

ECE 211 Test #3

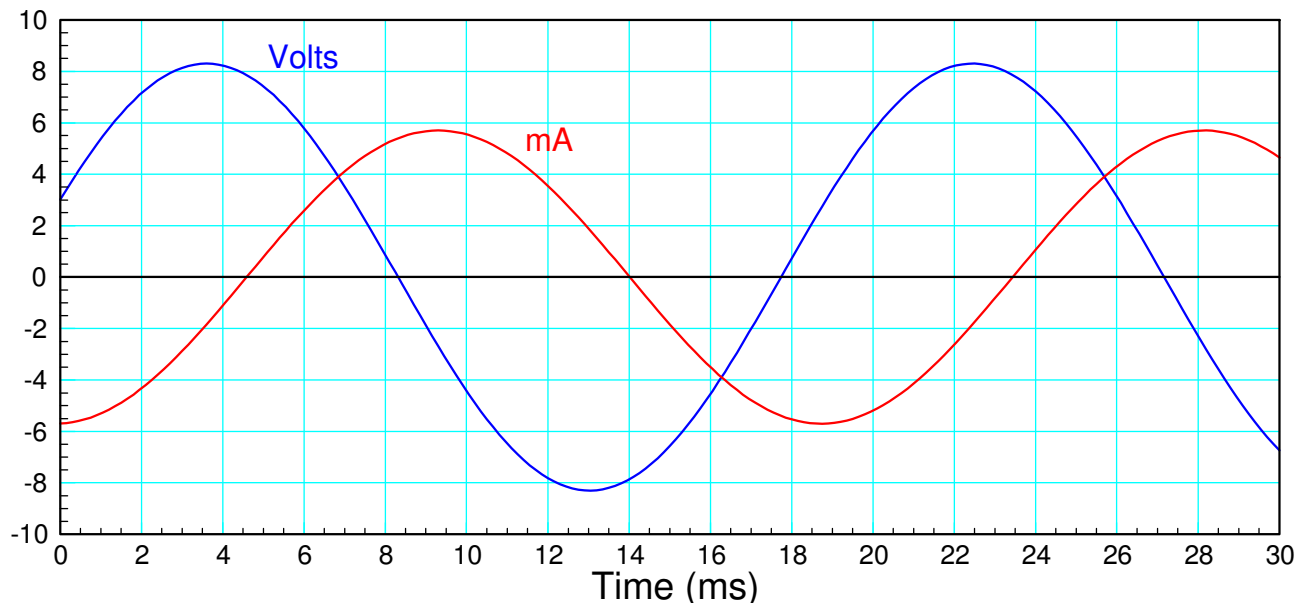
Phasors, Filters, Natural Response of RL and RC circuits

Phasor Voltages & Currents

The voltage and current through a device is shown below. Determine

- The frequency in Hz,
- The phasor representation for the voltage, and
- The phasor representation for the current

Frequency (Hz)	Volts (phasor notation)	Current (phasor notation)
54.05 Hz	$(8.2\angle -73.9^\circ)V$	$(5.8\angle -179^\circ)mA$



Period = 18.5ms

$$f = \frac{1}{T} = 54.05 \text{ Hz}$$

Voltage:

peak = 8.2V @ 3.8ms

$$\theta = -\left(\frac{3.8\text{ms}}{18.5\text{ms}}\right) 360^\circ = -73.9^\circ$$

Current:

peak = 5.8mA @ 9.2ms

$$\theta = -\left(\frac{9.2\text{ms}}{18.5\text{ms}}\right) 360^\circ = -179.0^\circ$$

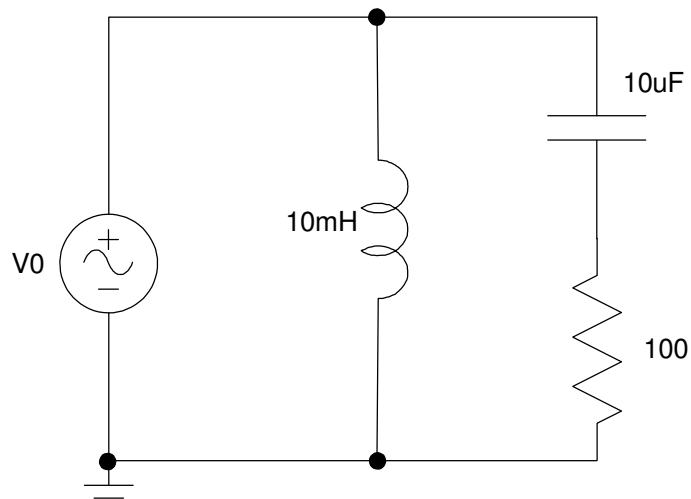
Phasors & Circuit Analysis

Assume V_0 is a sinusoid

$$V_0 = 2 \cos(7t) + 3 \sin(7t)$$

Determine the phasor representation for each element in the following circuit

Voltage Source	10mH inductor	10uF Capacitor	100 Ohm resistor
$2 - j3$	$j0.07 \text{ Ohms}$	$-j14,285 \text{ Ohms}$	100 Ohms



