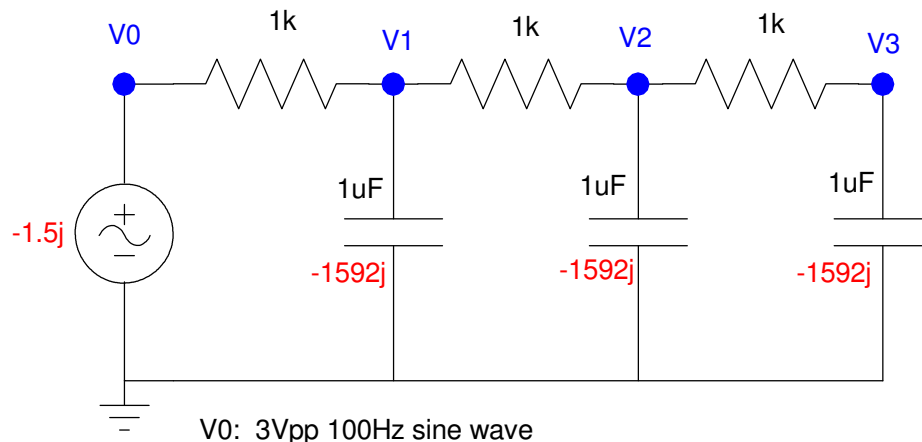


ECE 211: Homework #7

Phasors & Voltage Nodes / Current Loops / Max Power

Phasors & Voltage Nodes



Problems 1-6

- 1) Write the voltage node equations for the given RC circuit. Assume V0 is a 3Vpp 100Hz sine wave

$$V_0 = 1.5 \sin(628t)$$

Start by expressing the sources and impedances in phasor form

$$\omega = 628 \frac{\text{rad}}{\text{sec}}$$

$$V_0 = 0 - 1.5j \quad \text{real} = \text{cosine}, \quad -\text{imag} = \text{sine}$$

$$Z_C \rightarrow \frac{1}{j\omega C} = -1592j$$

Now write the four voltage node equations

$$V_0 = -1.5j$$

$$\left(\frac{V_1 - V_0}{1000} \right) + \left(\frac{V_1}{-1592j} \right) + \left(\frac{V_1 - V_2}{1000} \right) = 0$$

$$\left(\frac{V_2 - V_1}{1000} \right) + \left(\frac{V_2}{-1592j} \right) + \left(\frac{V_2 - V_3}{1000} \right) = 0$$

$$\left(\frac{V_3 - V_2}{1000} \right) + \left(\frac{V_3}{-1592j} \right) = 0$$

2) Solve for the voltages $\{V_1, V_2, V_3\}$

Group the terms

$$V_0 = -1.5j$$

$$-\left(\frac{1}{1000}\right)V_0 + \left(\frac{1}{1000} + \frac{1}{-1592j} + \frac{1}{1000}\right)V_1 - \left(\frac{1}{1000}\right)V_2 = 0$$

$$-\left(\frac{1}{1000}\right)V_1 + \left(\frac{1}{1000} + \frac{1}{-1592j} + \frac{1}{1000}\right)V_2 - \left(\frac{1}{1000}\right)V_3 = 0$$

$$-\left(\frac{1}{1000}\right)V_2 + \left(\frac{1}{1000} + \frac{1}{-1592j}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{1000}\right) & \left(\frac{1}{1000} + \frac{1}{-1592j} + \frac{1}{1000}\right) & \left(\frac{-1}{1000}\right) & 0 \\ 0 & \left(\frac{-1}{1000}\right) & \left(\frac{1}{1000} + \frac{1}{-1592j} + \frac{1}{1000}\right) & \left(\frac{-1}{1000}\right) \\ 0 & 0 & \left(\frac{-1}{1000}\right) & \left(\frac{1}{1000} + \frac{1}{-1592j}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1.5j \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> a1 = [1,0,0,0];
>> a2 = [-1/1000,1/1000+1/(-1592j)+1/1000,-1/1000,0];
>> a3 = [0,-1/1000,1/1000+1/(-1592j)+1/1000,-1/1000];
>> a4 = [0,0,-1/1000,1/1000+1/(-1592j)];
>> A = [a1 ; a2 ; a3 ; a4]

