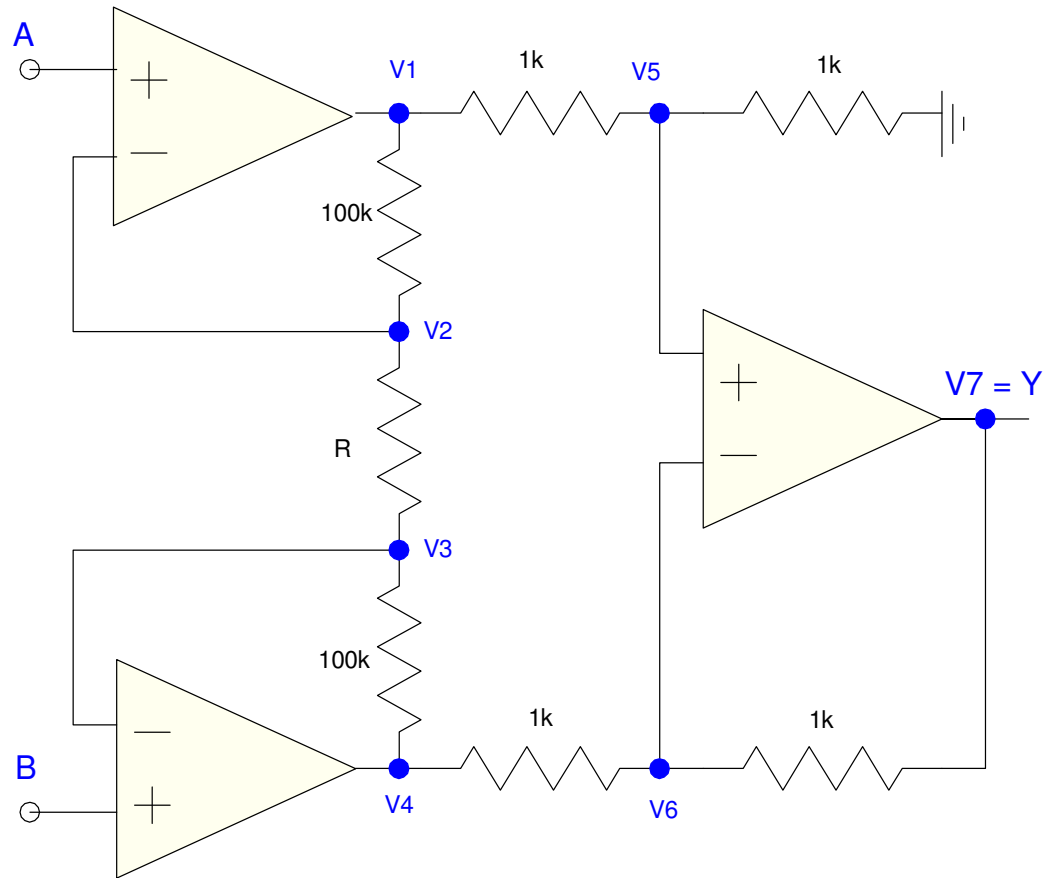
$$\left(\frac{V_2 - V_1}{100k} \right) + \left(\frac{V_2 - V_3}{R} \right) = 0$$

$$\left(\frac{V_3 - V_2}{R} \right) + \left(\frac{V_3 - V_4}{100k} \right) = 0$$

$$\left(\frac{V_5 - V_1}{1k} \right) + \left(\frac{V_5}{1k} \right) = 0$$

$$\left(\frac{V_6 - V_4}{1k} \right) + \left(\frac{V_6 - V_7}{1k} \right) = 0$$

Solve....



