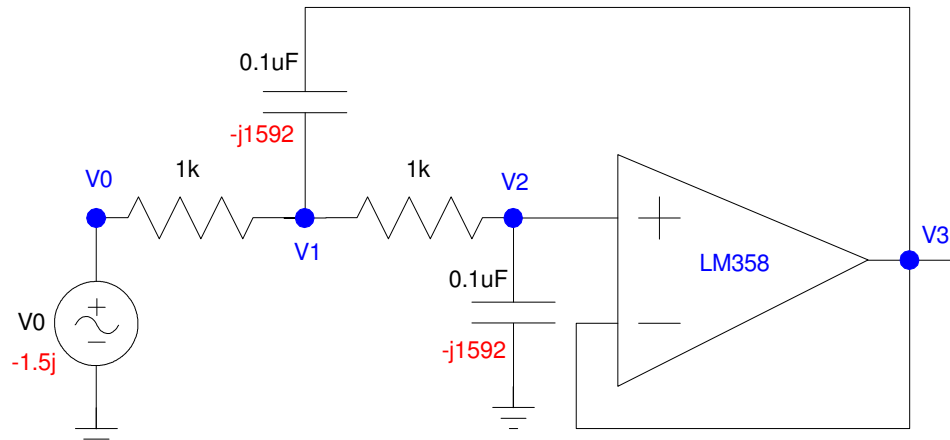


ECE 211: Homework #8

Phasors & Op-Amps, Active Filters, Superposition

Phasors & Op-Amps



Problems 1-3

1) Let V_0 be a 3Vpp 1kHz sine wave

$$V_0 = 1.5 \sin(6280t)$$

Find the phasor representation for each element in this circuit

$$V_0 = -1.5j$$

$$Z_c = \frac{1}{j\omega C} = -j1592$$

Write the voltage node equations.

Four nodes means we need four equations

$$V_0 = -1.5j$$

$$V_p = V_m$$

$$V_2 = V_3$$

Node V1

$$\left(\frac{V_1 - V_0}{1000} \right) + \left(\frac{V_1 - V_3}{-1592j} \right) + \left(\frac{V_1 - V_2}{1000} \right) = 0$$

Node V2

$$\left(\frac{V_2 - V_1}{1000} \right) + \left(\frac{V_2}{-1592j} \right) = 0$$

2) Solve for $\{V_1, V_2, V_3\}$

Group the terms

$$V_0 = -1.5j$$

$$V_2 - V_3 = 0$$

$$-\left(\frac{1}{1000}\right)V_0 + \left(\frac{1}{1000} + \frac{1}{-1592j} + \frac{1}{1000}\right)V_1 - \left(\frac{1}{1000}\right)V_2 - \left(\frac{1}{-1592j}\right)V_3 = 0$$

$$-\left(\frac{1}{1000}\right)V_1 + \left(\frac{1}{1000} + \frac{1}{-1592j}\right)V_2 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ \left(\frac{-1}{1000}\right) & \left(\frac{1}{1000} + \frac{1}{-1592j} + \frac{1}{1000}\right) & \left(\frac{-1}{1000}\right) & \left(\frac{1}{1592j}\right) \\ 0 & \left(\frac{-1}{1000}\right) & \left(\frac{1}{1000} + \frac{1}{-1592j}\right) & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1.5j \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> a1 = [1,0,0,0];
>> a2 = [0,0,1,-1];
>> a3 = [-1/1000, 1/1000-1/(1592j)+1/1000, -1/1000, 1/(1592j)];
>> a4 = [0, -1/1000, 1/1000-1/(1592j), 0];
>> A = [a1 ; a2 ; a3 ; a4]

    1.0000         0         0         0
         0         0         1.0000    -1.0000
   -0.0100    0.0200 + 0.0063i   -0.0100         0 - 0.0063i
         0    -0.0100    0.0100 + 0.0063i         0

>> B = [-1.5j ; 0 ; 0 ; 0]

    0 - 1.5000i
         0
         0
         0

>> V = inv(A)*B

V0    0 - 1.5000i
V1   -0.6760 - 1.0748i
V2   -0.9688 - 0.4655i
V3   -0.9688 - 0.4655i
```

meaning

$$\begin{aligned}V_0(t) &= 1.5000 \sin(6280t) \\V_1(t) &= -0.6760 \cos(6280t) + 1.0748 \sin(6280t) \\V_2(t) &= -0.9688 \cos(6280t) + 0.4655 \sin(6280t) \\V_3(t) &= -0.9688 \cos(6280t) + 0.4655 \sin(6280t)\end{aligned}$$

If you prefer polar form...