    1.0000         0         0         0
   -0.0010    0.0020 + 0.0006i   -0.0010         0
         0   -0.0010    0.0020 + 0.0006i   -0.0010
         0         0   -0.0010    0.0010 + 0.0006i

>> B = [-1.5j ; 0 ; 0 ; 0]

    0 - 1.5000i
         0
         0
         0

>> V = inv(A)*B

V0         0 - 1.5000i
V1   -0.4457 - 0.6797i
V2   -0.4645 - 0.1393i
V3   -0.3958 + 0.1094i
```

This gives the voltages in rectangular form

```
V0      0 - 1.5000i
V1 -0.4457 - 0.6797i
V2 -0.4645 - 0.1393i
V3 -0.3958 + 0.1094i
```

means

$$V_0 = 1.5 \sin(628t)$$

$$V_1 = -0.4457 \cos(628t) + 0.6797 \sin(628t)$$

$$V_2 = -0.4645 \cos(628t) + 0.1393 \sin(628t)$$

$$V_3 = -0.3958 \cos(628t) - 0.1094 \sin(628t)$$

If you prefer polar form

```
>> abs(V)
```

```
1.5000
0.8128
0.4849
0.4106
```

```
>> angle(V)*180/pi
```

```
-90.0000
-123.2562
-163.3101
164.5553
```

$$V_0 = 1.5 \cos(628t - 90^\circ)$$

$$V_1 = 0.8128 \cos(628t - 123^\circ)$$

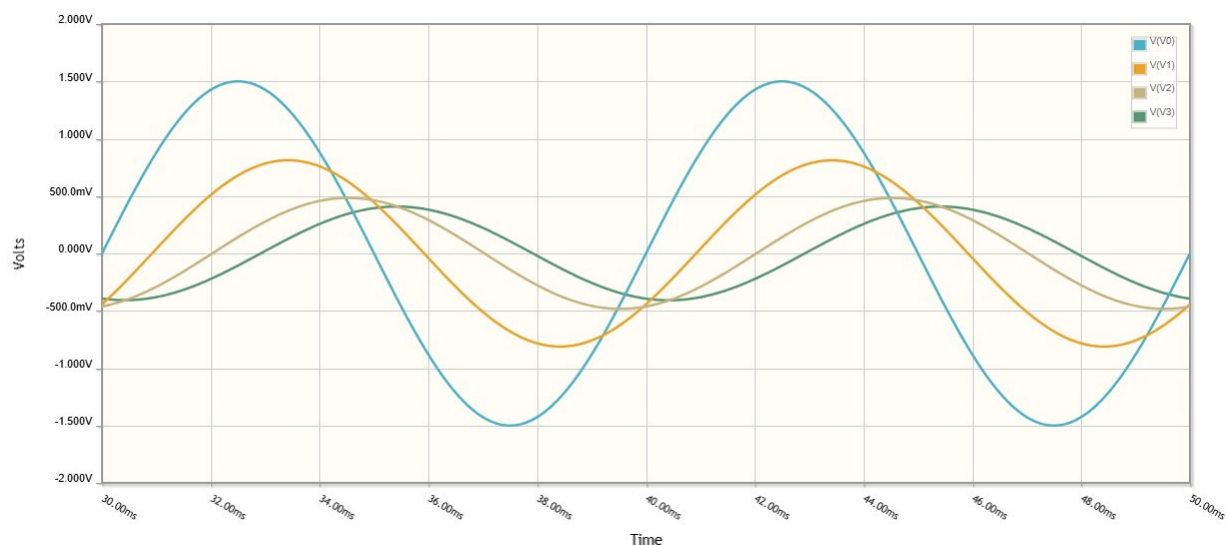
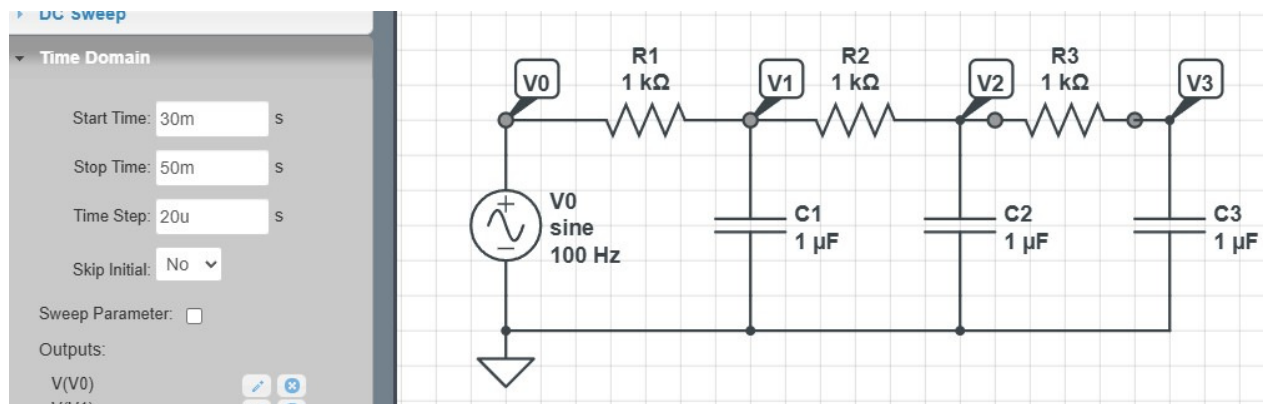
$$V_2 = 0.4849 \cos(628t - 163^\circ)$$

$$V_3 = 0.4060 \cos(628t + 164^\circ)$$