Calculations

- Take 2
- Let $B = 0$

$$V_1 = \left(1 + \frac{100k}{R}\right)A$$

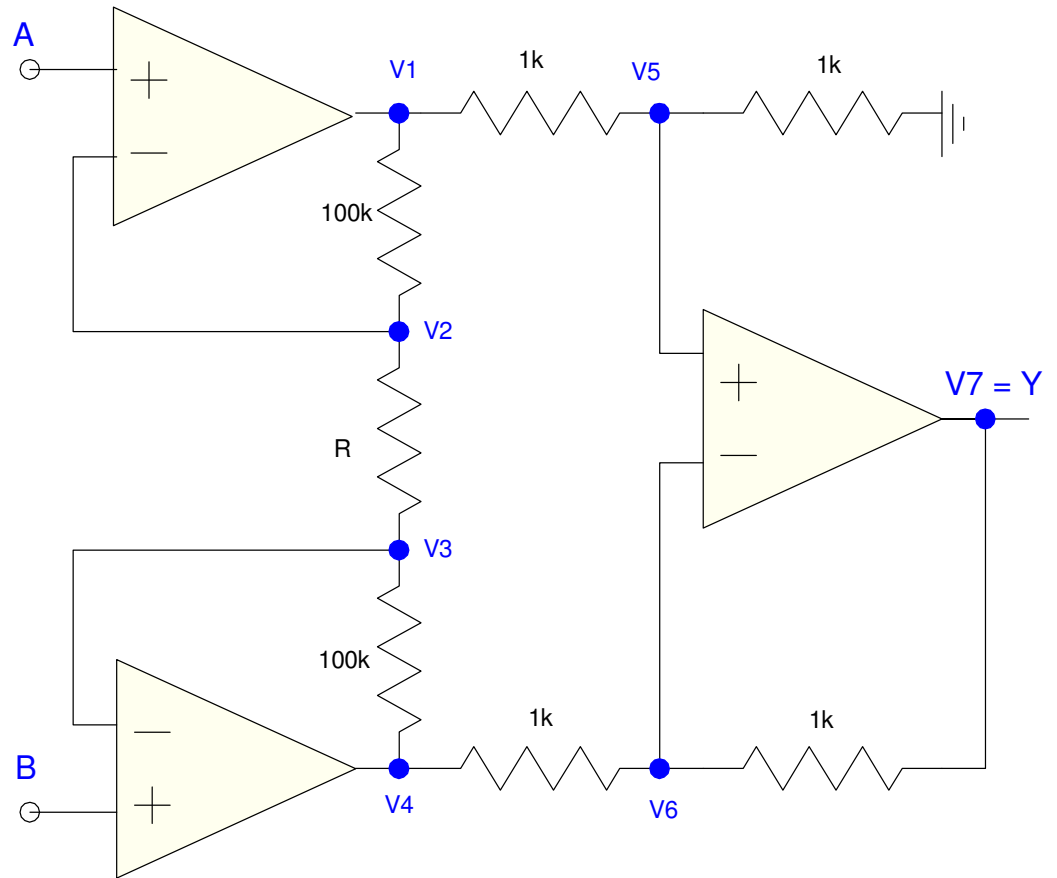
$$V_4 = -\left(\frac{100k}{R}\right)A$$

Observe that the right op-amp is a differential amplifier

$$V_7 = (V_1 - V_4)$$

Then

$$V_7 = \left(1 + \frac{200k}{R}\right)A$$



Calculations

- Take 2
- Let $A = 0$

$$V_4 = \left(1 + \frac{100k}{R}\right) B$$

$$V_1 = -\left(\frac{100k}{R}\right) B$$

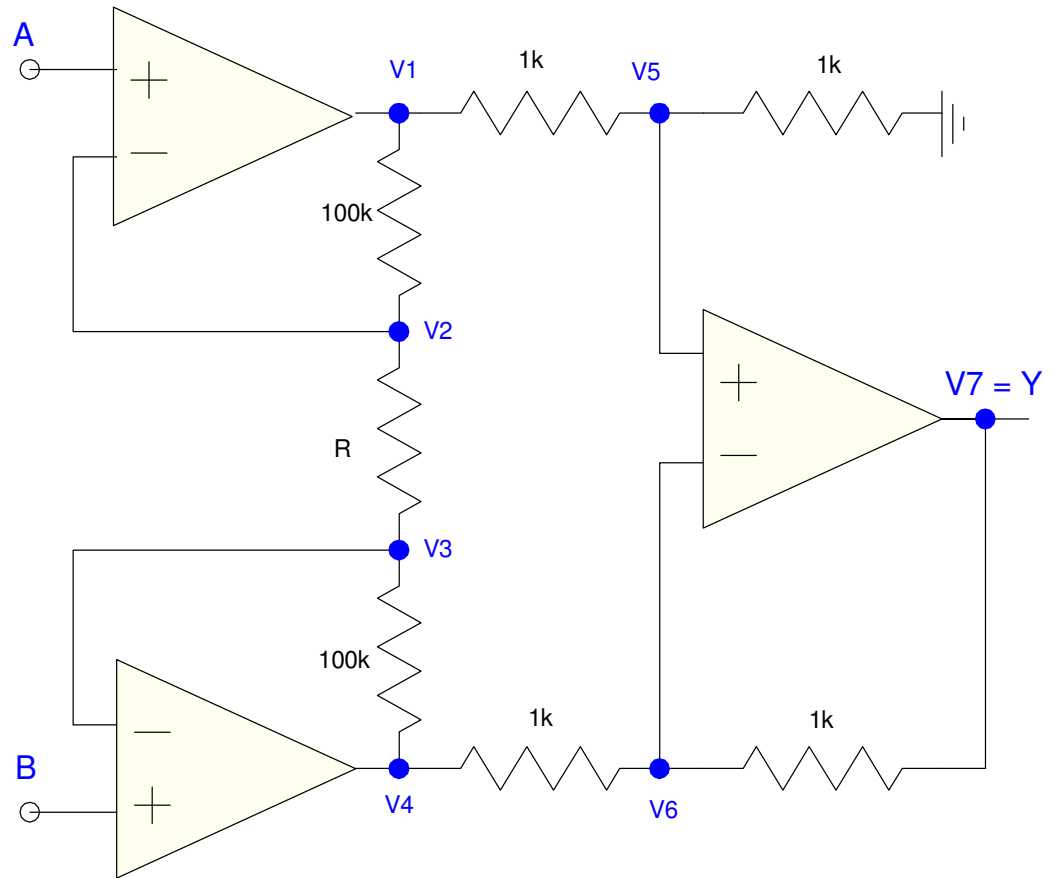
V7 is an instrumentation amplifier

$$V_7 = (V_1 - V_4)$$

$$V_7 = -\left(1 + \frac{200k}{R}\right) B$$

Together

