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# **Thevenin Equivalents**

**Dependent Sources & Max Power Transfer**

**ECE 211 Circuits I**

**Lecture #11**

Please visit [Bison Academy](#) for corresponding  
lecture notes, homework sets, and solutions

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# Thevenin & Norton Equivalents

Any linear circuit's output follows a load line

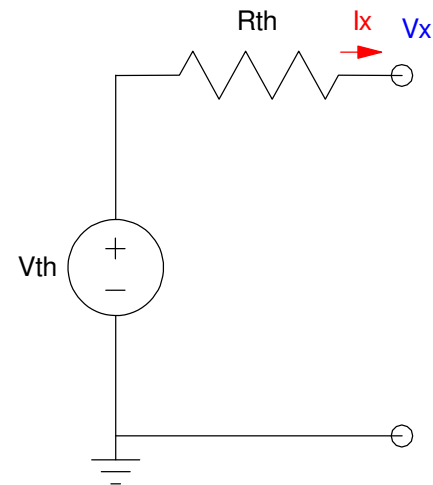
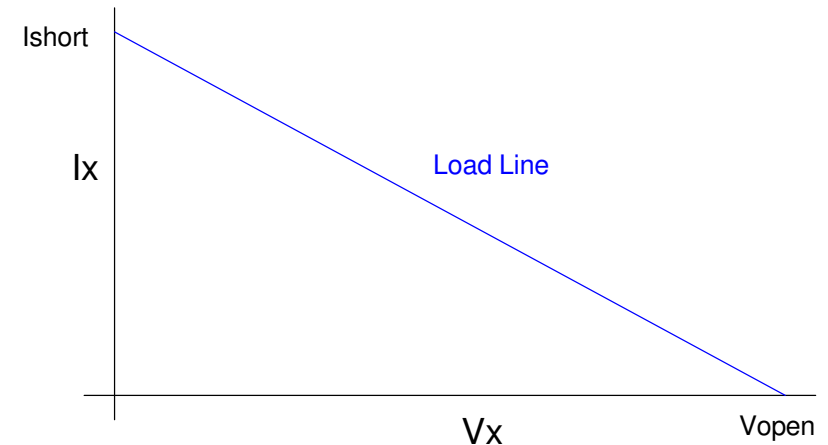
- $I_x$  varies linearly with  $V_x$

As far as the load is concerned

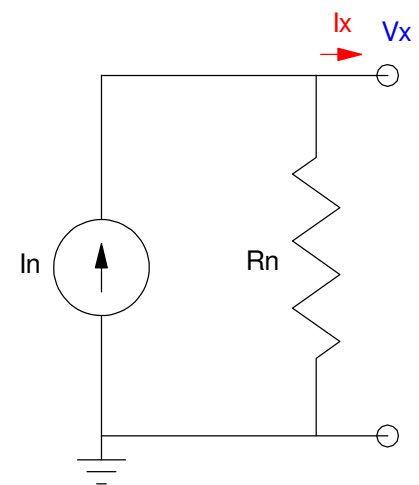
- All circuits with the same load line look alike

The simplest circuit to generate a given load line is

- Thevenin Equivalent
  - Voltage Source & Resistor in Series
- Norton Equivalent
  - Current Source & Resistor in Parallel



Thevenin Equivalent



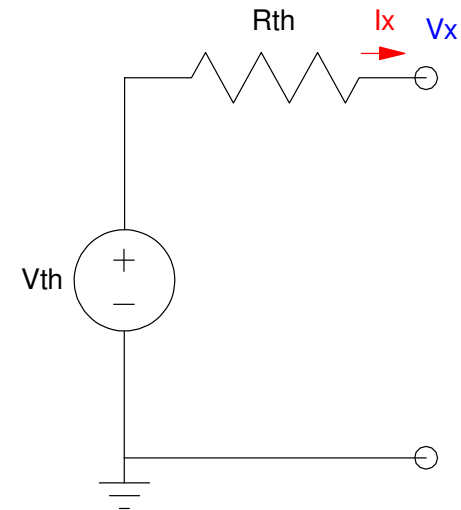
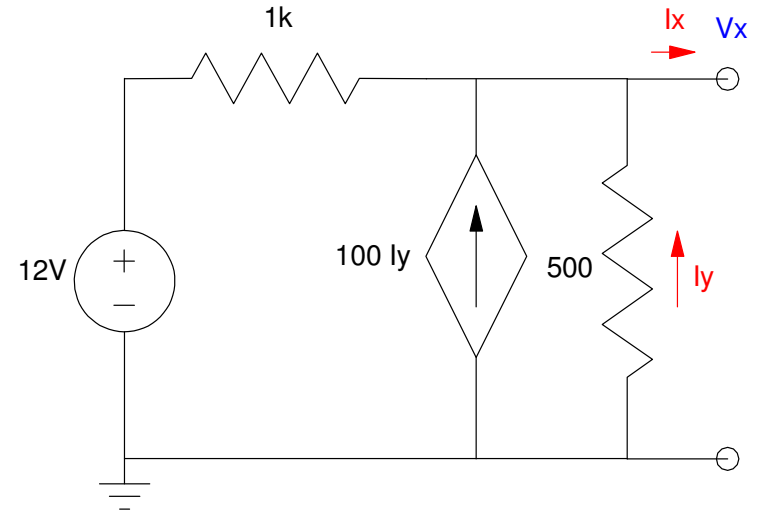
Norton Equivalent

## Previous Lecture

- Find the Thevenin & Norton equivalents
- For resistor circuits

## This Lecture

- Thevenin for Circuits with Dependent Sources
- Max Power Transfer



Thevenin Equivalent

# Types of Dependent Sources

Dependent sources are

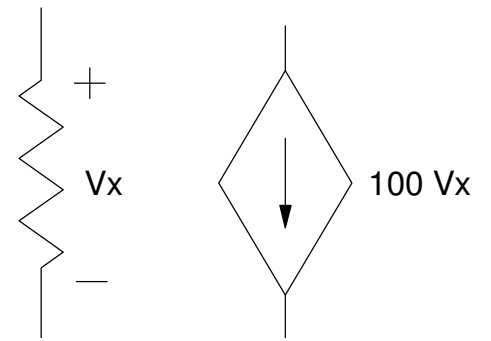
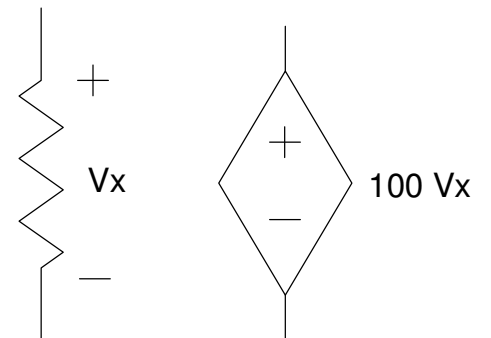
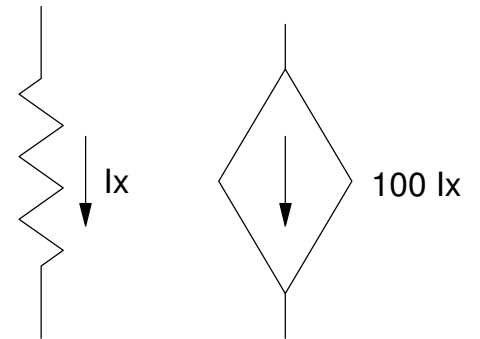
- Sources (voltage or current)
- Which depend upon another variable
- Dependent sources complicate analysis

Example:

- Current-controlled current source
  - BJT transistor
- Voltage-controlled voltage source
  - Amplifier
- Voltage-controlled current source
  - MOSFET

Devices which do this are covered in Electronics

- For this class, just treat these as sources

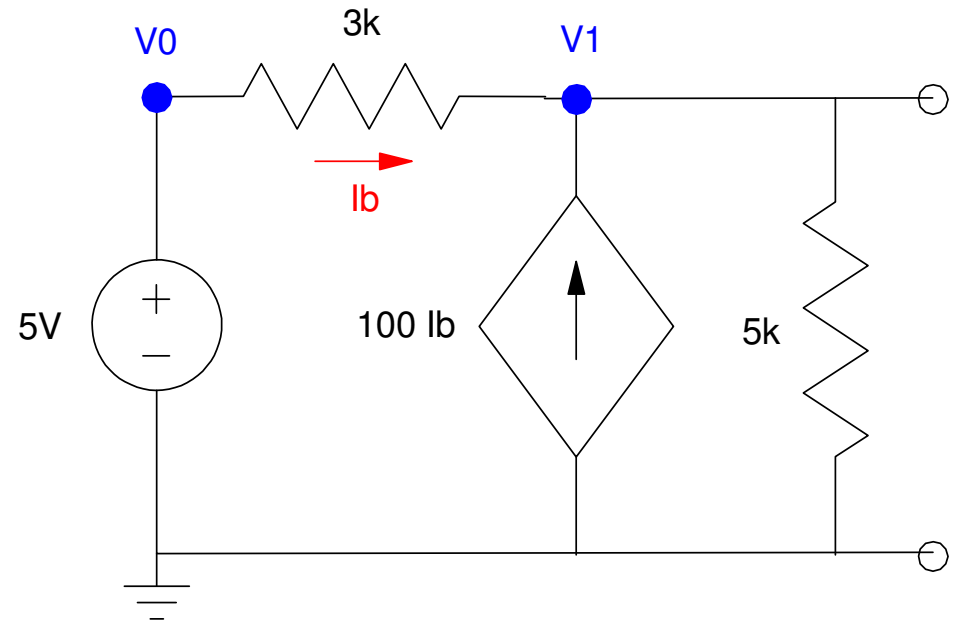


## Case 1: Current-Controlled Current Sources

- BJT Transistor

Find the Thevenin equivalent for the following circuit

- $V(\text{open}) = V_{th}$
- $I(\text{short}) = I_n$
- $R_{th}$



## **V(open) = Vth**

- Nothing changes with dependent sources
- Find the open-circuit voltage.

Write the voltage node equation at V1

$$I_b = \left( \frac{5 - V_1}{3k} \right)$$

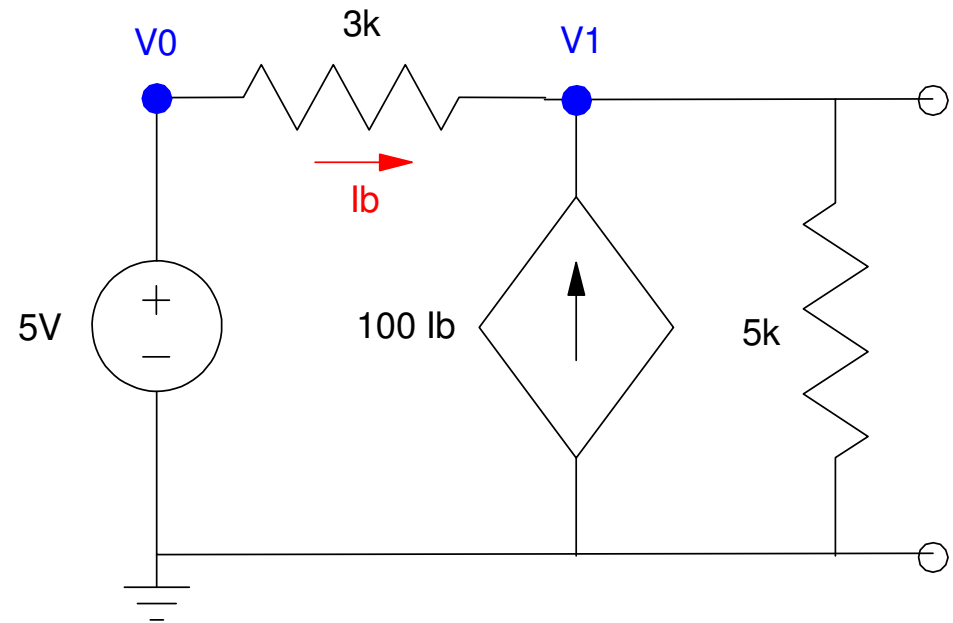
$$-I_b - 100I_b + \left( \frac{V_1}{5k} \right) = 0$$

Substitute and solve

$$V_1 = \left( \frac{\left( \frac{101}{3k} \right)}{\left( \frac{101}{3k} \right) + \left( \frac{1}{5k} \right)} \right) 5V = 4.9705V$$

This is  $V_{th}$ .

$$\mathbf{V(open) = Vth = 4.9705V}$$



## **I(short) = I<sub>n</sub>**

- Short the output
- Compute the current through the short

By observation

- $V_0 = 5V$
- $V_1 = 0V$

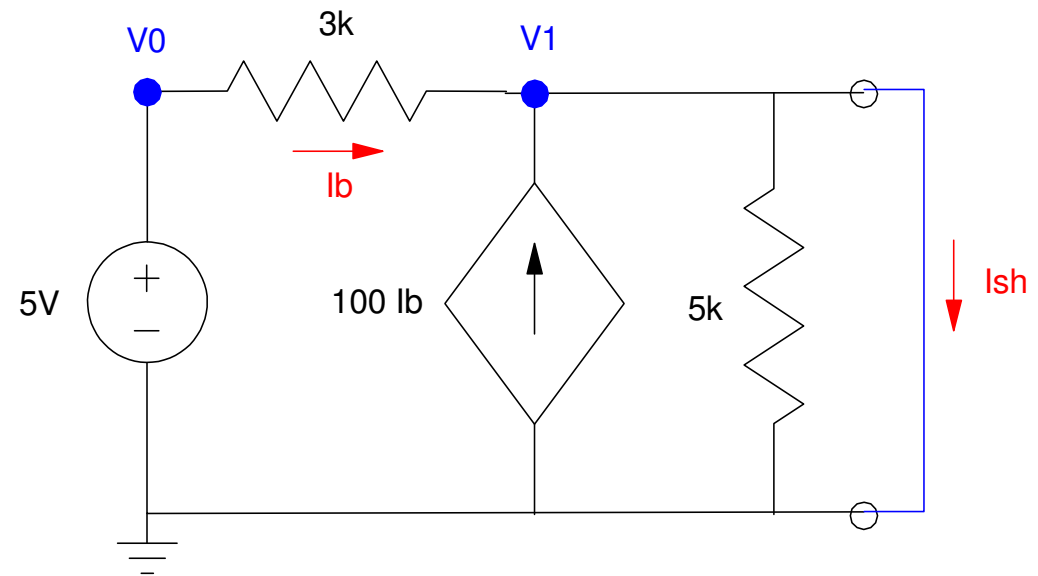
Conservation of Current at V1

$$I_b = \left( \frac{5V - 0V}{3k} \right) = 1.667mA$$

$$-I_b - 100I_b + \left( \frac{V_1}{5k} \right) + I_{short} = 0$$

$$I_{short} = 168.33mA$$

$$\mathbf{I_n = I(short) = 168.33mA}$$



**$R_{th}$ :**

Option 1:

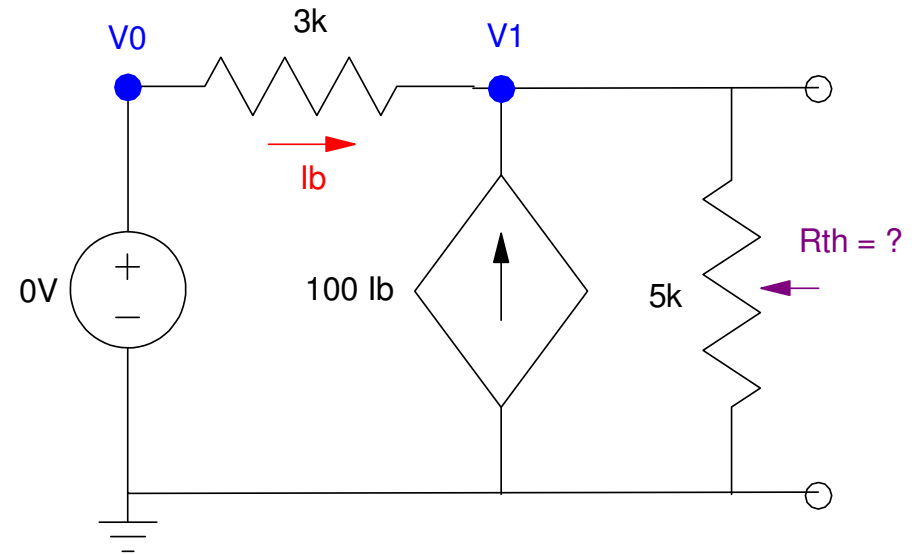
- Compute  $V(\text{open})$  &  $I(\text{short})$
- Then

$$R_{th} = \frac{V_{open}}{I_{short}} = 29.53\Omega$$

Option #2:

- Turn off the sources
- Compute the resistance looking in

With the dependent source, that's not obvious



## Finding Rth:

- Apply a 1V test voltage
- Compute the current draw ( $I_x$ )

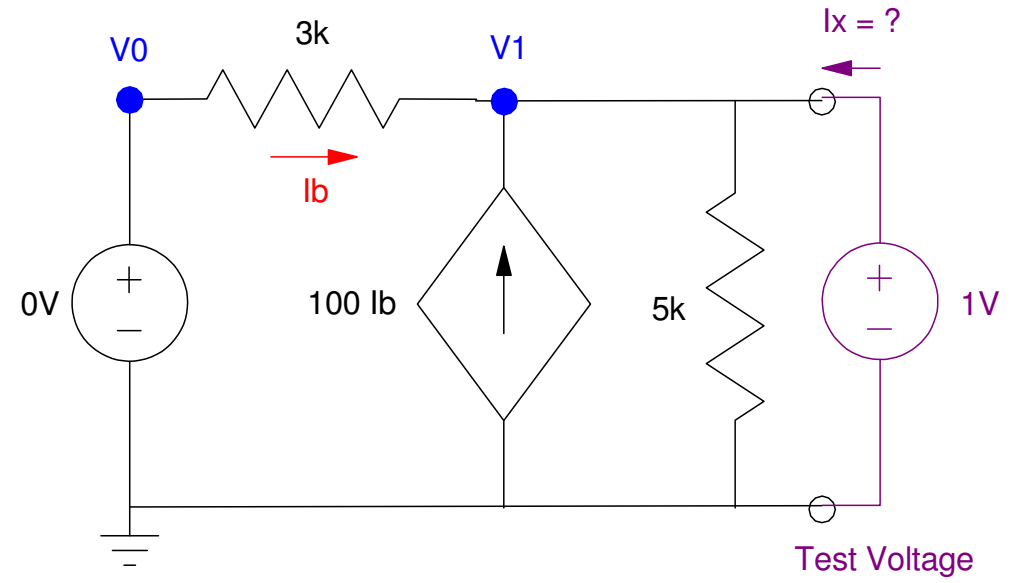
$$I_b = \left( \frac{0V - 1V}{3k} \right)$$

$$I_x = -I_b - 100I_b + \left( \frac{1V}{5k} \right)$$

$$I_{in} = 33.87mA$$

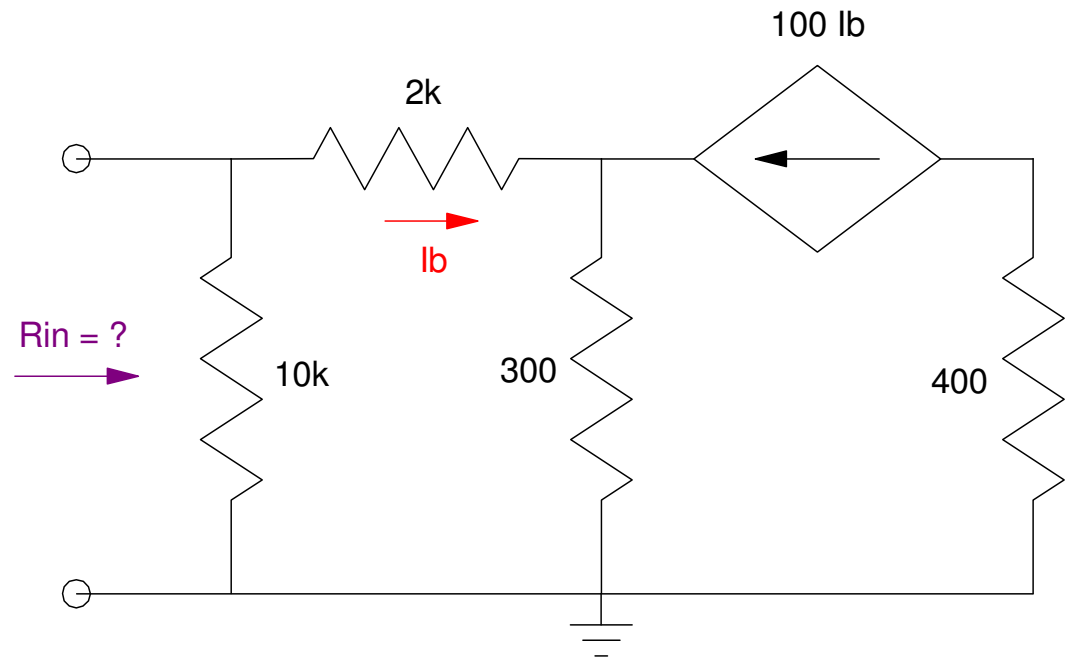
So

$$R_{th} = \frac{V_{in}}{I_{in}} = \frac{1V}{33.87mA} = 29.53\Omega$$



## Practice Problem:

Find the impedance looking in ( $R_{in}$ )