```
>> abs(V)

V0    1.5000
V1    1.2697
V2    1.0748
V3    1.0748

>> angle(V)*180/pi

V0   -90.0000
V1  -122.1671
V2 -154.3341
V3 -154.3341
```

meaning

$$V_0(t) = 1.5000 \cos(6280t - 90^\circ)$$

$$V_1(t) = 1.2697 \cos(6280t - 122.167^\circ)$$

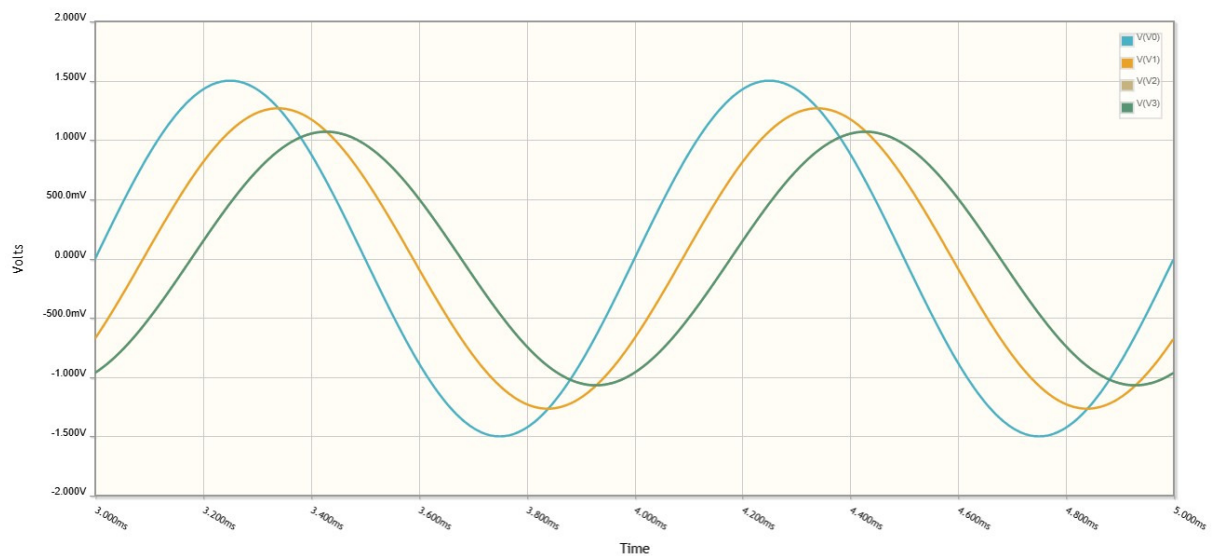
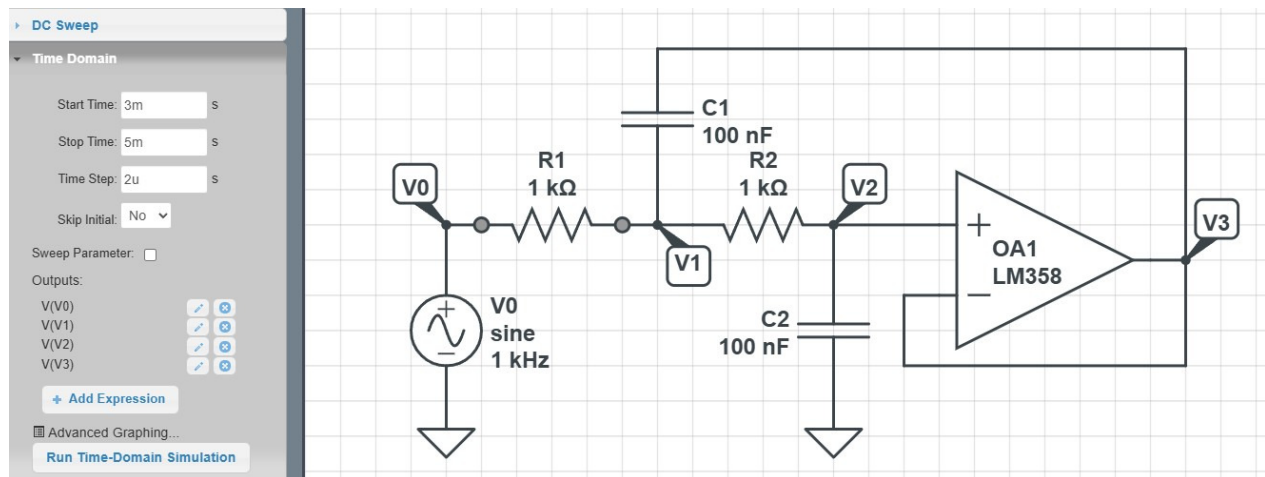
$$V_2(t) = 1.0748 \cos(6280t - 154.334^\circ)$$

$$V_3(t) = 1.0748 \cos(6280t - 154.334^\circ)$$