```
>>
```

3) Simulate this circuit using CircuitLab. Find the voltages at {V1, V2, V3}

- Both magnitude and phase



Pulling data off the graph

V1:

- peak = 812.2mV @ 33.44ms

$$\theta_1 = -\left(\frac{3.44ms}{10ms}\right) 360^\circ = -123.8^\circ$$

$$V_1 = 0.8122 \angle -123.8^\circ$$

Simulated

$$V_1 = 0.8128 \angle -123^\circ$$

Calculated

V2:

- peak = 483.7V @ 34.66ms

$$\theta_2 = -\left(\frac{4.66ms}{10ms}\right)360^0 = -167.8^0$$

$$V_2 = 0.4837\angle -167.8^0 \quad \text{circuitlab}$$

$$V_2 = 0.4849\angle -163^0 \quad \text{Matlab}$$

V3:

- peak = 407.6mV @ 35.52ms

$$\theta_3 = \left(\frac{5.532ms}{10ms}\right)360^0 = -199.2^0 = +160.8^0$$

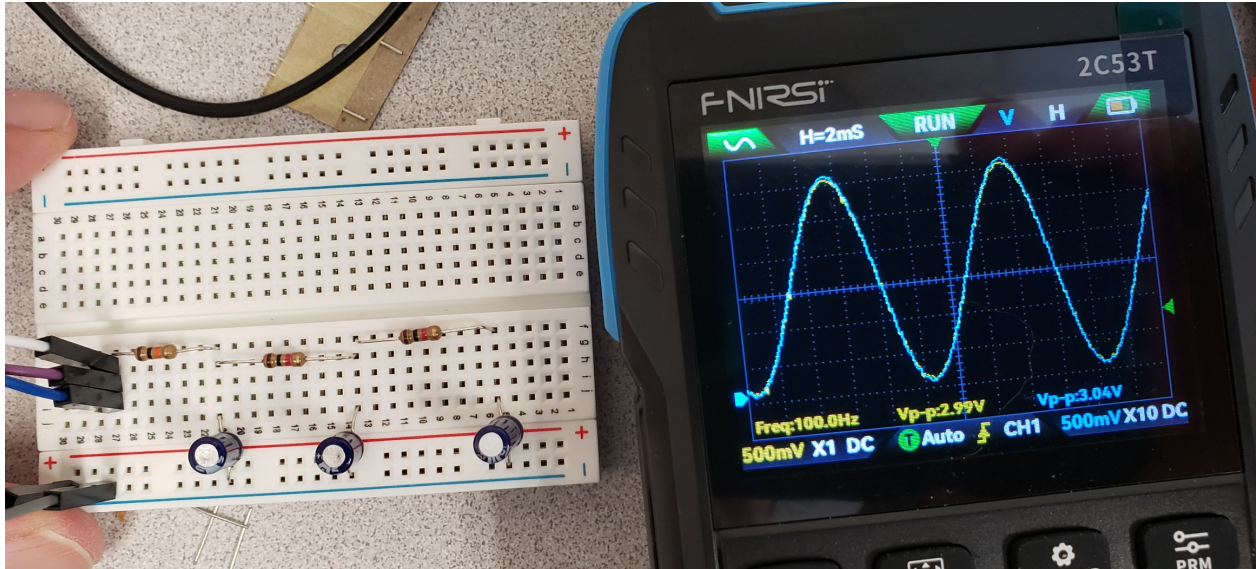
$$V_3 = 0.4076\angle +160.8^0 \quad \text{circuitlab}$$

$$V_3 = 0.4060\angle +164^0 \quad \text{Matlab}$$

Lab (part 1):

4) Build this circuit using your lab kit.

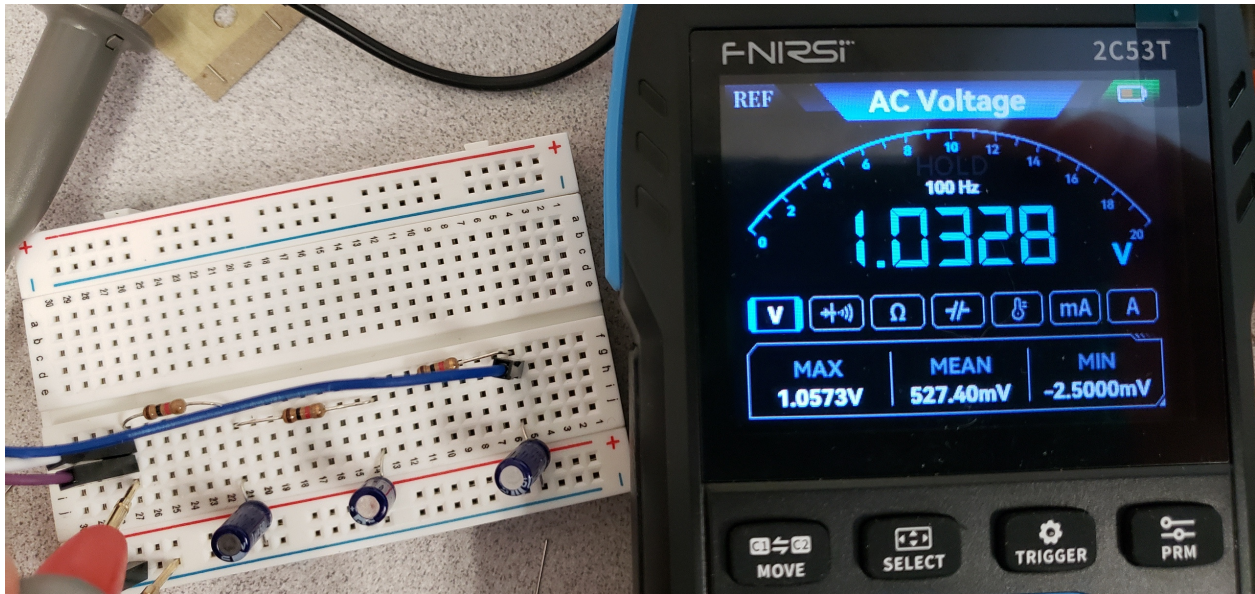
- Set V0 to be a 3Vpp 100Hz sine wave using your FNRSI meter