## Practice Problem:

- Apply a 1V source
- Compute the current draw

### Equations

$$I_b = \left( \frac{1 - V_1}{2k} \right)$$

$$-I_b - 100I_b + \frac{V_1}{300} = 0$$

### Solving

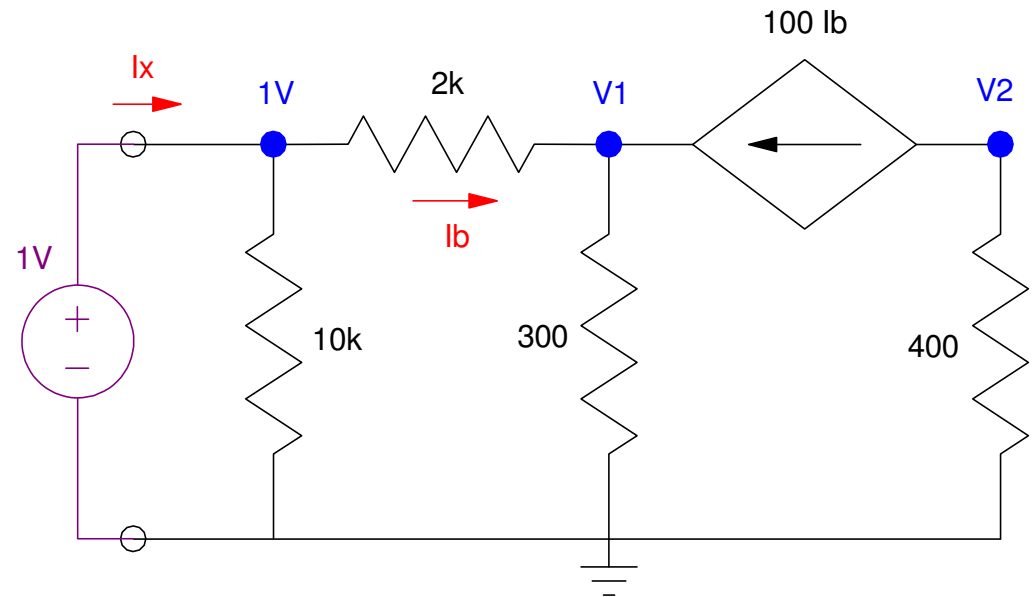
$$V_1 = 0.93808V$$

$$I_x = \left( \frac{1V}{10k} \right) + \left( \frac{1 - V_1}{2k} \right)$$

$$I_x = 130.9\mu A$$

Thus

$$R_{in} = \frac{1V}{130.9\mu A} = 7635\Omega$$

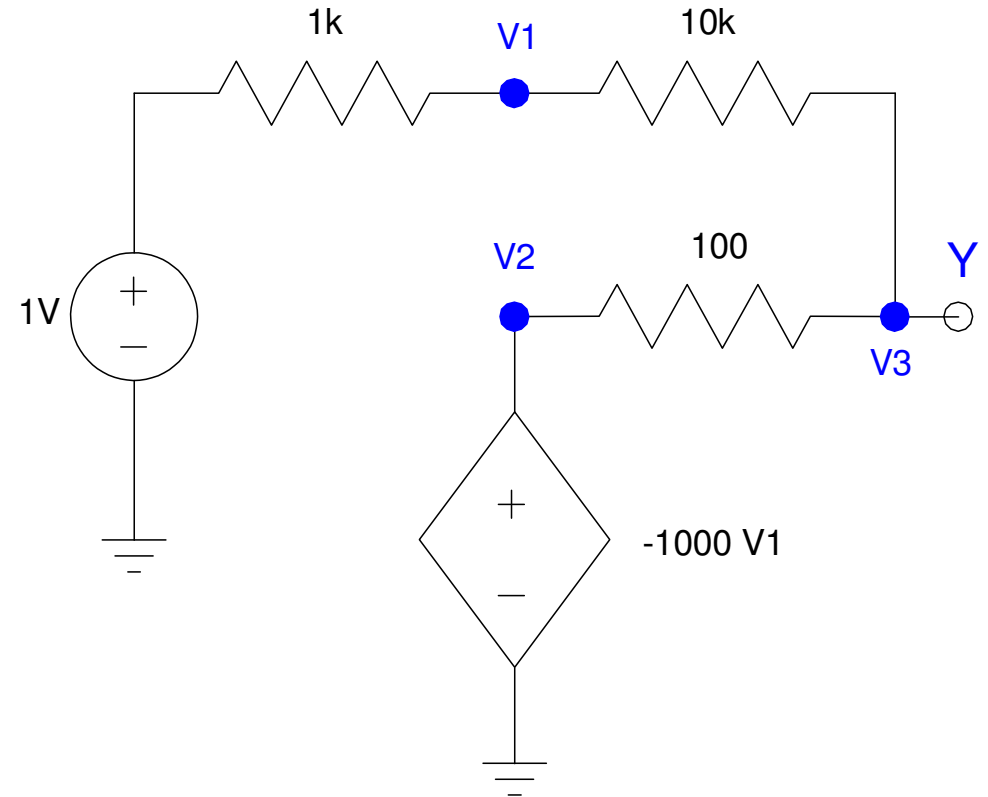


## Case 2: Voltage-Controlled Voltage Source

- Op-Amp

Find the Thevenin equivalent

- $V(\text{open}) = V_{th}$
- $I(\text{short}) = I_n$
- $R_{th} = R_n$



## Case 2: V(open)

- V<sub>th</sub>

Find V<sub>3</sub> (open circuit voltage)

$$V_2 = -1000V_1$$

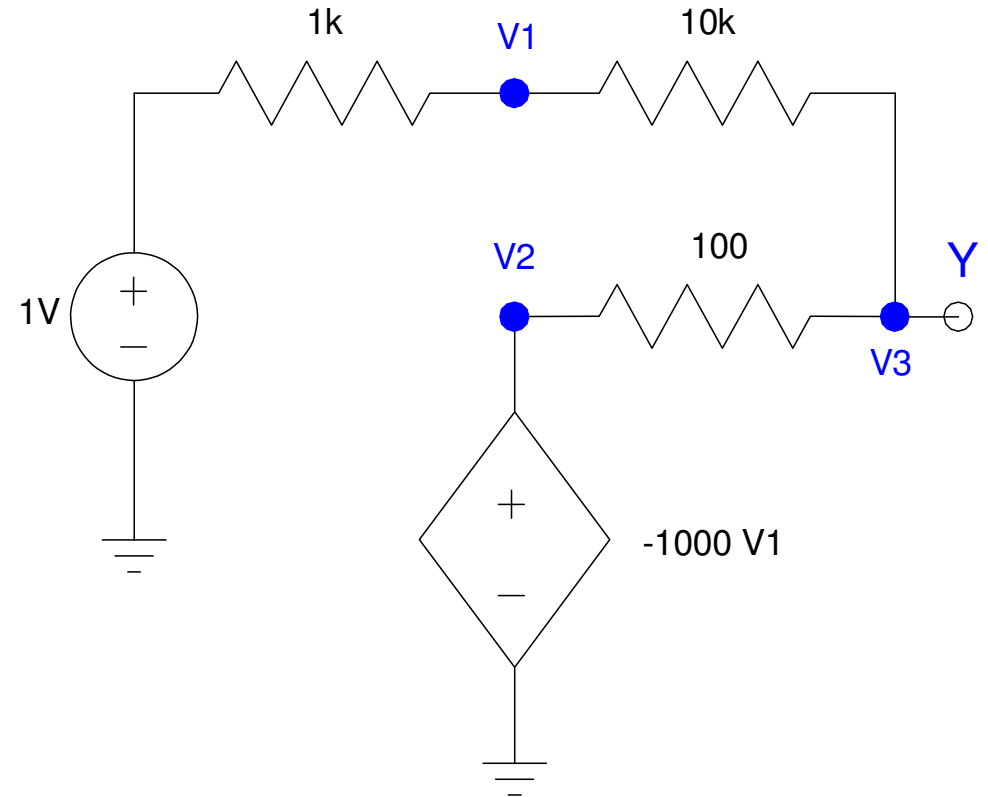
$$\left(\frac{V_1-1}{1k}\right) + \left(\frac{V_1-V_3}{10k}\right) = 0$$

$$\left(\frac{V_3-V_1}{10k}\right) + \left(\frac{V_3-V_2}{100}\right) = 0$$

Solve ( time passes.... )

```
A =  
[1000, 1, 0; 11, 0, -1; -1, -100, 101];  
B = [0; 10; 0];  
V = inv(A)*B
```

```
V1    0.0100  
V2   -9.9891  
V3   -9.8901    =  Vth
```