Voltages:

real = cosine

-imag = sine

Inductor

$$Z = j\omega L$$

$$Z = j\left(7 \frac{\text{rad}}{\text{sec}}\right)(10\text{mH})$$

$$Z = j0.07\Omega$$

Capacitor

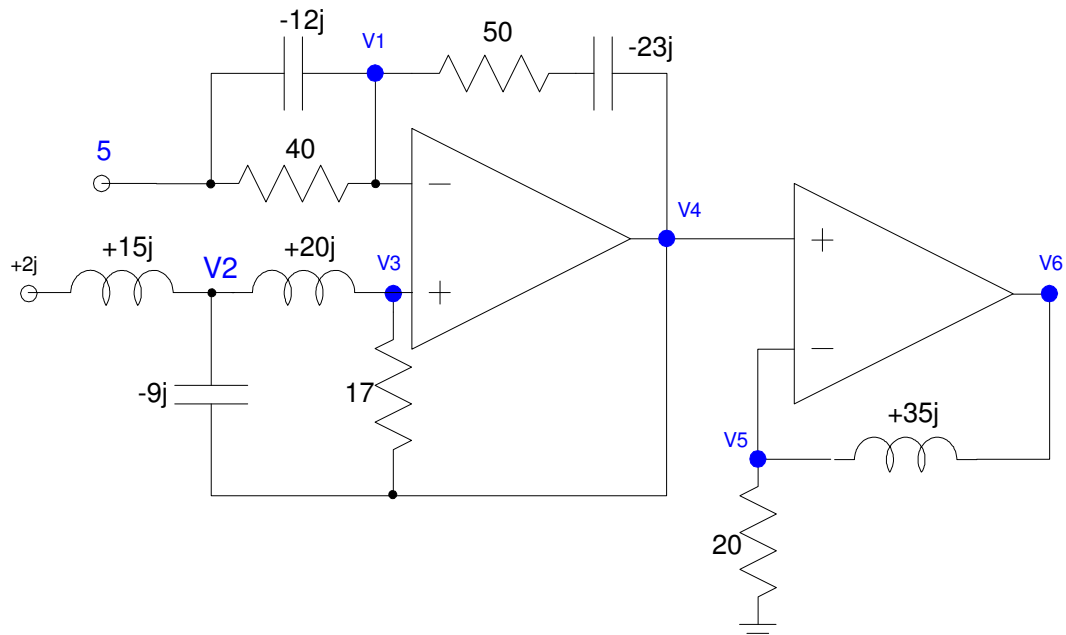
$$Z = \frac{1}{j\omega C}$$

$$Z = \left(\frac{1}{j\left(7 \frac{\text{rad}}{\text{sec}}\right)(10\mu\text{F})}\right) = -j14285\Omega$$

Phasors & Op-Amps

Write the voltage node equations for the following op-amp circuit

- Assume ideal op-amps.
- You don't need to solve (just write six equations for six unknowns)



$$V_1 = V_3$$

$$V_4 = V_5$$

$$\left(\frac{V_1 - 5}{-12j} \right) + \left(\frac{V_1 - 5}{40} \right) + \left(\frac{V_1 - V_4}{50 - 23j} \right) = 0$$

$$\left(\frac{V_2 - 2j}{15j} \right) + \left(\frac{V_2 - V_4}{-9j} \right) + \left(\frac{V_2 - V_3}{20j} \right) = 0$$