$$Y = \left(1 + \frac{200k}{R}\right) (A - B)$$



Applications

Case 1: Voltage Amplification

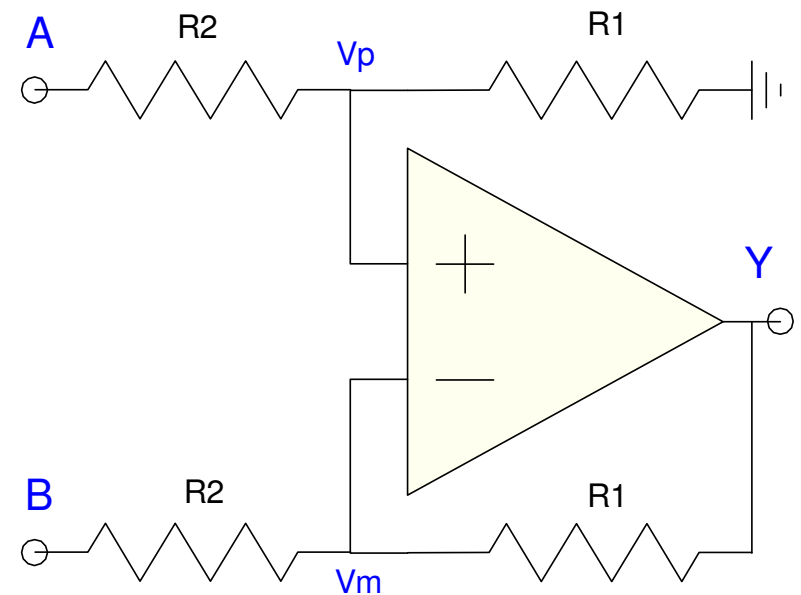
Design a circuit to convert 0..5V to -10V..+10V

Input

- $X = 0..5V$ analog voltage

Output

- $Y = -10V .. +10V$ analog voltage



Solution

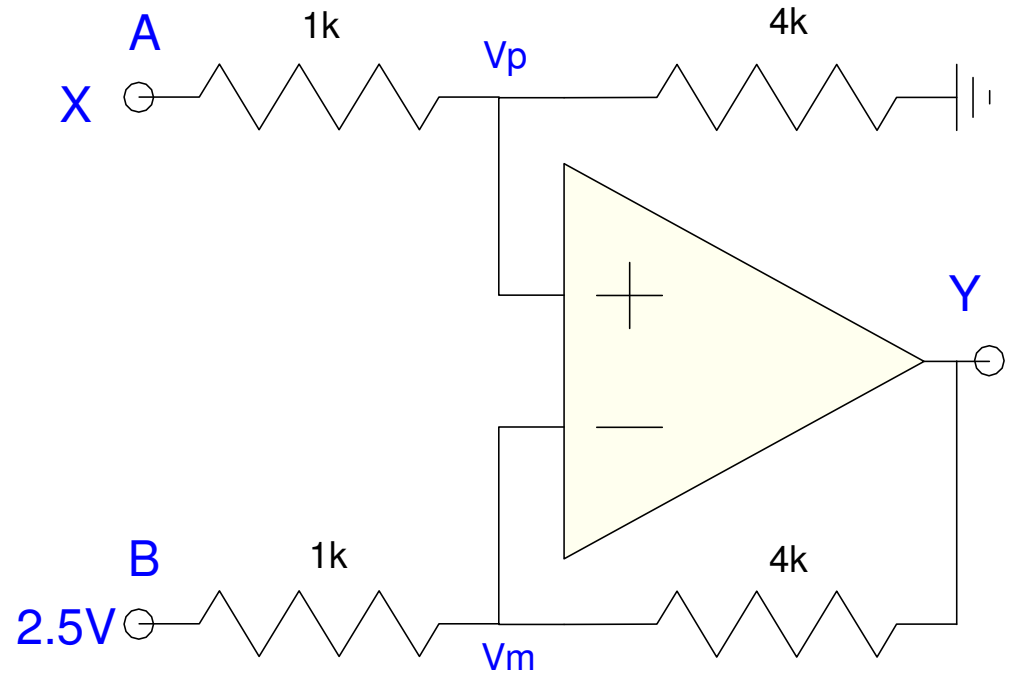
Express $y = f(x)$

$$y = 4x - 10$$

Rewrite as

$$Y = \left(\frac{R_1}{R_2} \right) (A - B)$$

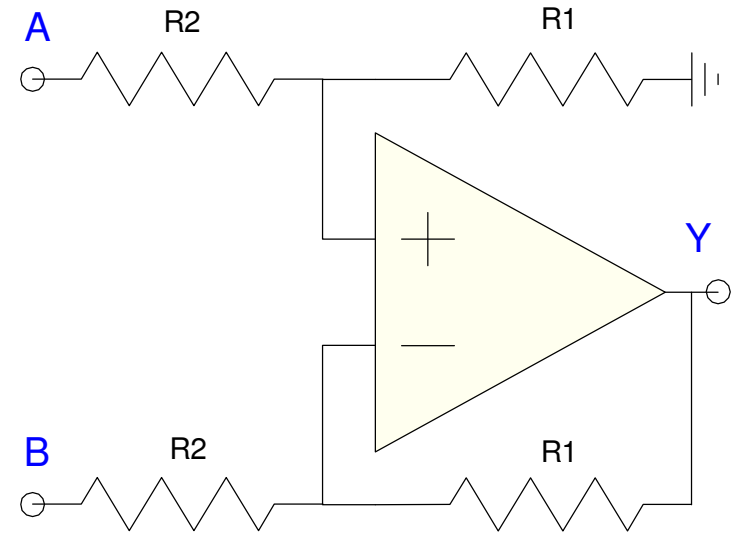
$$y = 4(x - 2.5)$$



Case 2: $V = f(R)$ (OhmMeter)

Output

- -10V when $R = 2000$ Ohms
- +10V when $R = 2200$ Ohms



Solution:

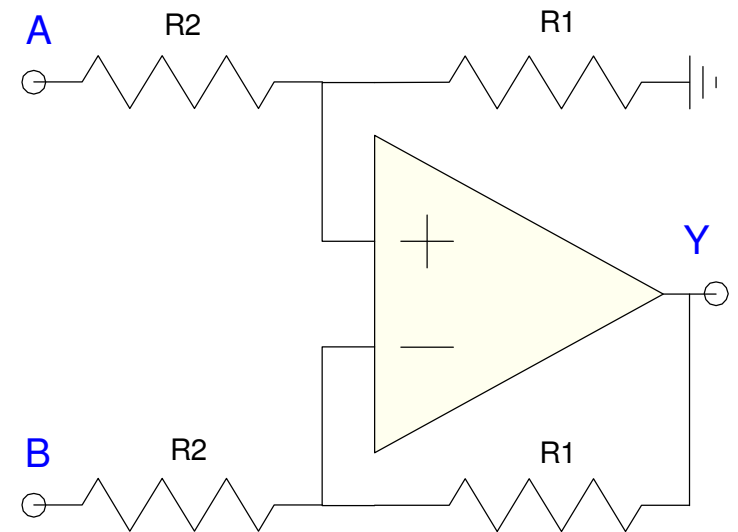
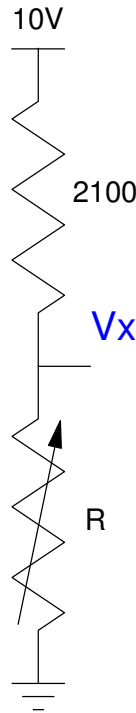
- Convert R to a voltage
- Assume a 2100 Ohm resistor

$R = 2000 \text{ Ohms}$ ($Y = -10V$)

$$V_x = \left(\frac{2000}{2000+2100} \right) 10V = 4.878V$$

$R = 2200 \text{ Ohms}$ ($Y = +10V$)

$$V_x = \left(\frac{2200}{2200+2100} \right) 10V = 5.116V$$

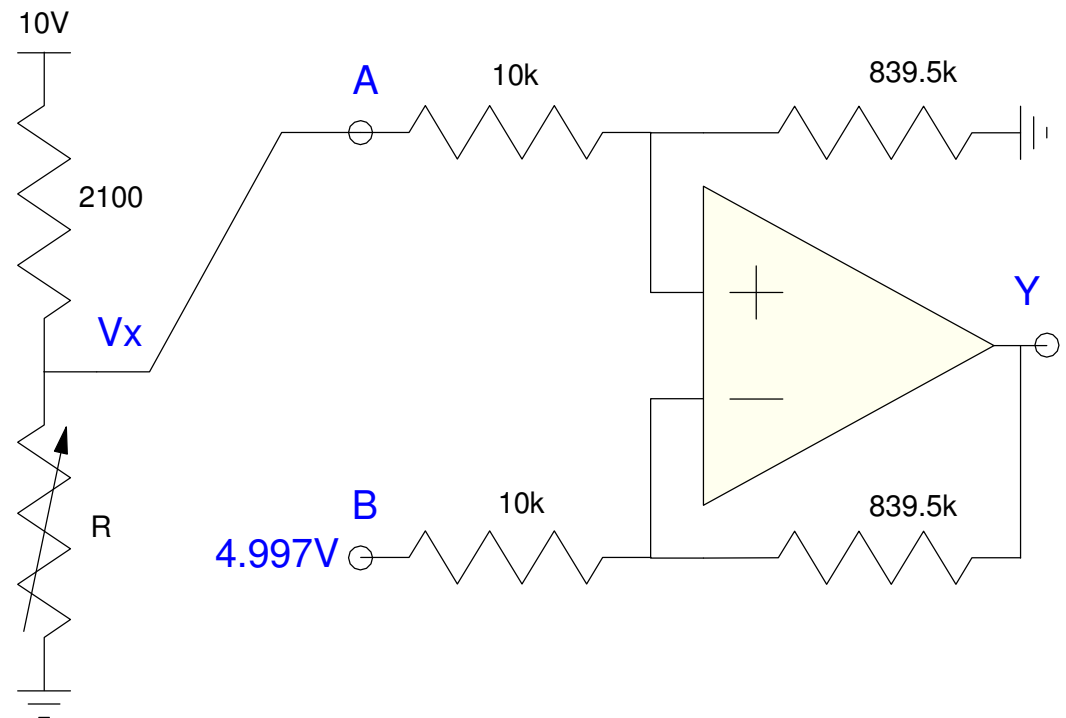


The gain you need is

$$gain = \left(\frac{\text{change in output}}{\text{change in input}} \right) = \left(\frac{10V - (-10V)}{5.116V - 4.878V} \right) = 83.95$$