## Case 2: I(short)

- In

Short the output

- Compute the current

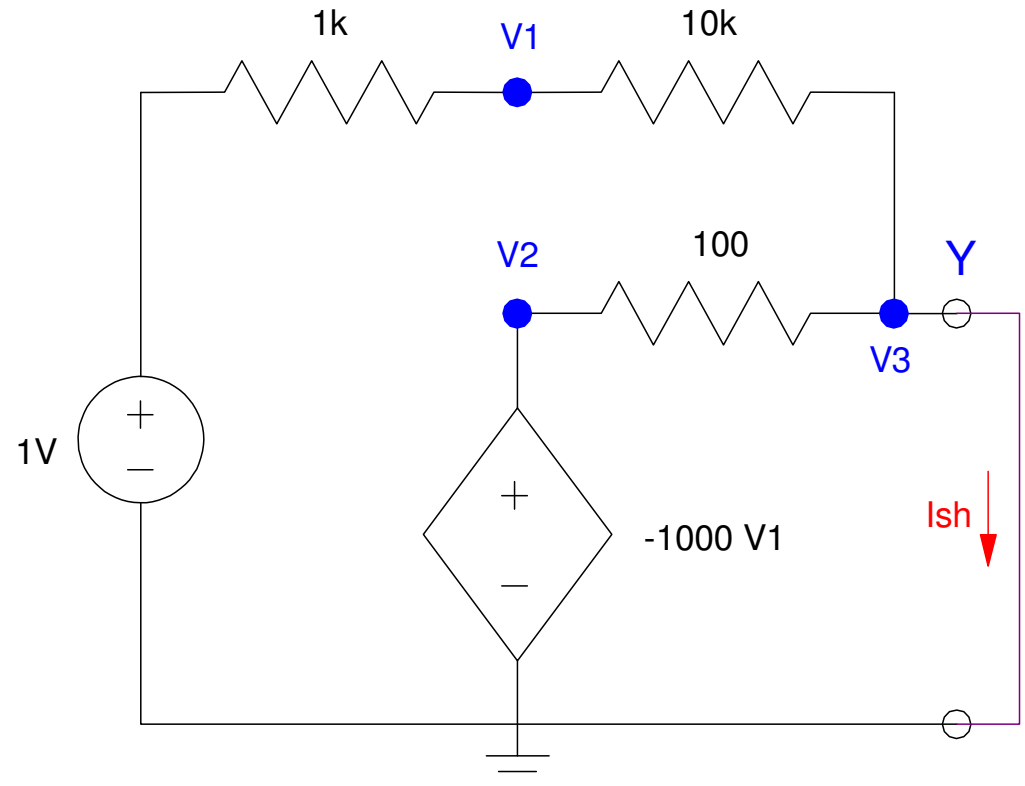
Equations

$$\left(\frac{V_1 - 1}{1k}\right) + \left(\frac{V_1 - 0}{10k}\right) = 0$$

$$V_1 = 0.90909V$$

$$I_{sh} = \left(\frac{1V}{11k}\right) - \left(\frac{1000V_1}{100}\right)$$

$$I_{sh} = -9.0908A$$



## Case 2: $R_{th}$ :

Option 1:

$$R_{th} = \frac{V_{open}}{I_{short}} = 1.088\Omega$$

Option 2:

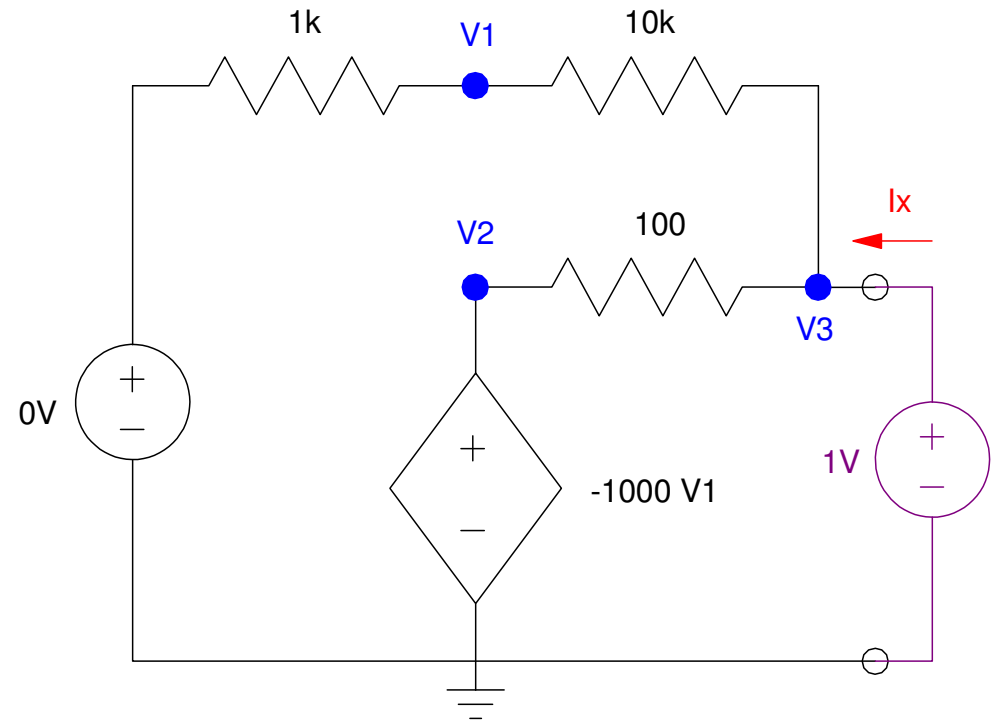
- Turn off voltage sources
- Apply a 1V test voltage
- Compute the current

$$V_1 = \left( \frac{1k}{1k+10k} \right) 1V = 90.91mV$$

$$V_2 = -1000V_1 = -90.91V$$

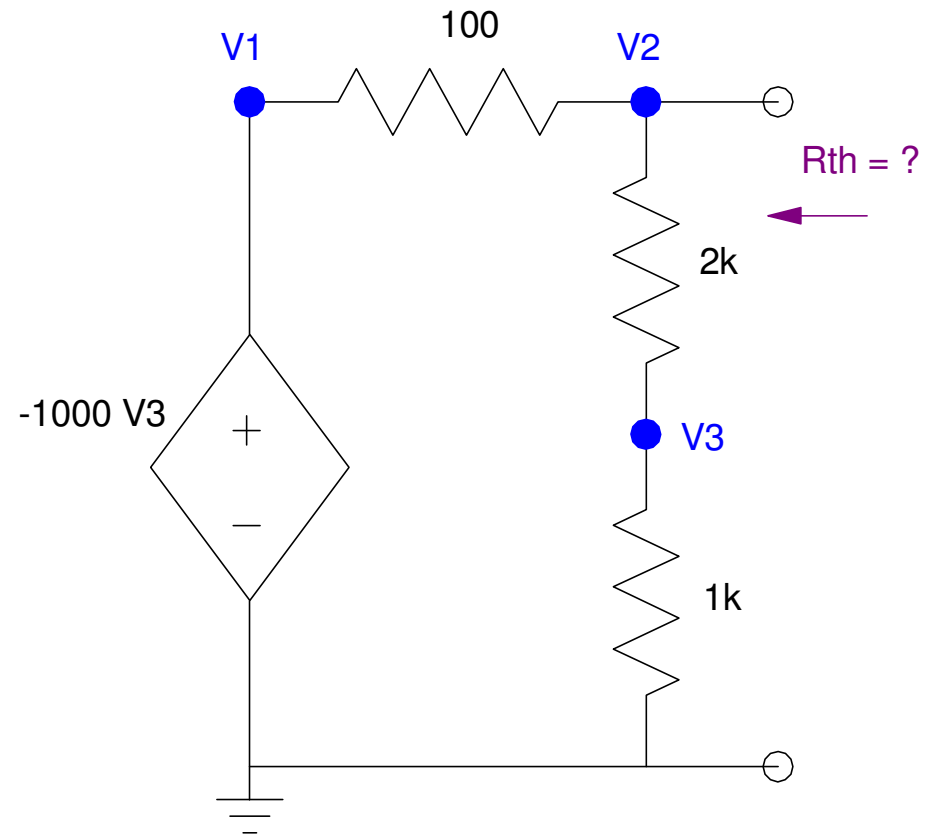
$$I_x = \left( \frac{1V}{11k} \right) + \left( \frac{1V-V_2}{100} \right) = 919.2mA$$

