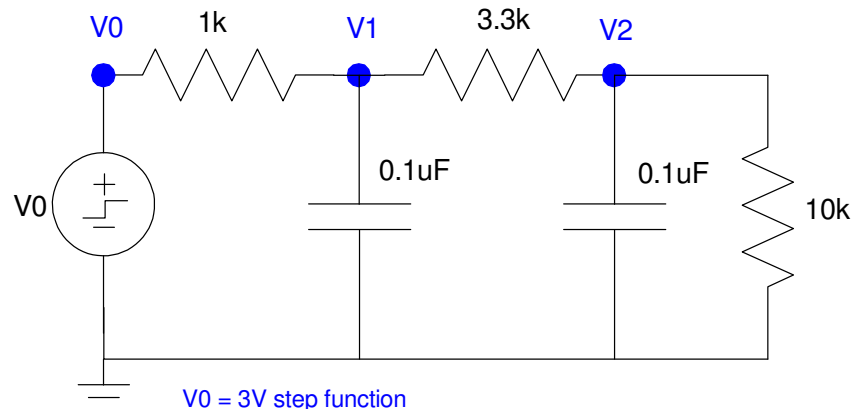


ECE 211: Homework #9

The p-operator. RC circuits. LC circuits.

RC Circuits



Problem 1-5

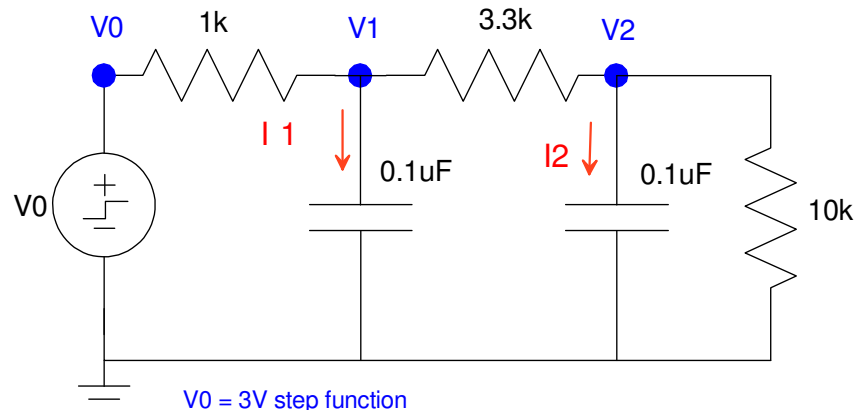
1) Assume V_0 is 3V for $t < 0$. Determine the voltages at V_1 and V_2 at $t = 0$

By voltage division, the voltages are

$$V_2(0) = \left(\frac{10k}{10k + 3.3k + 1k} \right) 3V = 2.0979V$$

$$V_1(0) = \left(\frac{13.3k}{10k + 3.3k + 1k} \right) 3V = 2.7902V$$

2) Give the differential equations which describe this two-stage RC circuit



Start with the VI characteristics for a capacitor

$$I = C \frac{dV}{dt} = CV'$$

For I1

$$I_1 = CV'_1 = \left(\frac{V_0 - V_1}{1k} \right) + \left(\frac{V_2 - V_1}{3.3k} \right)$$

For I2:

$$I_2 = CV'_2 = \left(\frac{V_1 - V_2}{3.3k} \right) - \left(\frac{V_2}{10k} \right)$$

Grouping terms

$$CV'_1 = \left(\frac{1}{1k} \right) V_0 - \left(\frac{1}{1k} + \frac{1}{3.3k} \right) V_1 + \left(\frac{1}{3.3k} \right) V_2$$

$$CV'_2 = \left(\frac{1}{3.3k} \right) V_1 - \left(\frac{1}{3.3k} + \frac{1}{10k} \right) V_2$$