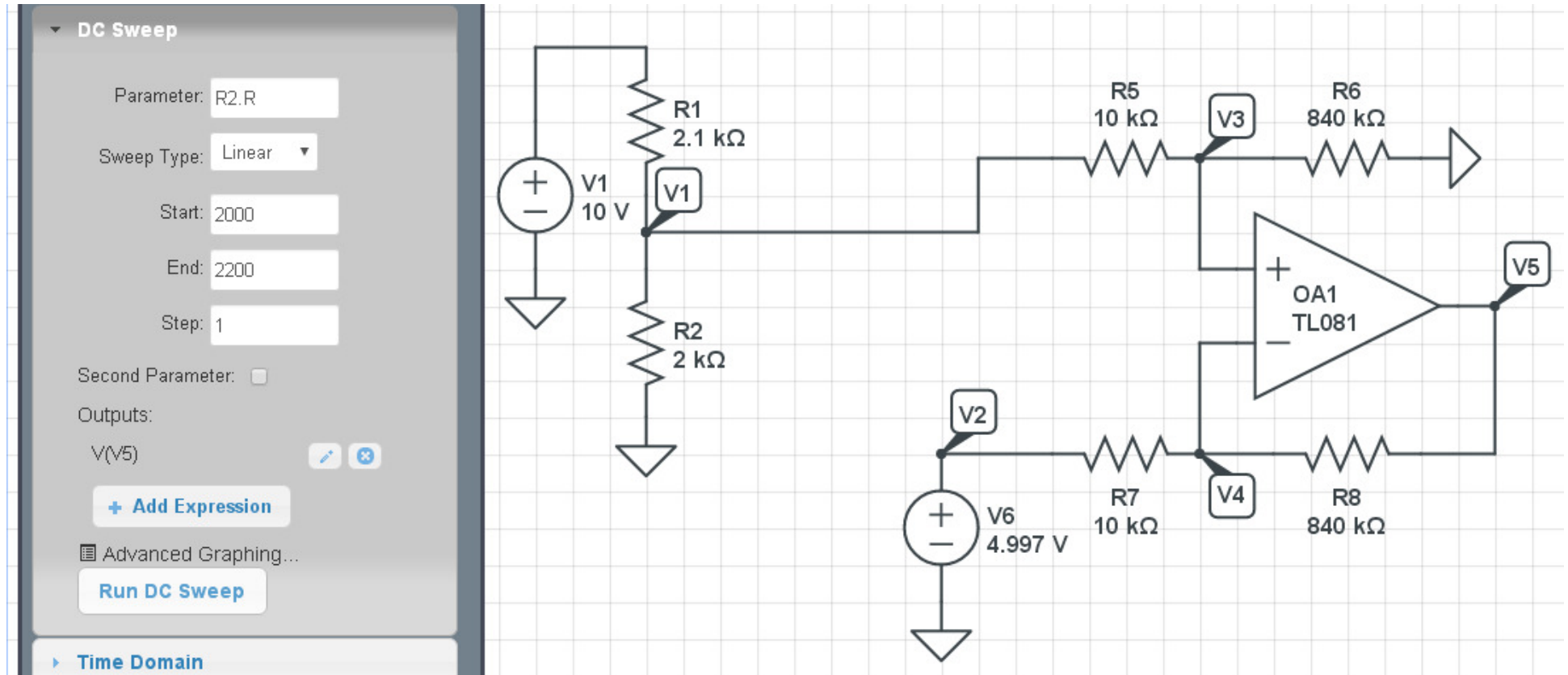
The output should be 0V (midband) when the input is midband

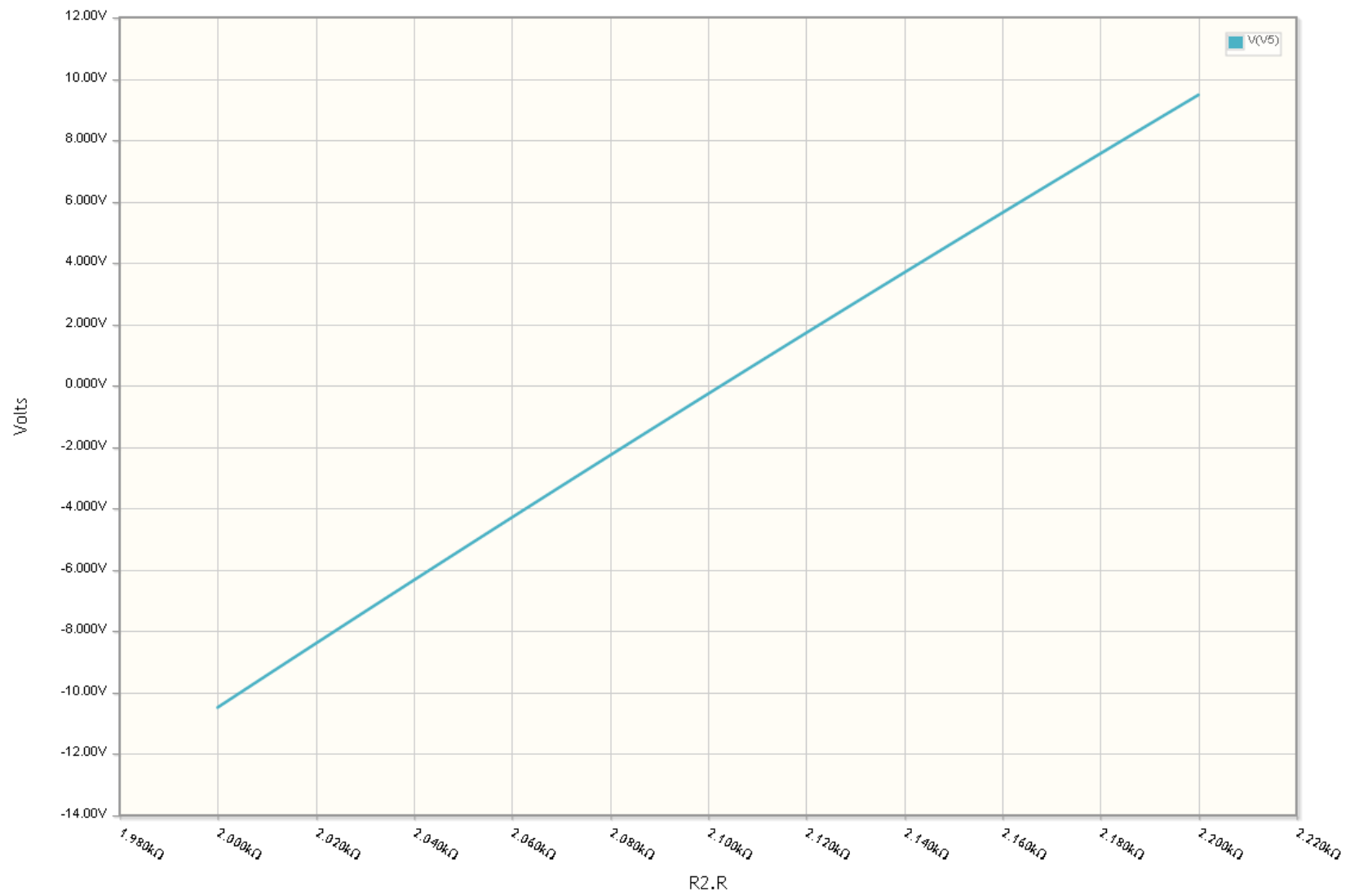
$$V_b = \left(\frac{4.878V + 5.116V}{2} \right) = 4.997V$$