$$R_{th} = \frac{V_{in}}{I_{in}} = \frac{1V}{919.2mA} = 1.088\Omega$$



## Practice Problem

Find the Thevenin resistance for the following circuit



## Solution

- Apply a 1V test voltage
- Compute the current draw

### Equations

$$V_3 = \left( \frac{1k}{1k+2k} \right) 1V$$

$$V_3 = 0.3333V$$

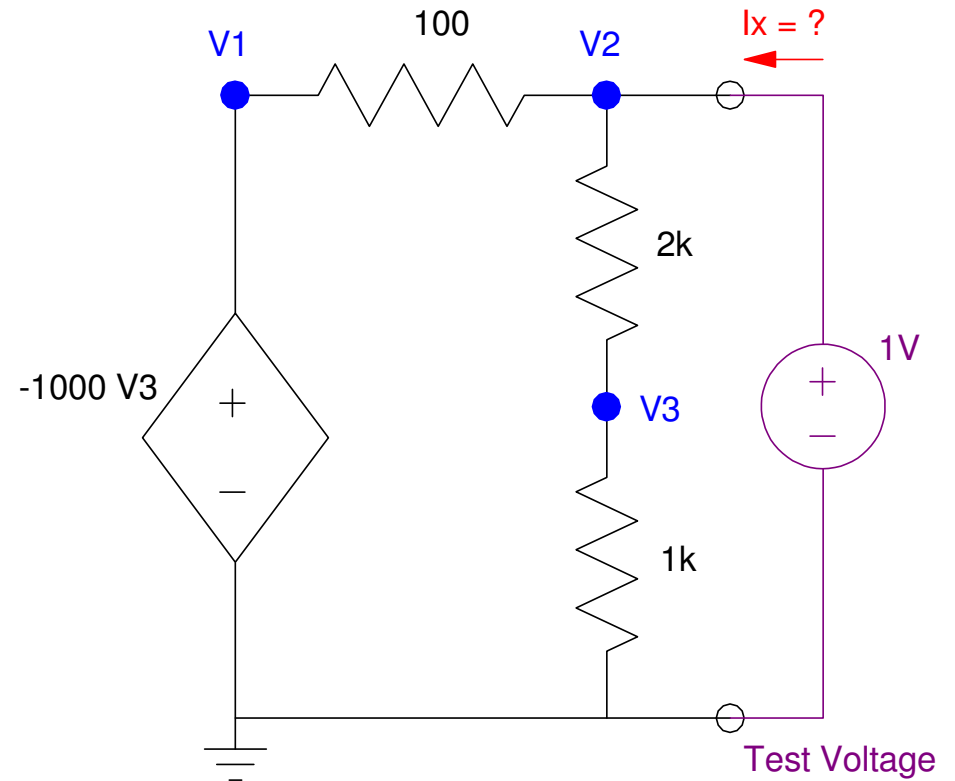
$$V_1 = -1000V_3$$

$$V_1 = -333.3V$$

$$I_x = \left( \frac{1V - V_1}{100} \right) + \left( \frac{1V}{3k} \right)$$

$$I_x = 3.34367A$$

$$R_{th} = \frac{1V}{3.34367A} = 0.299\Omega$$



# Max Power Transfer

- New Topic

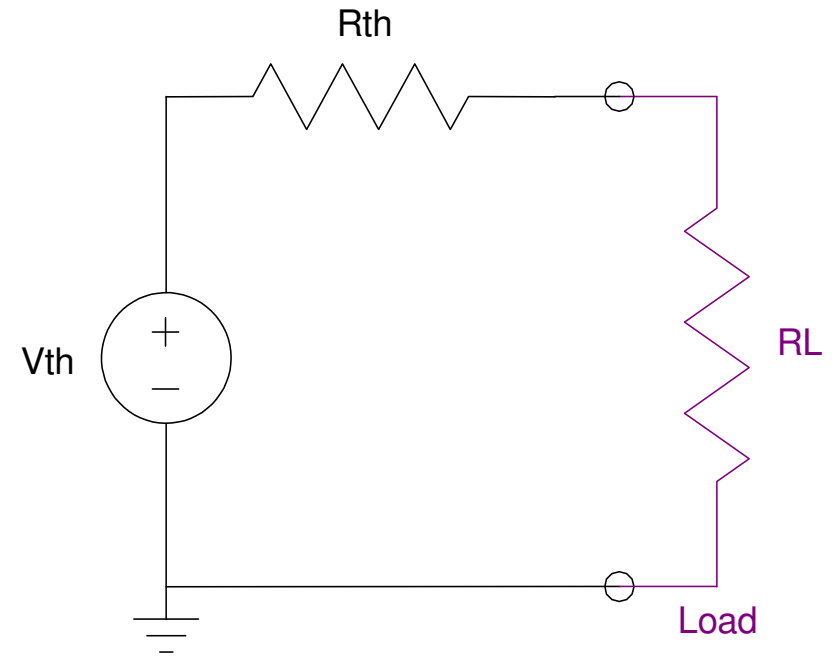
What resistance ( $R_L$ ) maximizes the power to the load?

- ( $V_{th}$ ,  $R_{th}$ ) models a solar panel.
  - What load maximizes the power the solar cell produces?
- ( $V_{th}$ ,  $R_{th}$ ) models a stereo.
  - What speaker ( $R_L$ ) maximizes the output power?

Note:

- $R_L = 0 \Rightarrow P = 0$ 
  - $V = 0$
- $R_L = \infty \Rightarrow P = 0$ 
  - $I = 0$