Plugging in numbers

$$V'_1 = 10,000V_0 - 13,030V_1 + 3030V_2$$

$$V'_2 = 3030V_1 - 4030V_2$$

3) Using the p-operator, determine the voltage $V_2(t)$ for $t > 0$

- Note how long it takes for $V_2(t)$ to decay to about 5% of its initial value

The differential equations from problem #2 are

$$V_1' = 10,000V_0 - 13,030V_1 + 3030V_2$$

$$V_2' = 3030V_1 - 4030V_2$$

Assuming $V_0 = 0$ and initial conditions of $V_1(0)$ and $V_2(0)$, these differential equations can be written with the p-operator as

$$pV_1 = 10,000V_0 - 13,030V_1 + 3030V_2$$

$$pV_2 = 3030V_1 - 4030V_2$$

Doing some algebra

$$(p + 13030)V_1 - (3030)V_2 = 0 \quad * (3030)$$

$$-(3030)V_1 + (p + 4030)V_2 = 0 \quad * (p + 13030)$$

----- add

$$(0)V_1 + \left((p + 4030)(p + 13030) - (3030)^2 \right) V_2 = 0$$

$$(p^2 + 17060p + 43330000)V_2 = 0$$

$$(p + 3105)(p + 13955)V_2 = 0$$

This tells you that either

- $V_2(t) = 0$ *the trivial solution*
- $p = \{-3105, -13955\}$

Going with the latter

$$V_2(t) = a \cdot e^{-3105t} + b \cdot e^{-13955t}$$

The initial condition is

$$V_1(0) = 2.7902V$$

$$V_2(0) = 2.0979V$$

From the differential equation

$$V_2' = 3030V_1 - 4030V_2$$

$$V_2'(0) = 3030V_1(0) - 4030V_2(0)$$

$$V_2'(0) = 0$$

Solve for {a, b}

$$V_2(0) = 2.0979V = a + b$$

$$V'_2(0) = 0 = -3105a - 13955b$$

Solving

```
>> A = [1,1 ; -3105,-13955]
```

```
      1      1  
    -3105    -13955
```

```
>> B = [2.0979 ; 0]
```

```
    2.0979  
         0
```

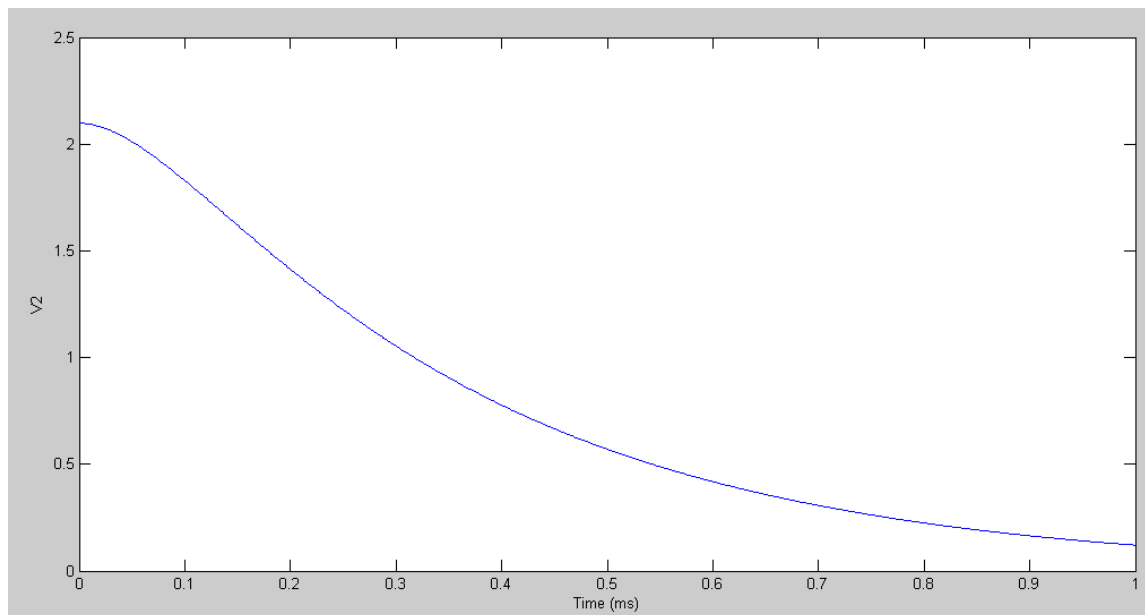
```
>> inv(A)*B
```

```
    2.6983  
   -0.6004
```