Verification: Use CircuitLab

- Sweep R2 from 2000 to 2200 Ohms
- V5 should vary from -10V to +10V





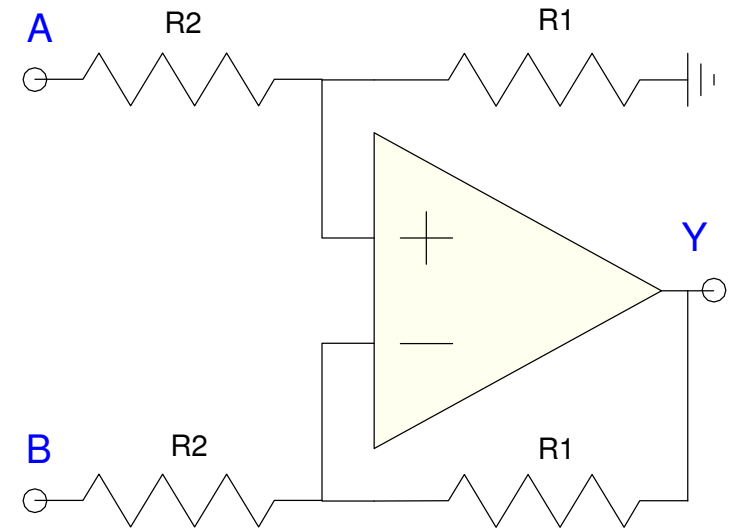
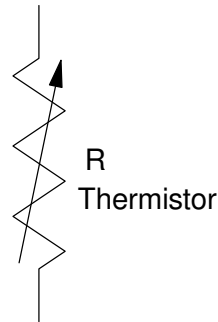
Case 3: Thermistor

Assume a thermistor: (T = degrees C)

$$R \approx 1000 \exp\left(\frac{3905}{T+273} - \frac{3905}{298}\right) \Omega$$

Design a circuit which outputs

- 0V at 0C
 - $R = 3320.125 \text{ Ohms}$
- +10V at +40C
 - $R = 533.664 \text{ Ohms}$



Solution:

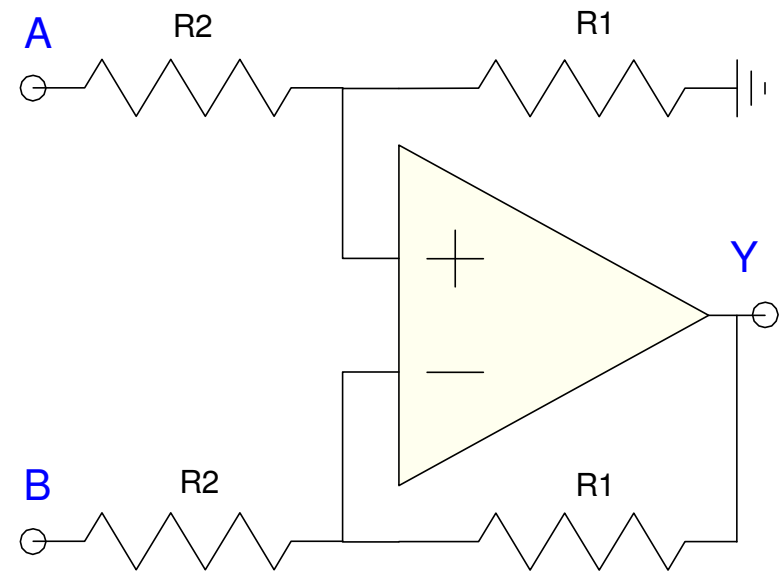
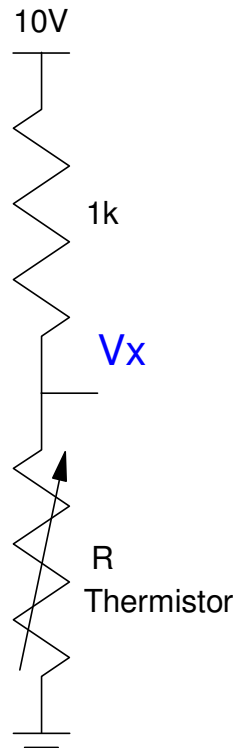
Use a voltage divider to convert R to V

At 0°C ($V_y = 0.00\text{V}$)

- $R = 3320.125\ \Omega$
- $V_x = 7.6853\ \text{V}$

At $+40^\circ\text{C}$ ($V_y = 10.00\text{V}$)