```
>>
```

3) Simulate this circuit using CircuitLab. Measure the signals at {V1, V2, V3}

- Magnitude and phase



From the graph,

V0:

- peak = 1.500V
- delay = 0.252ms
- period = 1.000ms (1kHz)

$$\theta_1 = -\left(\frac{0.252ms}{1ms}\right) 360^\circ = -90.72^\circ$$

$$V_0 = 1.500 \angle -90.72^\circ$$

Calculations

$$V_0 = 1.5 \angle -90^\circ$$

V1:

- Peak = 1.267V
- Delay to peak = 0.340ms

$$\theta_1 = -\left(\frac{0.340ms}{1ms}\right) 360^\circ = -122.4^\circ$$

$$V_1 = 1.267 \angle -122.4^\circ$$

Calculations:

$$V_1 = 1.2697 \angle -122.167^\circ$$

V2:

- peak = 1.069V
- Delay to peak = 0.432ms

$$\theta_2 = -\left(\frac{0.432ms}{1ms}\right) 360^\circ = -155.52^\circ$$

$$V_2 = 1.069 \angle -155.52^\circ$$

Calculations:

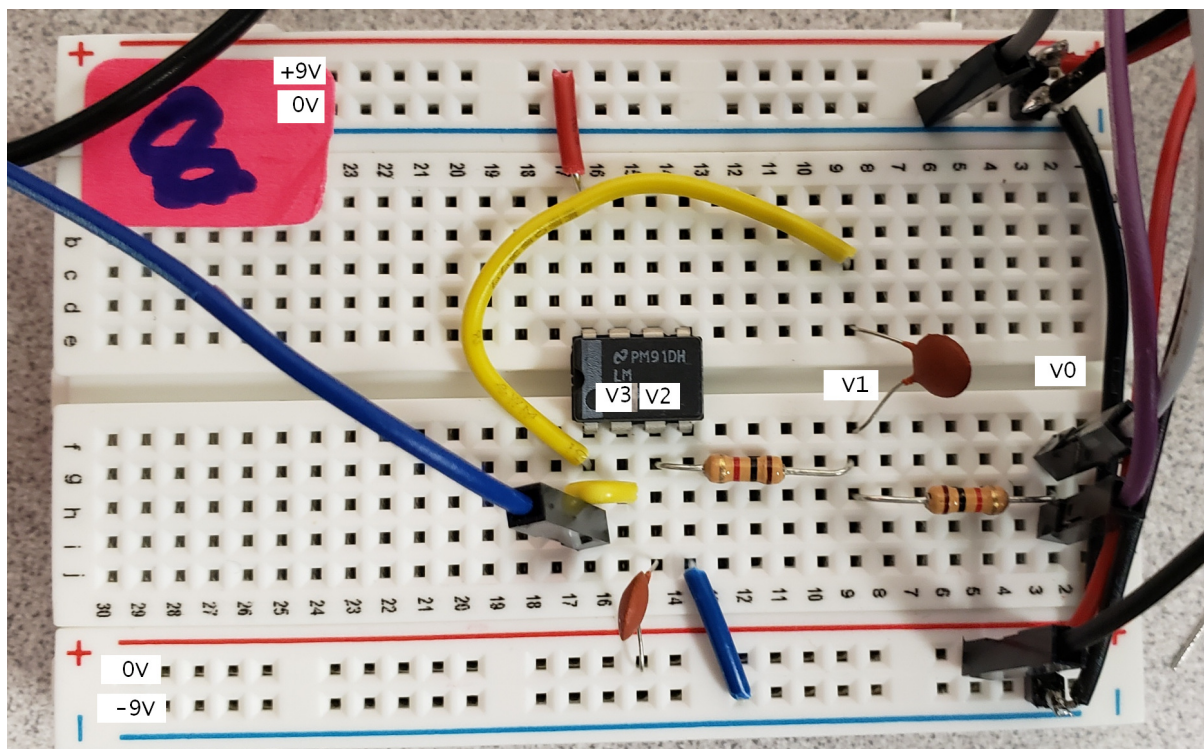
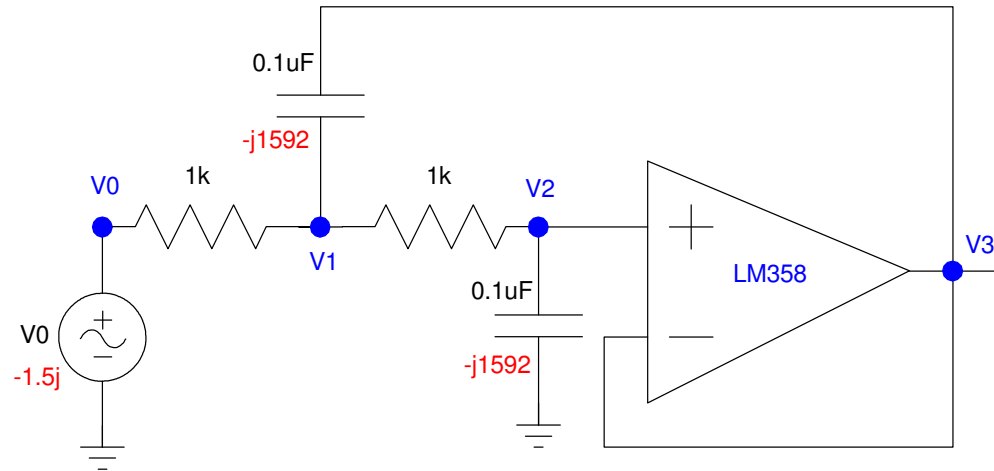
$$V_2 = 1.0748 \angle -154.334^\circ$$

Calculations match up with CircuitLab simulation results

Lab

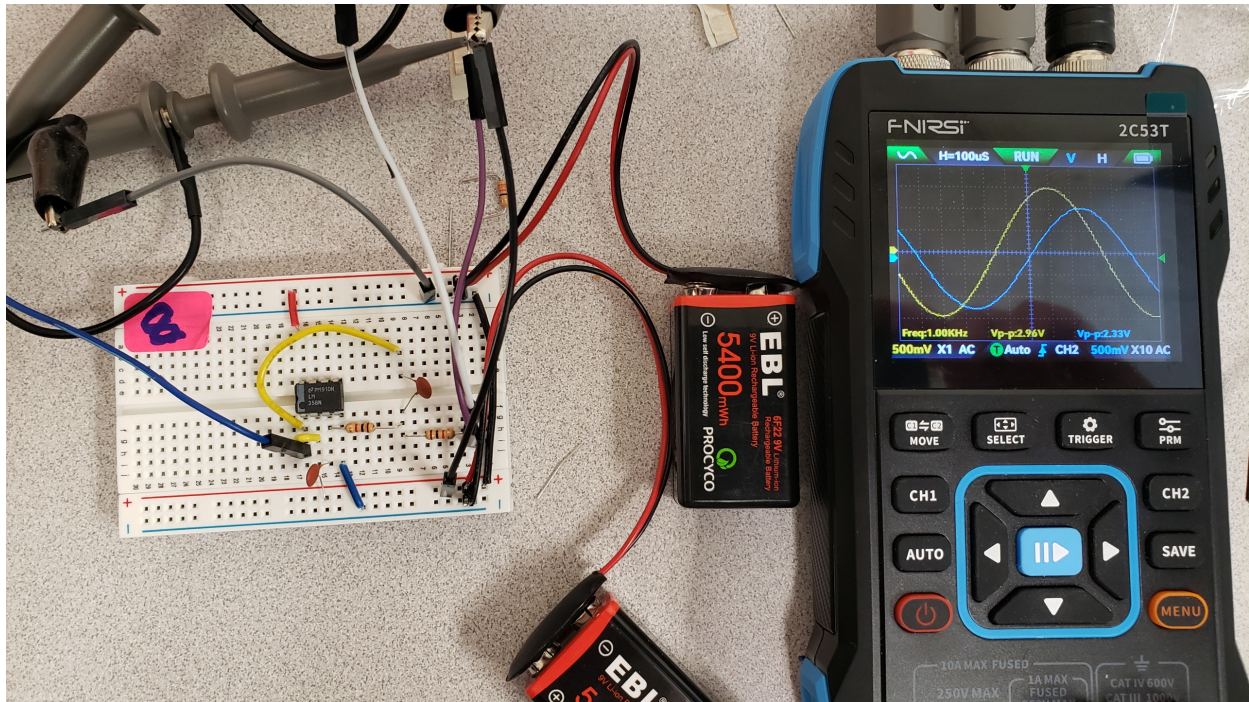
4) Build this circuit using your lab kit

- Set V0 to be a 3Vpp 1kHz sine wave using your FNRSI signal generator
- Include a photo of your finished breadboard



5) Meseasure the votlages at {V1, V2 V3}