$$V_2(t) = 2.6983 \cdot e^{-3105t} - 0.6004 \cdot e^{-13955t}$$

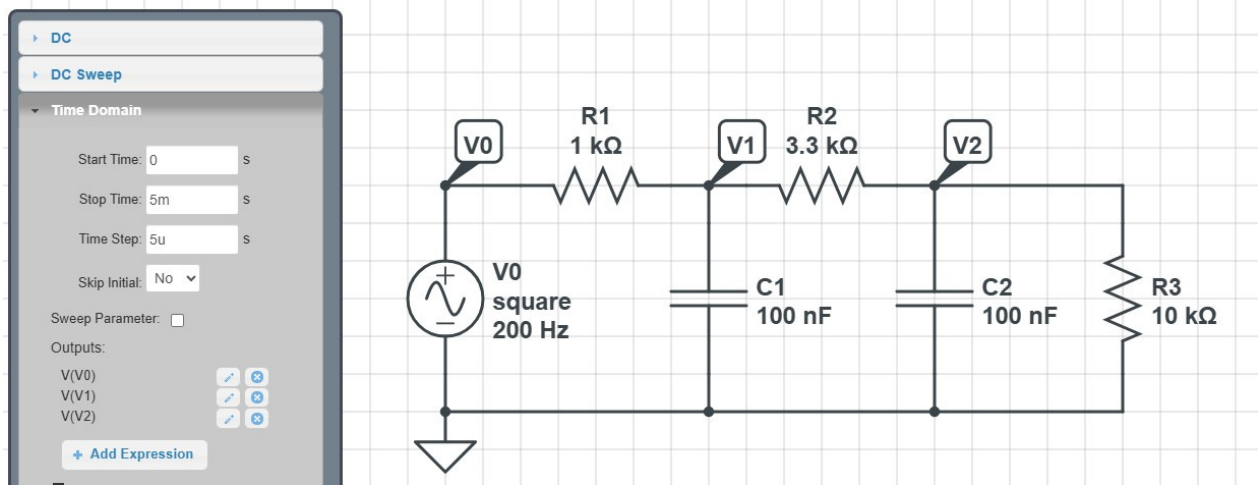
Plotting V2 in Matlab

```
>> t = [0:0.001:1]' * 0.001;  
>> V2 = 2.6983*exp(-3105*t) - 0.6004*exp(-13955*t);  
>> plot(t*1000,V2)  
>> xlabel('Time (ms)')  
>> ylabel('V2')
```



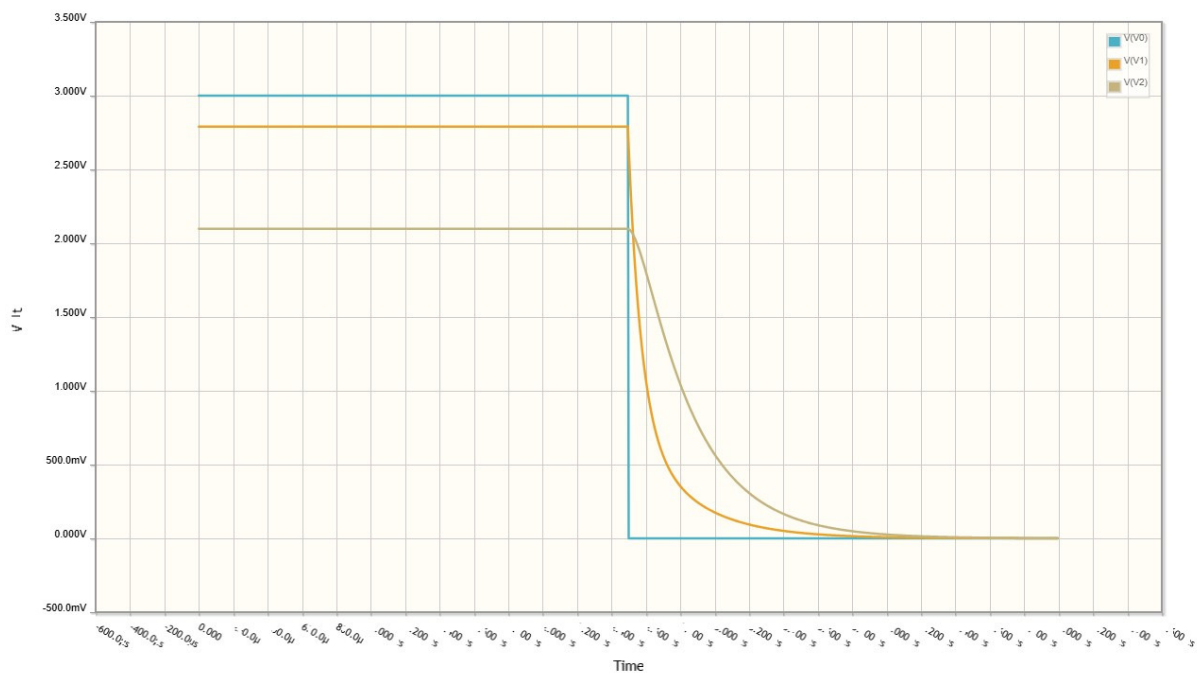
CircuitLab:

4) Simulate this circuit using CircuitLab. Set V0 to be a square wave which goes from 0V to 3V. Set the frequency long enough that V1 and V2 reach steady-state before switching



From the time-domain simulation (below)

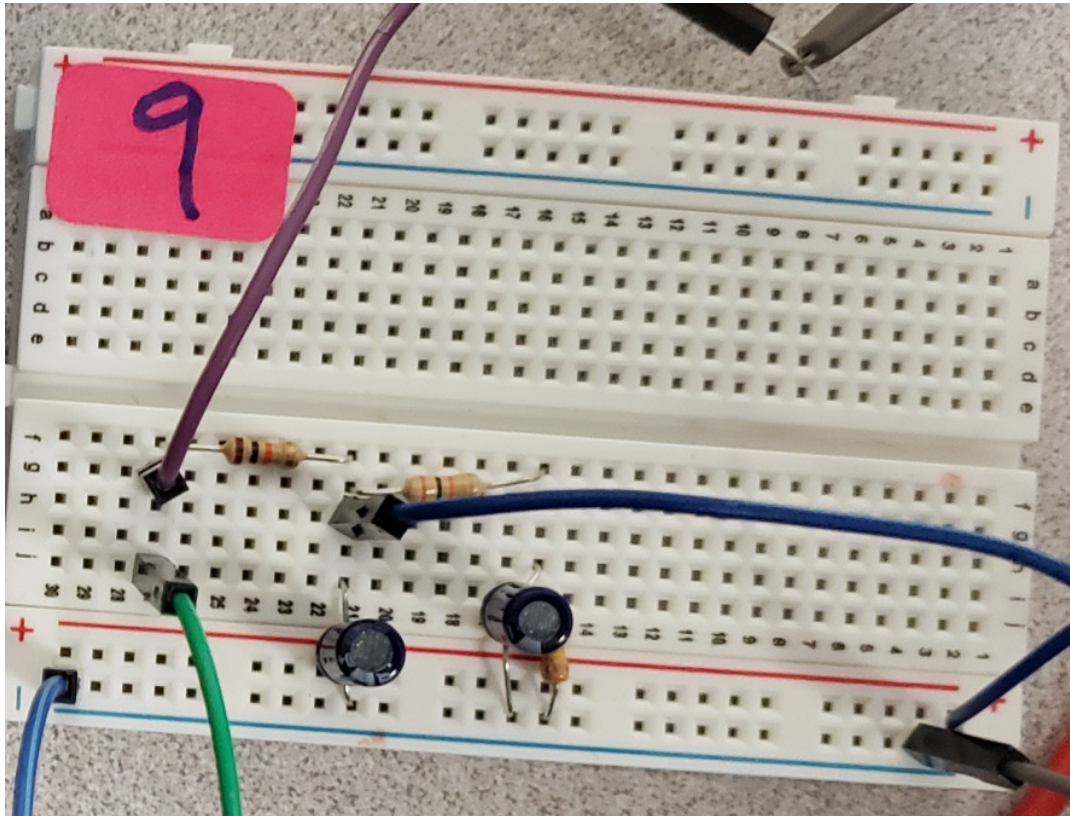
- $V1(0) = 2.790V$
 - 2.790V calculated
- $V2(0) = 2.098V$
 - 2.098V calculated
- 1.050ms for V2 to drop down to 5% of it's initial value (107mV)
 - Simulation = 1ms



Lab

5) Build this circuit with your breadboard.