- Include a photo in your homework set



Breadboard circuit along with the 3Vpp AC signal from the FNRSI meter.

- Both channel 1 and channel 2 are on the input
- Both probes are measuring the 3Vpp sine wave correctly

- 5) Measure the voltages at {V0, V1, V2, and V3} using your FNRSI multimeter
- AC voltages



Measuring the AC voltages at V0, V1, V2, V3

	Calculation	CircuitLab	Measured (rms)	Measured (Vp)
V0	1.500 Vp	1.500 Vp	1.0328 Vrms	1.4606 Vp
V1	0.8128 Vp	0.8122 Vp	0.5578 Vrms	0.7888 Vp
V2	0.4849 Vp	0.4837 Vp	0.3363 Vrms	0.4756 Vp
V3	0.4060 Vp	0.4076 Vp	0.2822 Vrms	0.3991 Vp

note: All three voltages match up: calculated, simulated, & measured

$$V_p = \sqrt{2} \cdot V_{rms}$$

6) Measure the voltages at {V0, V1, V2, and V3} using your FNRSI oscilloscope

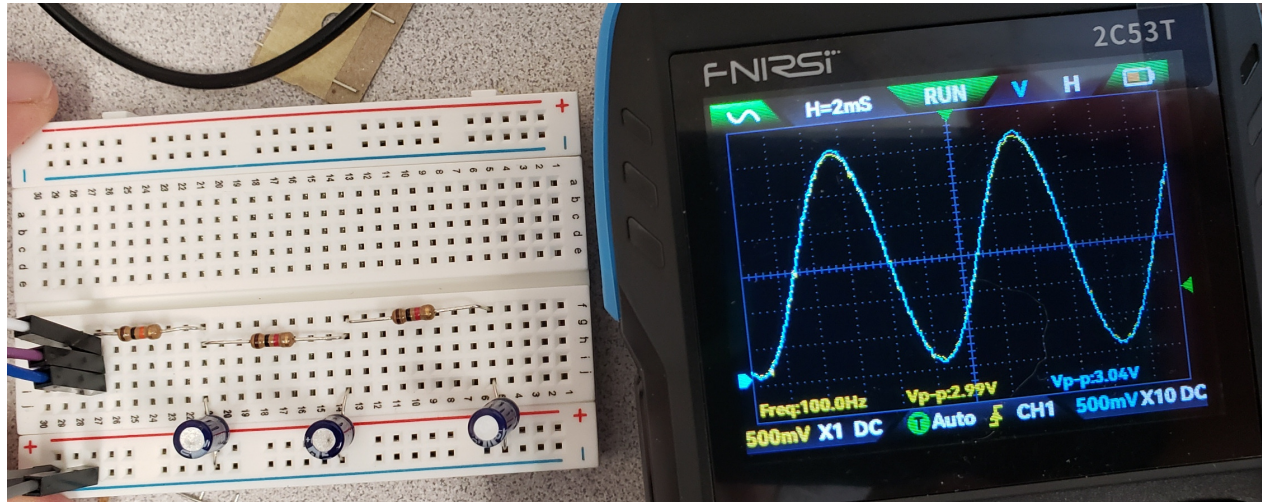
- Amplitude
- Phase relative to V0

Voltage V0:

- Channel 1 (yellow): V0
- Channel 2 (blue) V0