- Magnitude & phase



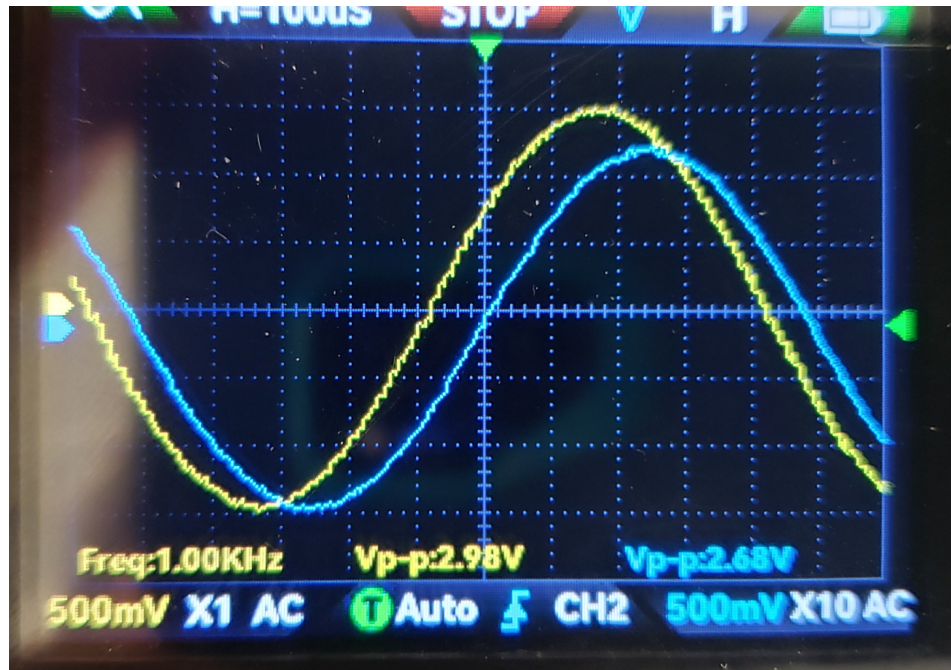
Using the FNRSI meter to output a sine wave (V0) and measure the signals at V0 (yellow) and V3 (blue)

Net Result

	V0	V1	V2
Calculated	$1.5000 \angle -90^0$	$1.2697 \angle -122.167^0$	$1.0748 \angle -154.334^0$
CircuitLab	$1.5000 \angle -90^0$	$1.267 \angle -122.4^0$	$1.069 \angle -155.52^0$
Measured	$1.49 \angle -90^0$	$1.34 \angle -118.8^0$	$1.17 \angle -154.8^0$

Calculations match CircuitLab match measurements (with a slight error in measurements)

V0 & V1: AC coupling (DC offset is removed)



V0 (yellow) & V1 (blue)

Doing some calculations:

V0: (yellow plot)

- Sine wave
- $V_0 = 2.98V_{pp}$
 - should be $3.00V_{pp}$
- Sine wave ($t=0$ is at the zero crossing - somewhat arbitrary)

V1: (blue plot)

- $V_1 = 2.68V_{pp} = 1.34V_p$
- Zero crossing is delayed $80\mu s$ from V0 (-90 degrees)
 - Period = $1ms$

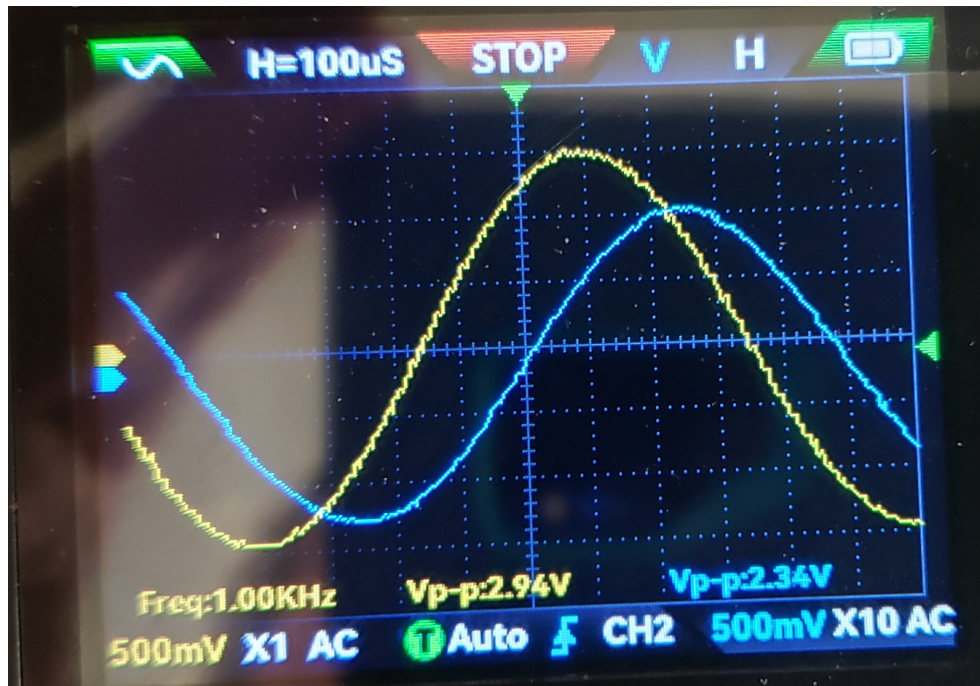
$$\theta_1 = -90^\circ - \left(\frac{80\mu s}{1ms} \right) 360^\circ = -118.8^\circ$$

$$V_1 = 1.34 \angle -118.8^\circ$$

Calculated value

$$V_1 = 1.2697 \angle -122.167^\circ$$

V0 & V2:



V0 (yellow)

- $V_0 = 2.94V_{pp}$