- Use your FNRSI signal generator for V0. Set it to be a 3V square wave with the period long enough that the voltages at V1 and V2 reach steady-state before V0 switches.
- Include a photo of your circuit

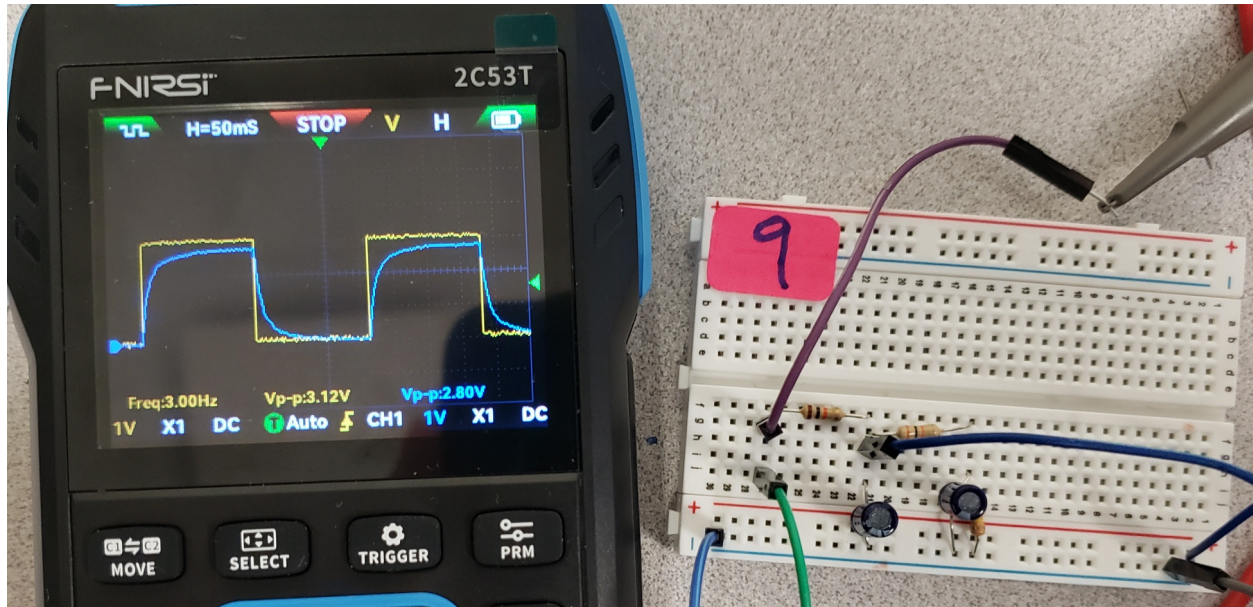


Problem 6)

6) Define $t=0$ to be the time when V_0 goes from 3V to 0V. Measure $V_1(t=0)$, $V_2(t=0)$, and the settling time

$V_1(t)$:

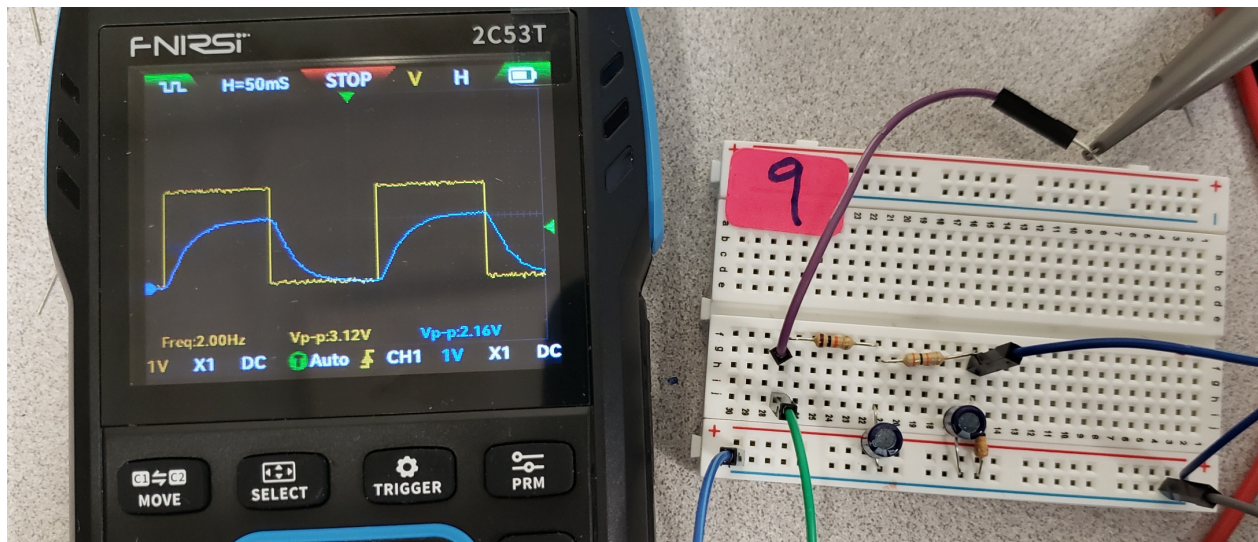
- $t = 0$ is when the input goes from 3V to 0V (falling edge)
- $V_1(0) = 2.42\text{V}$ (measured)
 - 2.786V (calculated)
- Settling time = 1ms (approx)
 - about 1ms calculated