Phasor voltage : Assume $t=0$ is at the zero crossing for channel 1 (making V1 a sine wave)

$$V_0 = 1.50 \angle -90^\circ$$



Voltage V1:

- Channel 1 (yellow): V_{in}
- Channel 2 (blue): V_1

Treat the zero crossing of V_{in} to be $t=0$

$$V_1 = 1.44 \angle -90^\circ \quad \text{battery is getting low?}$$

V_1 :

$$V_p = 0.805V \quad (1/2 V_{pp})$$

Peak is delayed by 0.3 divisions

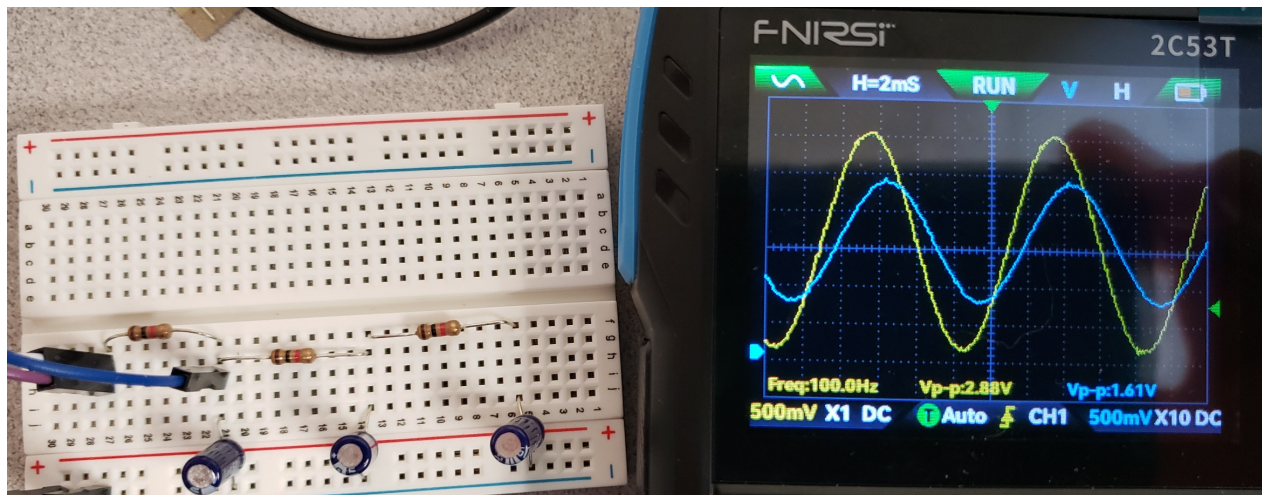
$$\text{Period} = 5 \text{ divisions} \quad (10\text{ms})$$

$$\theta_1 = -90^\circ - \left(\frac{0.3 \text{ divisions}}{5 \text{ divisions}} \right) 360^\circ = -111.6^\circ$$

$$V_1 = 0.805 \angle -111.6^\circ$$

Calculated value:

$$V_1 = 0.8128 \angle -123^\circ$$



Channel 1 = V_0 , Channel 2 = V_1

Voltage V2: (blue)

$$V_p = 478\text{mV}$$

Delay = 1 division from V0

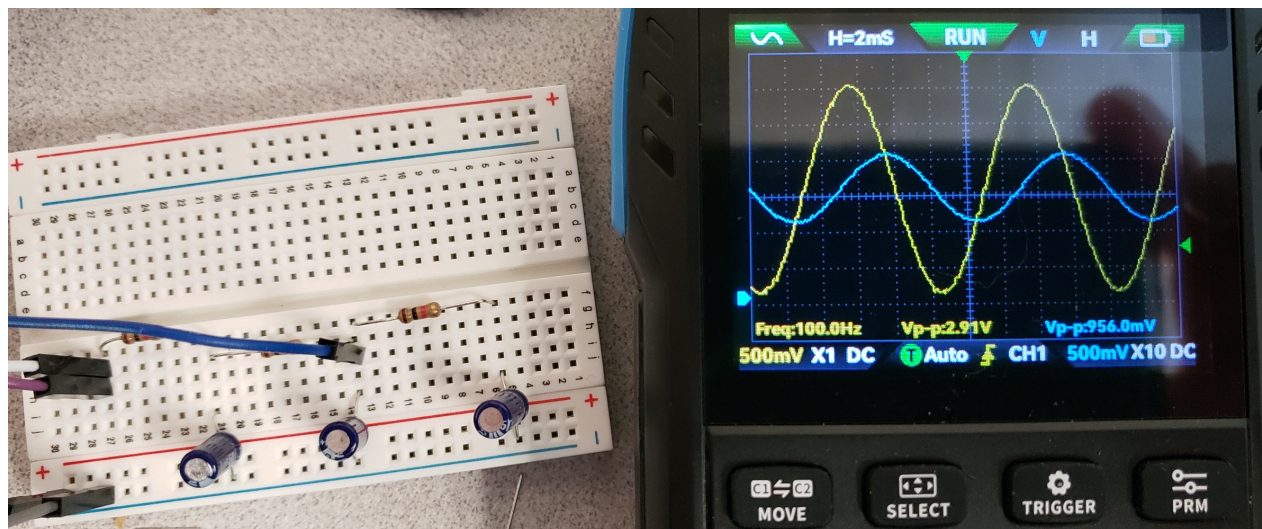
Period = 5 divisions (10ms)

$$\theta_2 = -90^\circ - \left(\frac{1 \text{ division}}{5 \text{ divisions}} \right) 360^\circ = -162^\circ$$