- $R = 533.664\ \Omega$
- $V_x = 3.4797\text{V}$



Solution

As V_b goes down, Y goes up

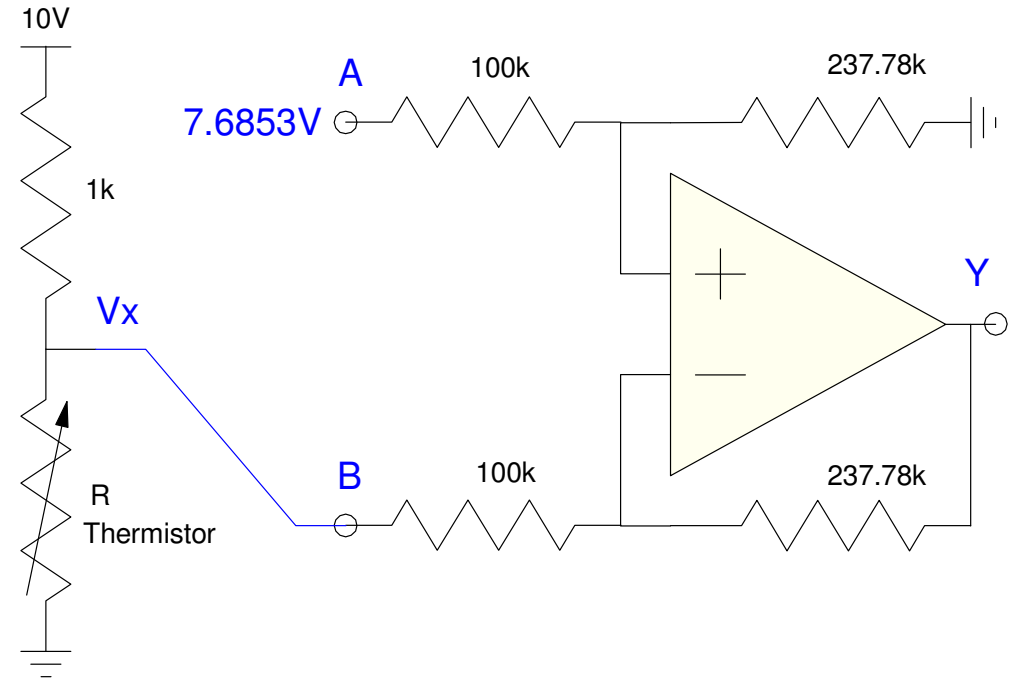
- Connect V_b to the minus input (B)

The output is 0V when $V_b = 7.6853V$

- Make $A = 7.6853V$

The gain needed is

$$gain = \left(\frac{10V - 0V}{3.4797V - 7.6853V} \right) = -2.3778$$



Checking in Matlab.

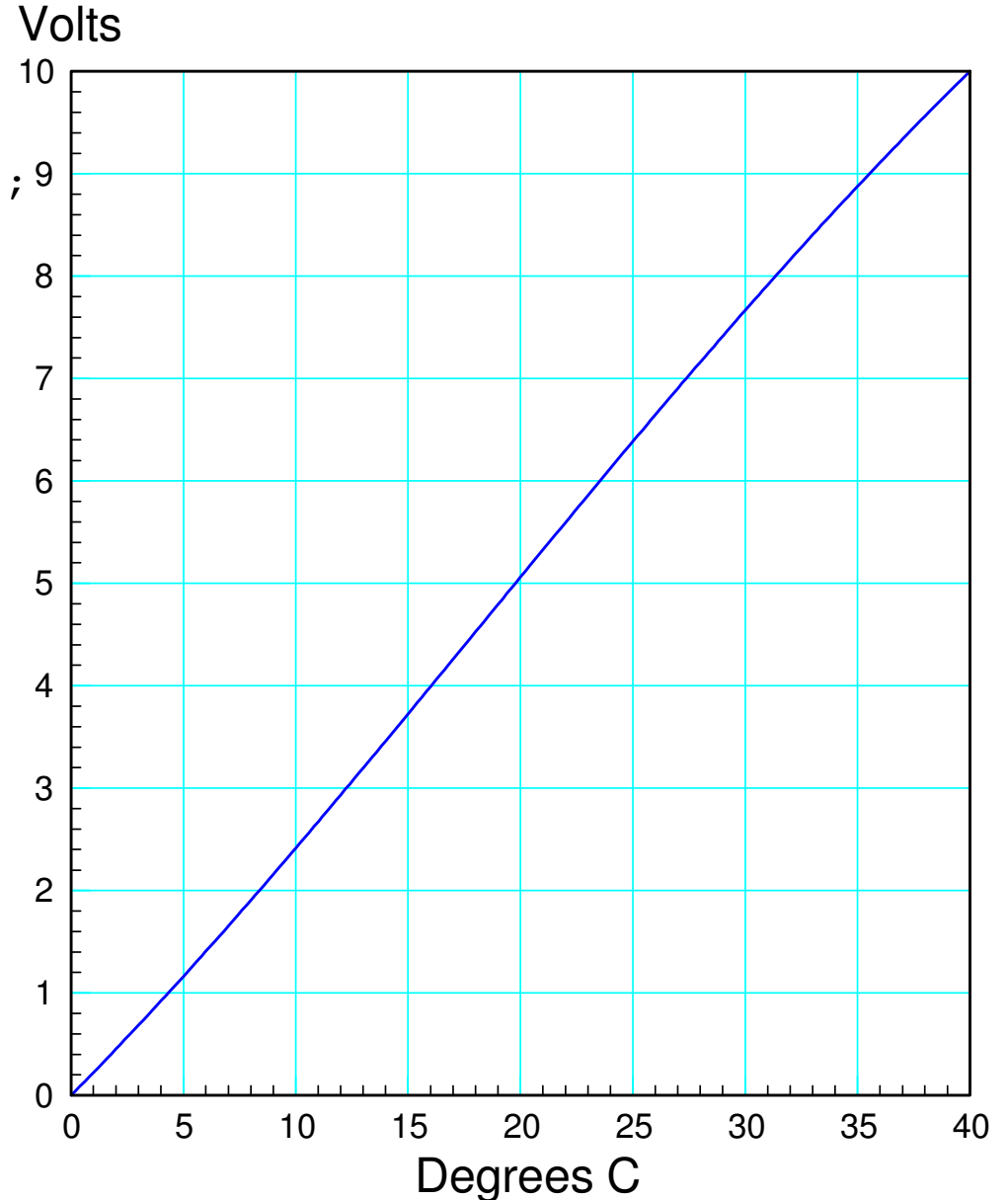
```
T = [0:0.2:40]';  
R = 1000*exp(3905./(T+273) - 3905/298);  
Va = R ./ (1000 + R) * 10;  
Y = 2.3778 * (7.6853 - Va);  
plot(T,Y);
```