 - should be $3.00V_{pp}$
- Phase = -90 degrees (from the problem)

V2 (blue)

- $V_2 = 2.34V_{pp} = 1.17V_p$
- Zero crossing is delayed 180us from V0 (-90 degrees)
 - Period = 1ms

$$\theta_2 = -90^\circ - \left(\frac{180\mu s}{1ms} \right) 360^\circ = -154.8^\circ$$

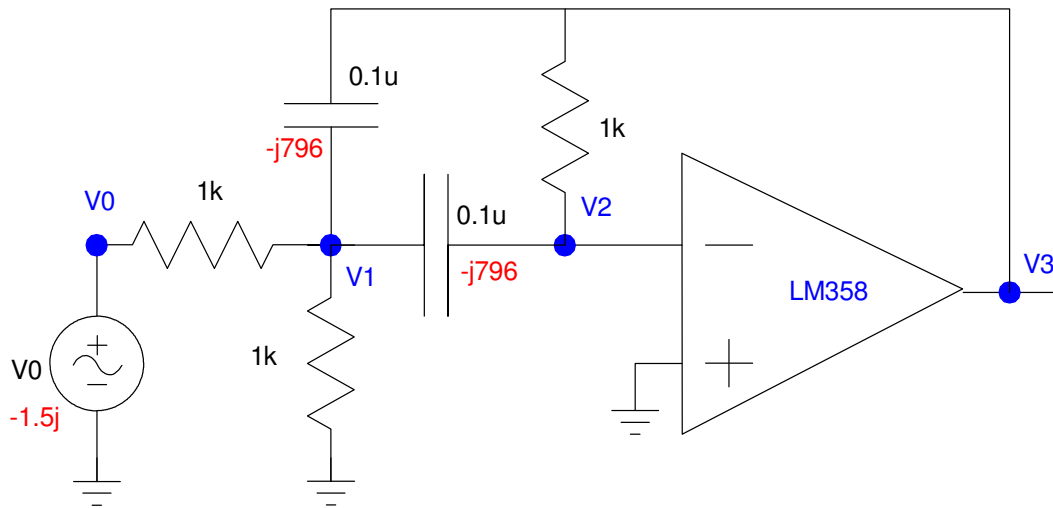
$$V_2 = 1.17 \angle -154.8^\circ$$

Calculated = -154 degrees

$$V_2 = 1.0748 \angle -154.334^\circ$$

Measured values match calculations and simulation results

Active Filters



6) Let V_0 be a $3V_{pp}$ $2kHz$ sine wave

$$V_0 = 1.5 \sin(12566t)$$

- Find the phasor representation for each element in this circuit
- Write the voltage node equations

The phasor representation is:

$$V_0 = -1.5j$$

$$Z_c = \frac{1}{j\omega C} = -796j$$

Write the voltage node equations

$$V_0 = -1.5j$$

$$V_2 = 0$$

$$\left(\frac{V_1 - V_0}{1000} \right) + \left(\frac{V_1}{1000} \right) + \left(\frac{V_1 - V_2}{-796j} \right) + \left(\frac{V_1 - V_3}{-796j} \right) = 0$$

$$\left(\frac{V_2 - V_1}{-796j} \right) + \left(\frac{V_2 - V_3}{1000} \right) = 0$$

7) Solve for $\{V_1, V_2, V_3\}$

Group terms

$$V_0 = -1.5j$$

$$V_2 = 0$$