$$V_2 = 0.478 \angle -162^\circ$$

Calculated value:

$$V_2 = 0.4837 \angle -167.8^\circ$$



Voltage V3 (blue):

$$V_p = 405\text{mV}$$

delay = 1.4 divisions from V0 (yellow)

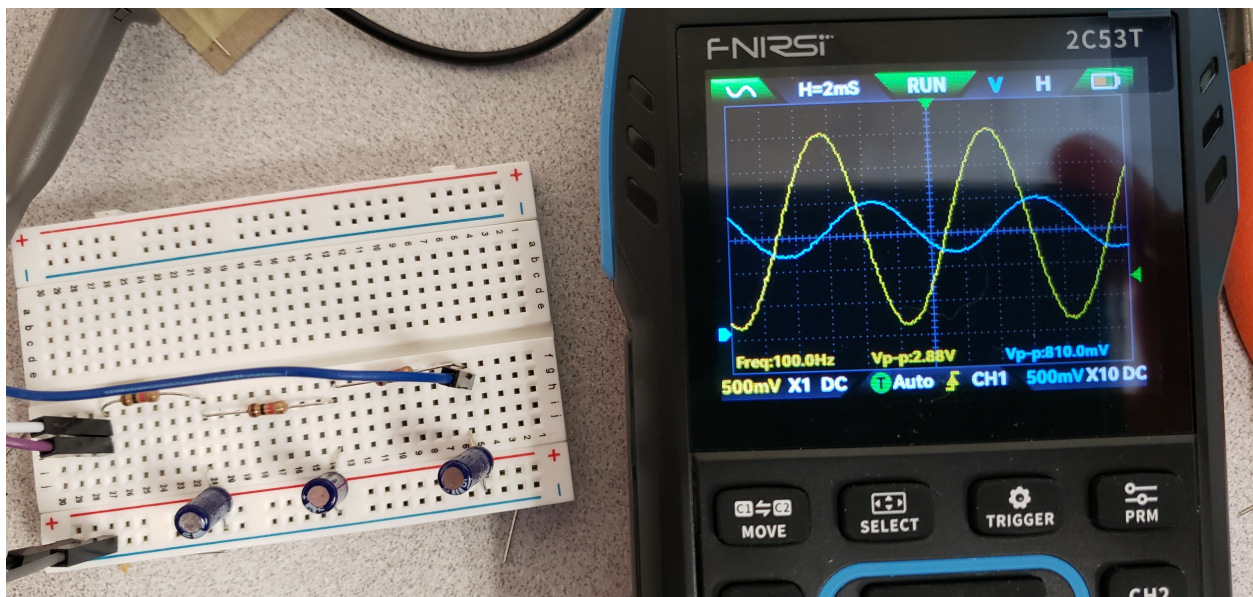
period = 5 divisions (10ms)

$$\theta_3 = -90^\circ - \left(\frac{1.4}{5}\right) 360^\circ = -190.8^\circ$$

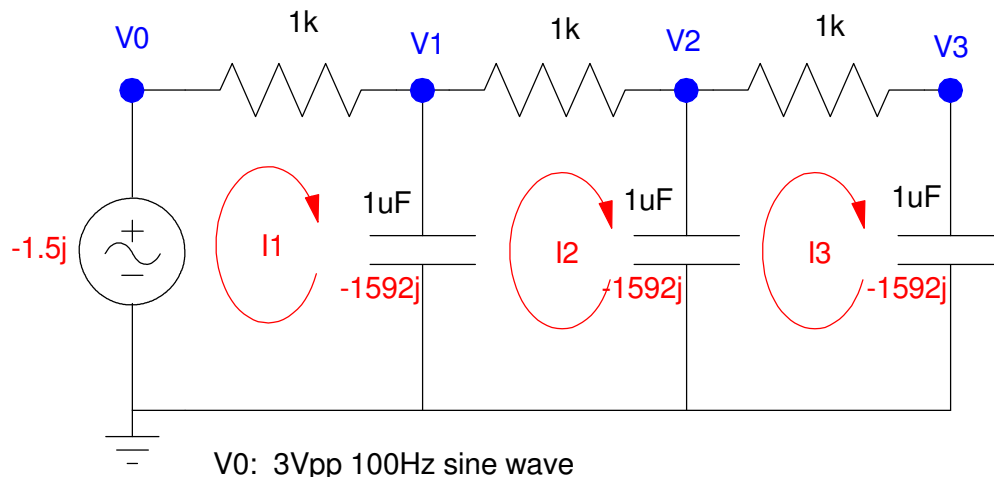
$$V_3 = 0.405 \angle -190.8^\circ$$

Calculations

$$V_3 = 0.4076 \angle -199.2^\circ$$



Phasors & Current Loops



Problems 7-12

7) Write the current loop equations for the given RC circuit. Assume V0 is a 3Vpp 100Hz sine wave

$$V_0 = 1.5 \sin(628t)$$

Step 1: Convert to phasors (use peak voltages and currents)

$$V_0 = -1.5j$$

$$Z_C \rightarrow \frac{1}{j\omega C} = -1592j$$

Step 2: Write the current loop equations

Loop I1

$$-(-1.5j) + 1000(I_1) - 1592j(I_1 - I_2) = 0$$

Loop I2

$$-1592j(I_2 - I_1) + 1000(I_2) - 1592j(I_2 - I_3) = 0$$

Loop I3