Note:

- 0V @ 0C
- 10V @ 40C

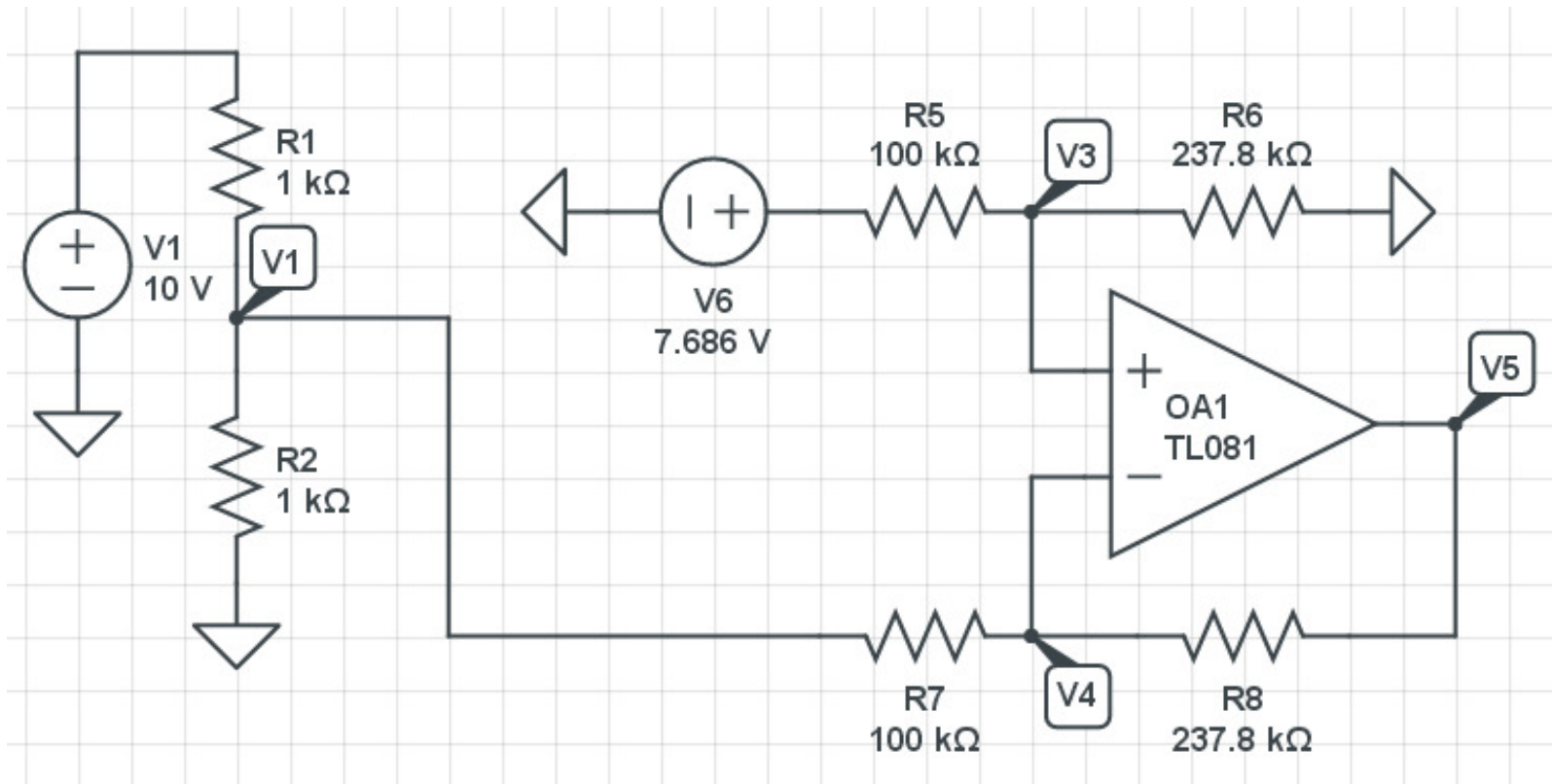
Not quite linear

- The thermistor is nonlinear
- Voltage division is nonlinear
- Result is nonlinear



Checking in CircuitLab. Check

- The left endpoint (0C or $R2 = 3320.125 \text{ Ohms}$)
- The midpoint (20C or $R2 = 1250.593 \text{ Ohms}$)
- The right endpoint (40C or $R2 = 533.664 \text{ Ohms}$)



V(V1)	7.668 V		
V(V3)	5.410 V		
V(V4)	5.410 V		
V(V5)	42.32 mV		

Output (V5) at 0C = 0.04232V (0V ideally)

V(V1)	5.556 V		
V(V3)	5.410 V		
V(V4)	5.410 V		
V(V5)	5.064 V		

Output (V5) at +20C = 5.064V (5.000V ideally)

V(V1)	3.486 V		
V(V3)	5.410 V		
V(V4)	5.410 V		
V(V5)	9.985 V		

Output (V5) at +40C = 9.985V (10.000V ideally)

Summary

Instrumentation amplifiers are differential amplifiers

$$Y = k(A - B)$$

Several variations exist

- More op-amps generally give better designs
- Single op-amp version is usually used in Circuits

With an instrumentation amplifier, you can

- Implement $y = ax + b$
- Output -10V to +10V as a resistor varies
- Output -10V to +10V as temperature varies