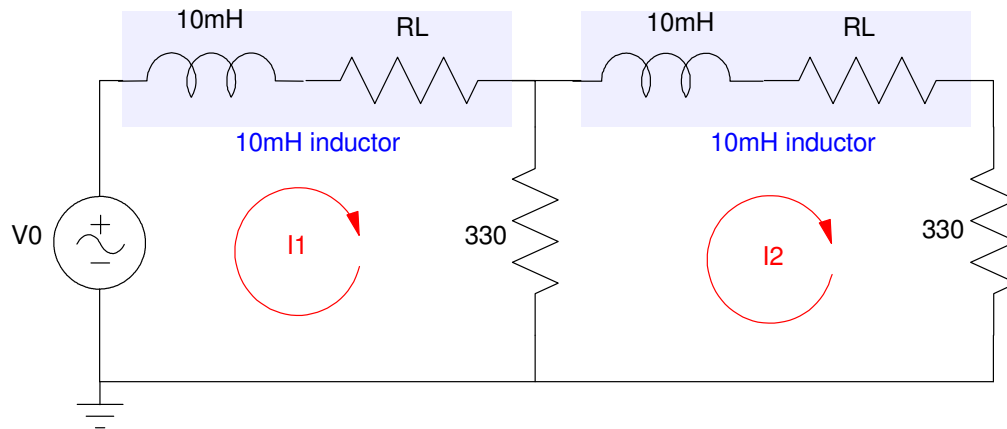
$V_2(t)$

- $t = 0$ is the falling edge of V_{in} (yellow line)
- $V_2(0) = 1.89V$ (measured)
 - 2.143V calculated
- Settling time = 1ms (two division) measured
 - about 1ms calculated

It's harder to take measurements from these waveforms than with sine wave signals, but the hardware results are similar to calculations and CircuitLab



RL Circuits



Problem 7-13

7) Measure the DC resistance of your 10mH inductors in your lab kit. This is RL in the circuit above. (RL isn't added to the circuit: it's the series resistance of the 10mH inductor).

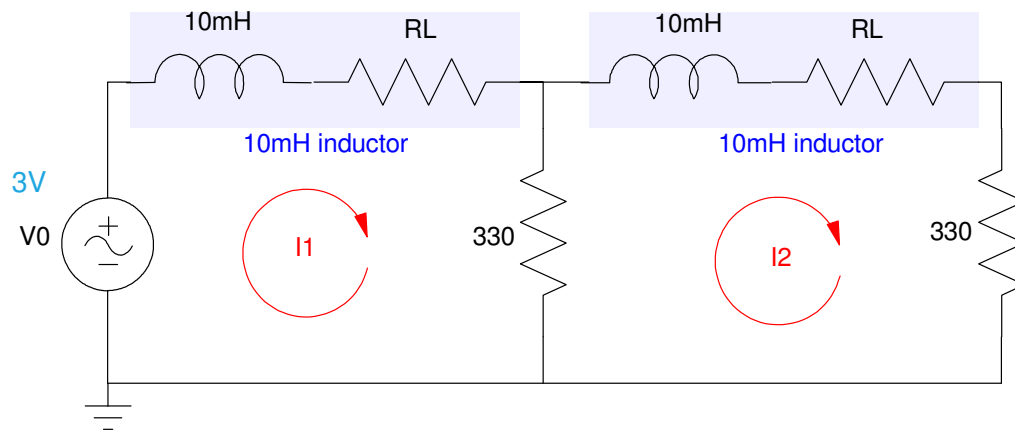
- Resistance: $7.24\ \Omega$ *
- Inductance: Assume 10mH (FNRSI meter doesn't have an inductance scale)



* note: The inductor measured $7.19\ \Omega$ when making up these solutions (probably a different inductor). The rest of these solution likewise use $7.19\ \Omega$ rather than $7.24\ \Omega$ for the inductor.