$$-\left(\frac{1}{1000}\right)V_0 + \left(\frac{2}{1000} + \frac{2}{-796j}\right)V_1 - \left(\frac{1}{-796j}\right)V_2 - \left(\frac{1}{-796j}\right)V_3 = 0$$

$$-\left(\frac{1}{-796j}\right)V_1 + \left(\frac{1}{1000} + \frac{1}{-796j}\right)V_2 - \left(\frac{1}{1000}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \left(\frac{-1}{1000}\right) & \left(\frac{2}{1000} + \frac{2}{-796j}\right) & \left(\frac{1}{796j}\right) & \left(\frac{1}{796j}\right) \\ 0 & \left(\frac{1}{796j}\right) & \left(\frac{1}{1000} - \frac{1}{796j}\right) & \left(\frac{-1}{1000}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1.5j \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> a1 = [1,0,0,0];
>> a2 = [0,0,1,0];
>> a3 = [-1/1000, 2/1000-2/796j, 1/796j, 1/796j];
>> a4 = [0, 1/796j, 1/1000-1/796j, -1/1000];
>> A = [a1;a2;a3;a4]
```

```
1.0000      0      0      0
      0      0      1.0000      0
-0.0010      0.0020 + 0.0025i      0 - 0.0013i      0 - 0.0013i
      0      0 - 0.0013i      0.0010 + 0.0013i     -0.0010
```

```
>> B = [-1.5j ; 0 ; 0 ; 0]
```

```
0 - 1.5000i
0
0
0
```

```
>> V = inv(A)*B
```

```
v0      0 - 1.5000i
v1 -0.5806 - 0.0975i
v2      0
v3 -0.1224 + 0.7294i
```