$$-1592j(I_3 - I_2) + 1000(I_3) - 1592j(I_3) = 0$$

8) Solve for the currents $\{I_1, I_2, I_3\}$

Group terms

$$(1000 - 1592j)I_1 + (1592j)I_2 = -1.5j$$

$$(1592j)I_1 + (1000 - 3184j)I_2 + (1592j)I_3 = 0$$

$$(1592j)I_2 + (1000 - 3184j)I_3 = 0$$

Place in matrix form

$$\begin{bmatrix} (1000 - 1592j) & (1592j) & 0 \\ (1592j) & (1000 - 3184j) & (1592j) \\ 0 & (1592j) & (1000 - 3184j) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -1.5j \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> a1 = [1000-1592j, 1592j, 0];
>> a2 = [1592j, 1000-3184j, 1592j];
>> a3 = [0, 1592j, 1000-3184j];
>> A = [a1;a2;a3]

1.0e+003 *

    1.0000 - 1.5920i      0 + 1.5920i      0
      0 + 1.5920i    1.0000 - 3.1840i      0 + 1.5920i
      0              0 + 1.5920i    1.0000 - 3.1840i

>> B = [-1.5j ; 0 ; 0]

      0 - 1.5000i
      0
      0

>> I = inv(A)*B;
>> mA = I*1000
```

```
I1    0.4457 - 0.8203i    mA
I2    0.0188 - 0.5404i    mA
I3   -0.0687 - 0.2486i    mA
```