8) Assume V_0 is 3V for $t < 0$. Determine the DC currents I_1 and I_2 at $t = 0$

- $R_L = 7.19$ Ohms assumed from hereon



Write the loop equations

$$-3 + 7.19I_1 + 330(I_1 - I_2) = 0$$

$$330(I_2 - I_1) + 7.19I_2 + 330I_2 = 0$$

Group terms

$$(337.19)I_1 - (330)I_2 = 3$$

$$-(330)I_1 + (667.19)I_2 = 0$$

Place in matrix form

$$\begin{bmatrix} 337.19 & -330 \\ -330 & 667.19 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

Solve

```
>> A = [337.19, -330; -330, 667.19]
```

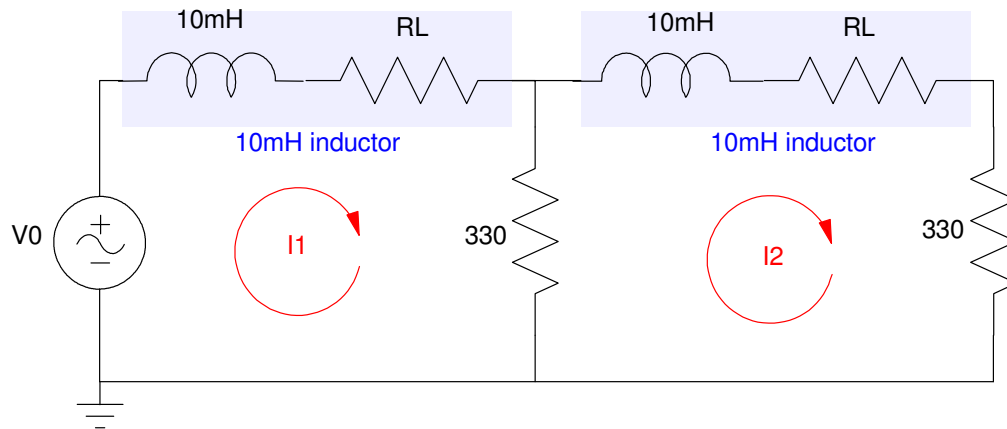
```
    337.1900  -330.0000
   -330.0000   667.1900
```

```
>> B = [3; 0];
>> I = inv(A)*B;
>> mA = I*1000
```

```
I1    17.2445 mA
I2     8.5294 mA
```

note: The FNRSI signal generator can only generate 10mA - so this won't work well in lab. Plus, you only have one inductor in your lab kit. Part of the reason there's no lab section for this circuit.

9) Give the differential equations which describe this two-stage RL circuit



Write the loop equations

$$-V_0 + 0.01I_1' + 7.19I_1 + 330(I_1 - I_2) = 0$$

$$330(I_2 - I_1) + 0.01I_2' + 7.19I_2 + 330I_2 = 0$$

Simplifying

$$I_1' = 100V_0 - 33,719I_1 + 33,000I_2$$

$$I_2' = 33,000I_1 - 66,719I_2$$

10) Using the p-operator, determine the current $I_2(t)$ for $t > 0$

- Note how long it takes for $I_2(t)$ to decay to about 5% of its initial value

$$I_1' = 100V_0 - 33,719I_1 + 33,000I_2$$

$$I_2' = 33,000I_1 - 66,719I_2$$

switch to the p operator

$$pI_1 = 100V_0 - 33,719I_1 + 33,000I_2$$

$$pI_2 = 33,000I_1 - 66,719I_2$$

Do some algebra, note that $V_0 = 0$ for $t > 0$

$$(p + 33724)I_1 - (33000)I_2 = 0 \quad * 33000$$

$$-(33000)I_1 + (p + 66,724)I_2 = 0 \quad * (p + 33724)$$

----- add

$$(0)I_1 + \left((p + 66719)(p + 33719) - (33000)^2 \right) I_2 = 0$$

$$(p^2 + 100,438p + 1,160,697,961)I_2 = 0$$

$$(p + 13,323)(p + 87,114)I_2 = 0$$

So I_2 is of the form

$$I_2(t) = a \cdot \exp(-13,323t) + b \cdot \exp(-87,114t)$$

Plug in the initial conditions

$$\begin{array}{ll} I_1 & 17.2445 \text{ mA} \\ I_2 & 8.5294 \text{ mA} \end{array}$$