Max power is in the range  $0 < R_L < \infty$



## Case 1: $R_{th}$ is fixed.

Calculations

$$I = \left( \frac{V_{th}}{R_{th} + R_L} \right)$$

$$P = I^2 R_L$$

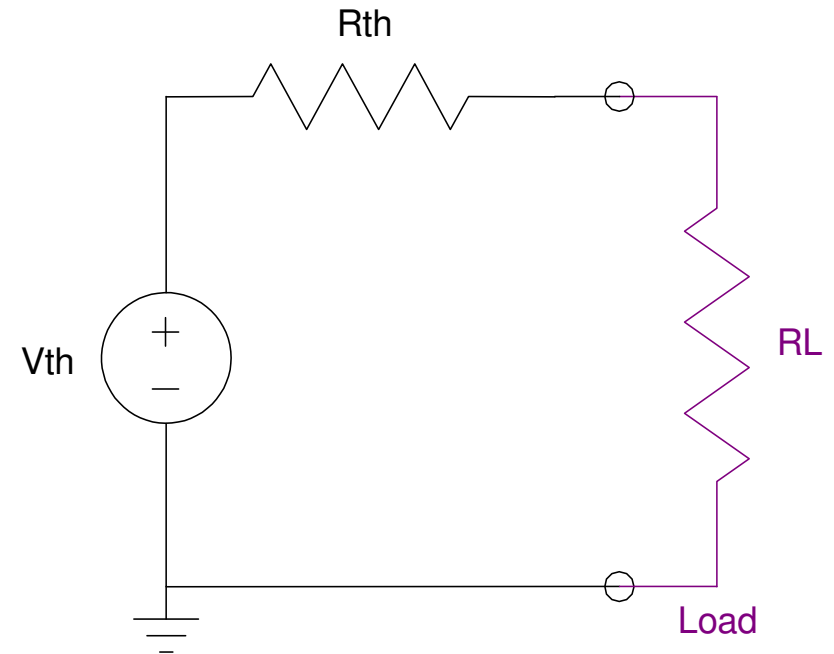
$$P = \left( \frac{R_L}{(R_{th} + R_L)^2} \right) V_{th}^2$$

$$\frac{d}{dR_L} \left( \frac{R_L}{(R_{th} + R_L)^2} \right) = 0$$

$$(R_L + R_{th})(R_{th} - R_L) = 0$$

$$R_L = R_{th} \quad \text{maximum}$$

$$R_L = -R_{th} \quad \text{minimum}$$



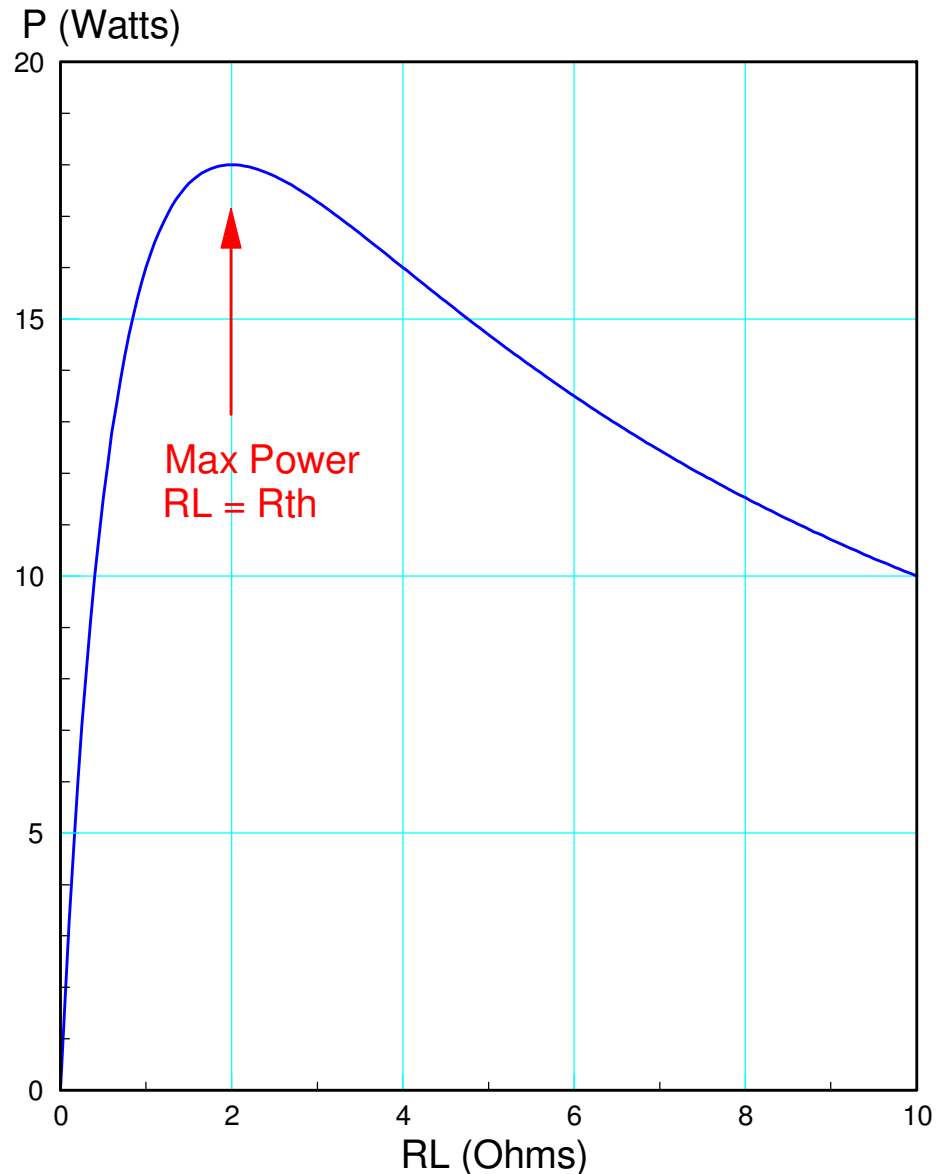
# Matlab Calculations

- Let  $V_{th} = 12V$  and  $R_{th} = 2\text{ Ohms}$ :

```
>> Vth = 12;  
>> Rth = 2;  
>> RL = [0:0.05:10]';  
>> I = Vth ./ (Rth + RL);  
>> V = RL ./ (Rth + RL) * Vth;  
>> P = V .* I;  
>> plot(RL, P);
```

## Note

- $RL = R_{th}$  for max power
- At that point, you're at 50% efficiency



## Case 2: $R_L$ is fixed.

Find  $R_{th}$  to maximize the power to the load.

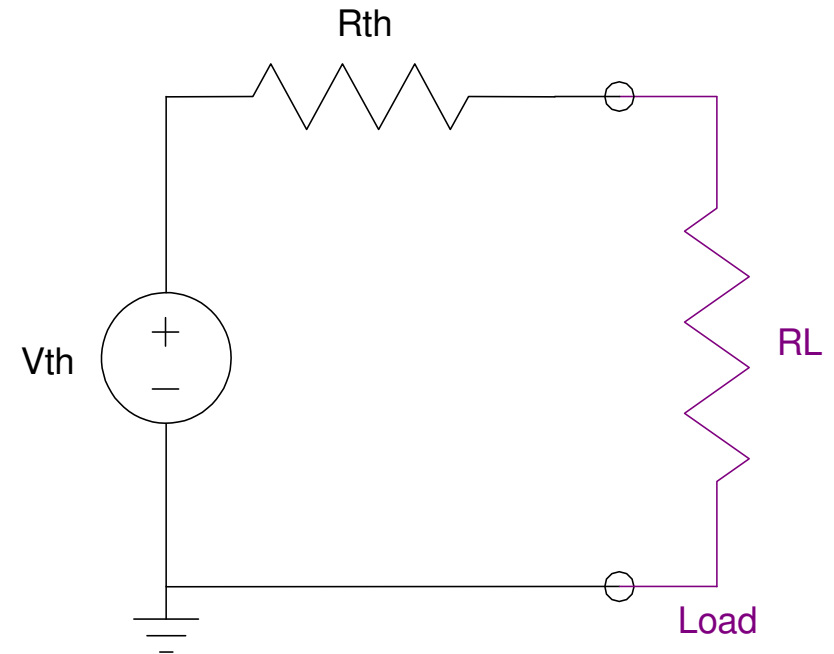
$$P = \left( \frac{R_L}{(R_{th} + R_L)^2} \right) V_{th}^2$$