or if you prefer polar form...

```
>> abs(V)

|V0|    1.5000
|V1|    0.5888
|V2|     0
|V3|    0.7397

>> angle(V)*180/pi

V0q   -90.0000
V1q  -170.4712
V2q     0
V3q   99.5288
```

or

$$V_0 = 1.5 \cos(\omega t - 90^\circ)$$

$$V_1 = 0.5888 \cos(\omega t - 170.47^\circ)$$

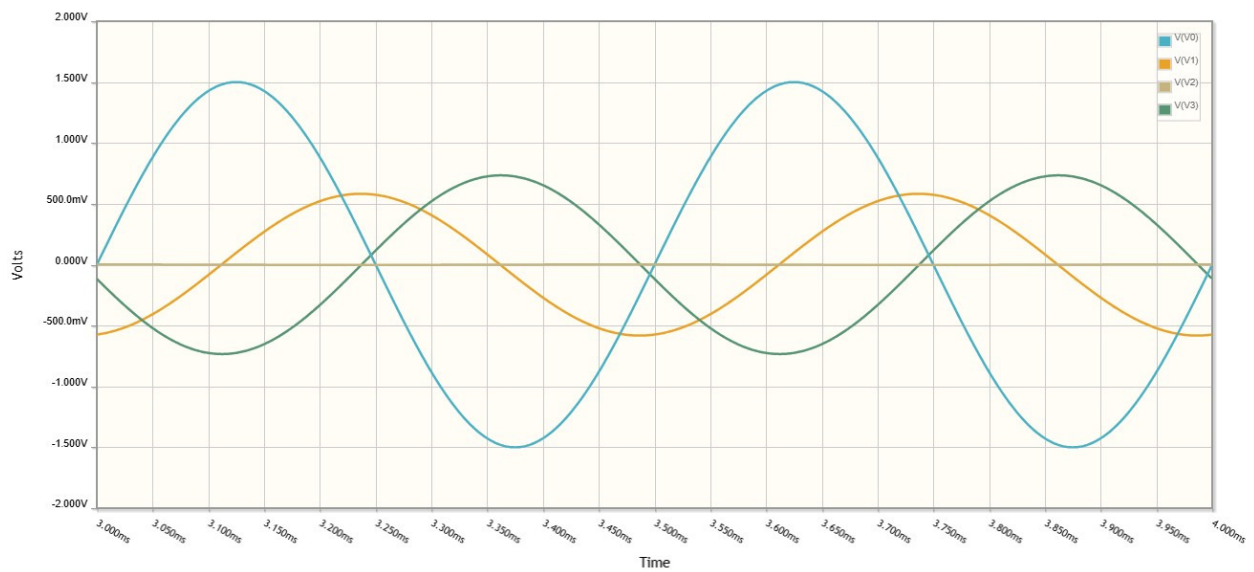
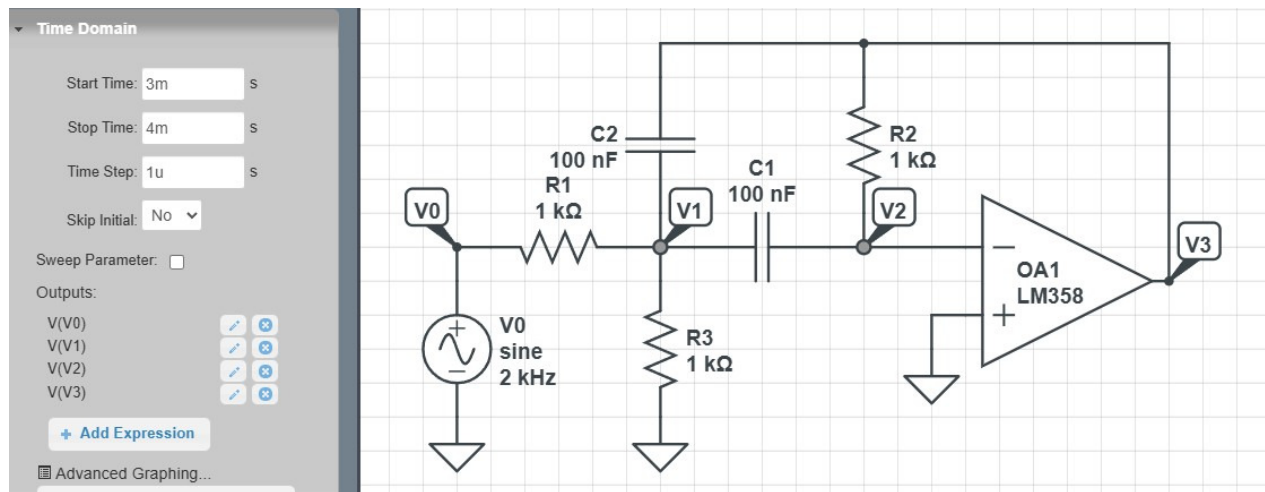
$$V_2(t) = 0$$

$$V_3(t) = 0.7397 \cos(\omega t + 99.53^\circ)$$

	V1	V2	V3
Calculated	$0.5888 \angle -170.47^\circ$	0	$0.7397 \angle +99.53^\circ$
CircuitLab	-	-	-
Measured	-	-	-

8) Simulate this circuit using CircuitLab. Measure the signals at {V1, V2, V3}

- Magnitude and phase



From the graph

- $V_0 = 1.5 \angle -90^\circ$
- (same as calculations)

V1:

- peak = 582.1mV
- zero crossing 0.111ms delayed from V0 (-90 degrees)
- period = 0.5ms

$$\theta_1 = -\left(\frac{0.111ms}{0.5ms}\right) 360^\circ - 90^\circ = -169.92^\circ$$

$$V_1 = 0.582 \angle -169.92$$

V3:

- peak = 733.8mV
- zero crossing 0.236ms delayed from V0 (-90 degrees)
- period = 0.5ms

$$\theta_3 = -\left(\frac{0.236ms}{0.5ms}\right) 360^\circ = -259.92^\circ = +100.08^\circ$$

$$V_3 = 0.7338 \angle +100.8^\circ$$

Calculations match CircuitLab results

	V1	V2	V3
Calculated	$0.5888 \angle -170.47^\circ$	0	$0.7397 \angle +99.53^\circ$
CircuitLab	$0.582 \angle -169.92$	0	$0.7338 \angle +100.8^\circ$
Measured	-	-	-