Use the differential equation to find $I_2'(0)$

$$I_2' = 33,000I_1 - 66,719I_2$$

$$I_2'(0) = 0$$

Solve for {a, b}

$$I_2(0) = 8.5294 \text{ mA} = a + b$$

$$I_2'(0) = 0 = -13323a - 87114b$$

$$>> A = [1, 1 ; -13323, -87114]$$

$$\begin{array}{cc} 1 & 1 \\ -13323 & -87114 \end{array}$$

$$>> B = [8.5294 ; 0]$$

$$\begin{array}{c} 8.5294000000000001 \\ 0 \end{array}$$

```
>> inv(A)*B
```

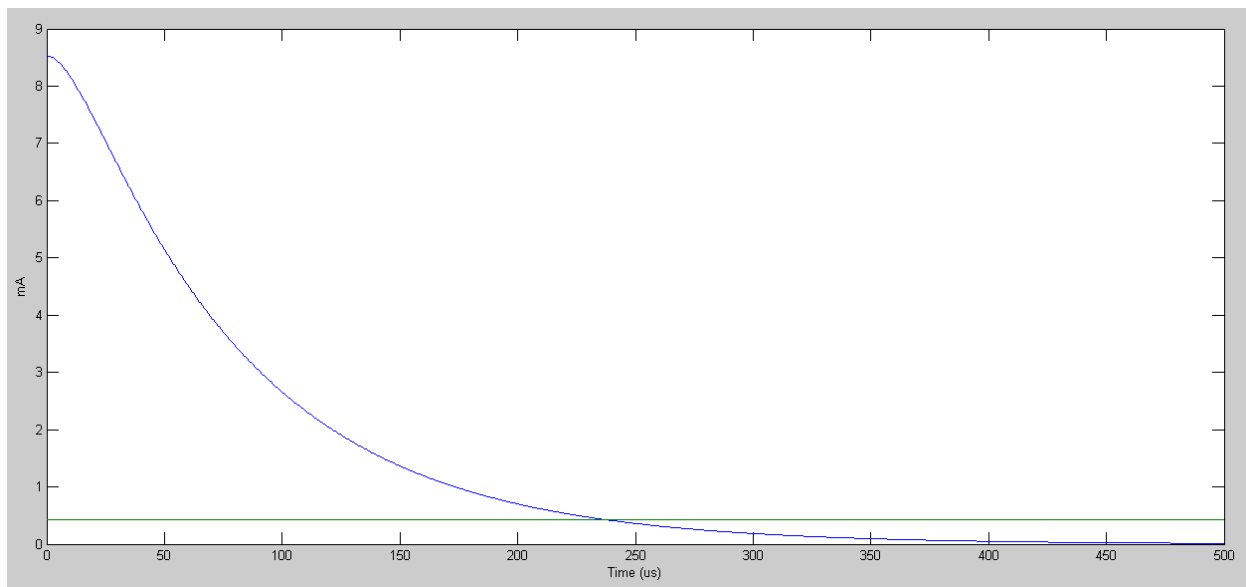
```
a 10.069387209822336  
b -1.539987209822336
```

So

$$I_2(t) = 10.069 \cdot \exp(-13,323t) - 1.5400 \cdot \exp(-87,114t)$$

```
>>
```

```
>> t = [0:0.001:1]' * 500e-6;  
>> I2 = 10.069*exp(-13323*t) - 1.54*exp(-87114*t);  
>> plot(t*1e6, I2, t*1e6, 0*t + 0.05*I2(0))  
>> xlabel('Time (us)')  
>> ylabel('mA')
```



It takes 237us for I_2 to decay down to 5% of its initial value

CircuitLab:

11) Simulate this circuit using CircuitLab. Set V0 to be a wave which goes from 0V to 3V. Set the frequency long enough that I1 and I2 reach steady-state before switching.

- $I_1(0) = 17.24\text{mA}$
- $I_2(0) = 8.529\text{mA}$
- It takes $238\mu\text{s}$ for I2 to decay down to 5% of its initial value