Find the maximum with respect to  $R_{th}$

$$\frac{dP}{dR_{th}} = 0 = \left( \frac{-2(R_{th} + R_L)R_L}{(R_{th} + R_L)^3} \right) V_{th}^2$$

Maximum is when  $R_{th} = -R_L$

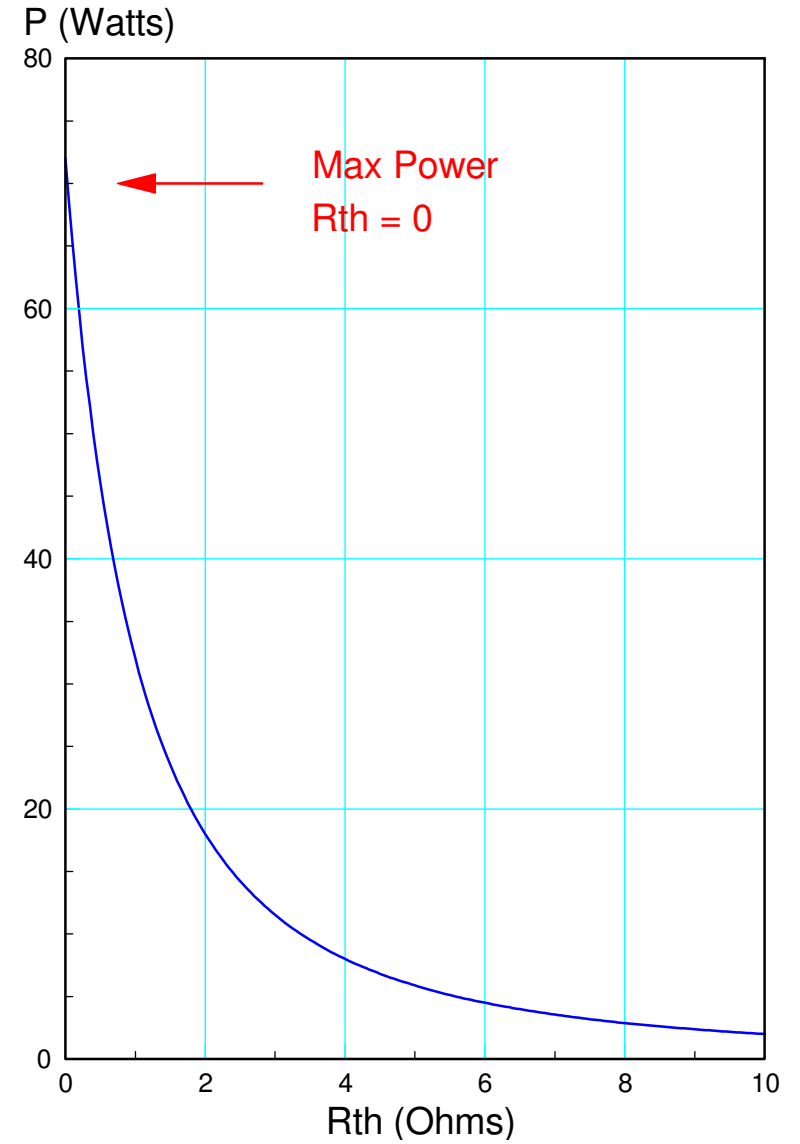
- Make  $R_{th}$  as small as possible
  - Zero is good
  - Negative  $R_{th}$  is better



# Matlab Calculations

- $V_{th} = 12V$  and  $R_L = 2 \text{ Ohms}$

```
>> Vth = 12;  
>> RL = 2;  
>> Rth = [0:0.05:10]';  
>> I = Vth ./ (Rth + RL);  
>> V = RL ./ (Rth + RL) * Vth;  
>> P = V .* I;  
>> plot(Rth, P);
```



### Case 3: Find $R_L$ to maximize efficiency

Efficiency is the power to the load divided by the total power dissipated.

$$eff = \left( \frac{P_{Load}}{P_{total}} \right)$$

or

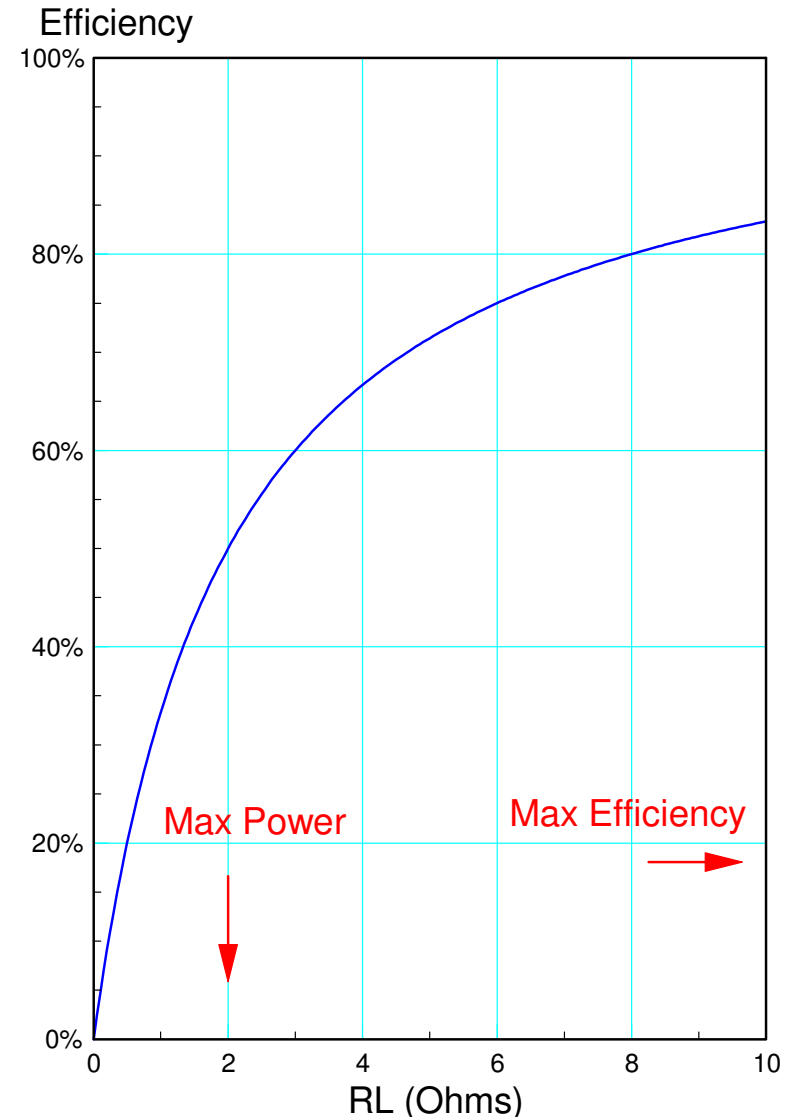
$$eff = \frac{I^2 R_L}{I^2 (R_{th} + R_L)} = \left( \frac{R_L}{R_L + R_{th}} \right)$$

Max efficiency is

- $R_L = \text{infinity}$
- Efficiency = 100%
- Power = 0

That's one of the problems of delivering power

- For high efficiency, you want  $R_L \gg R_{th}$
- For maximum power, you want  $R_L = R_{th}$



# Summary

Dependent sources complicate calculations for  $R_{th}$

Trick:

- Turn off independent sources
  - Same as before
- Apply a 1V test voltage
  - New trick
- Compute the current draw,  $I_x$

$$R_{th} = \frac{1V}{I_x}$$

The Thevenin resistance is important for calculating the max-power to a load

- Max power is when  $R_L = R_{th}$
- Efficiency = 50% at that point