$$\left(\frac{V_3 - V_2}{20j} \right) + \left(\frac{V_3 - V_4}{17} \right) = 0$$

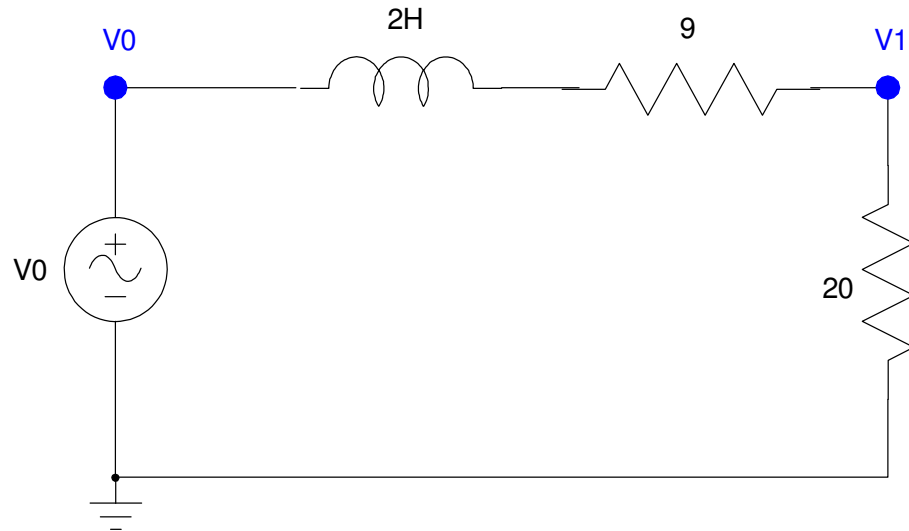
$$\left(\frac{V_5}{20} \right) + \left(\frac{V_5 - V_6}{35j} \right) = 0$$

Superposition

Assume

$$V_0 = 5 \cos(20t) + 7 \sin(50t)$$

Determine $V_1(t)$



$V_0 = 5 \cos(20t)$. In phasor notation

$$V_0 = 5 + j0$$

$$L \rightarrow j\omega L = j40$$

$$V_1 = \left(\frac{20}{20+9+j40} \right) (5 + j0)$$

$$V_1 = 1.1880 - 1.6387j$$

$$v_1(t) = 1.1880 \cos(20t) + 1.63 \sin(20t)$$

$V_1 = 7 \sin(50t)$

$$V_0 = 0 - j7$$

$$L \rightarrow j\omega L = j100$$

$$V_1 = \left(\frac{20}{20+9+j100} \right) (0 - j7)$$

$$V_1 = -1.2914 - 0.3745j$$

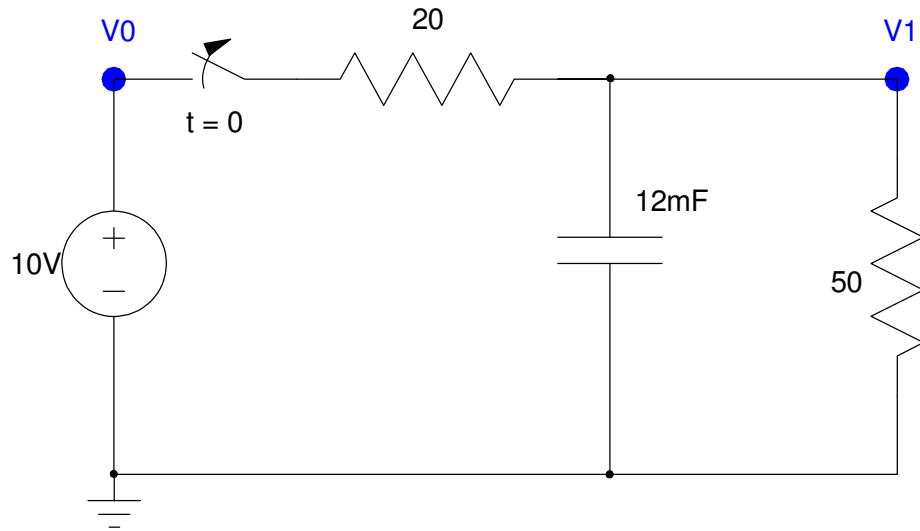
$$v_1(t) = -1.2914 \cos(50t) + 0.3745 \sin(50t)$$

The total answer is then

$$v_1(t) = 1.1880 \cos(20t) + 1.63 \sin(20t) - 1.2914 \cos(50t) + 0.3745 \sin(50t)$$

Natural Response of RC Circuits

Assume the switch opens at $t=0$. Find $V_1(t)$ for $t > 0$.



Start with the initial condition. At $t=0$

$$V_1 = \left(\frac{50}{50+20} \right) 10V = 7.1429V$$

The voltage node equation for $t > 0$ is

$$0.012V_1' + \left(\frac{V_1}{50} \right) = 0$$

$$V_1' + 1.667V_1 = 0$$

Using the p-operator

$$pV_1 + 1.667V_1 = 0$$

$$(p + 1.667)V_1 = 0$$

$$p = -1.667$$

$$V_1(t) = a \cdot e^{-1.667t}$$

Plug in the initial condition

$$V_1(t) = 7.1429 \cdot e^{-1.667t} \quad \text{for } t > 0$$