in polar form...

```
>> abs(mA)

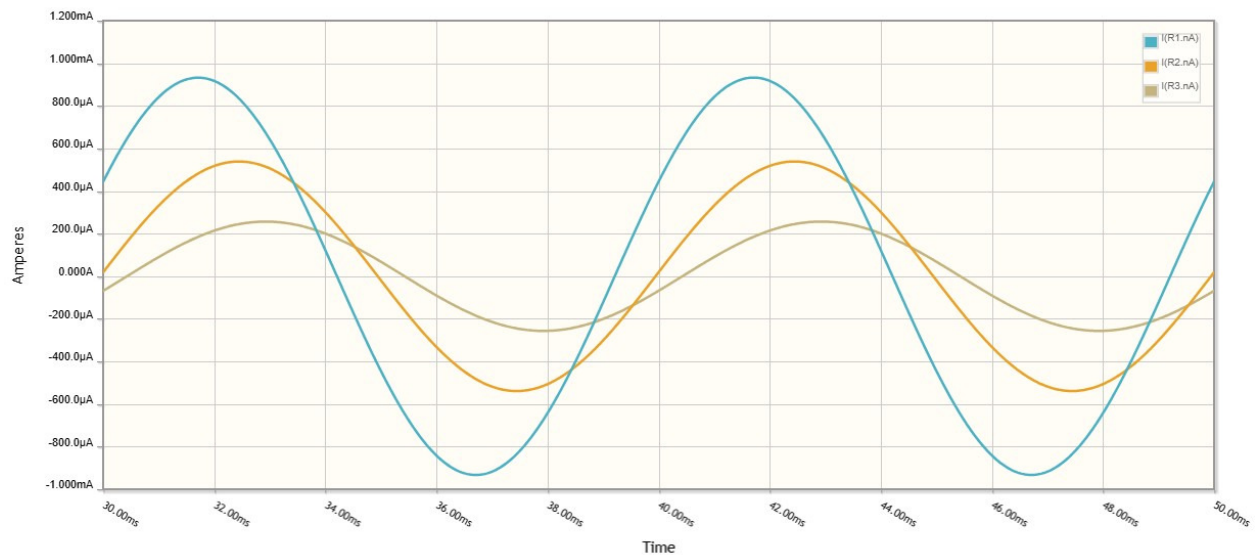
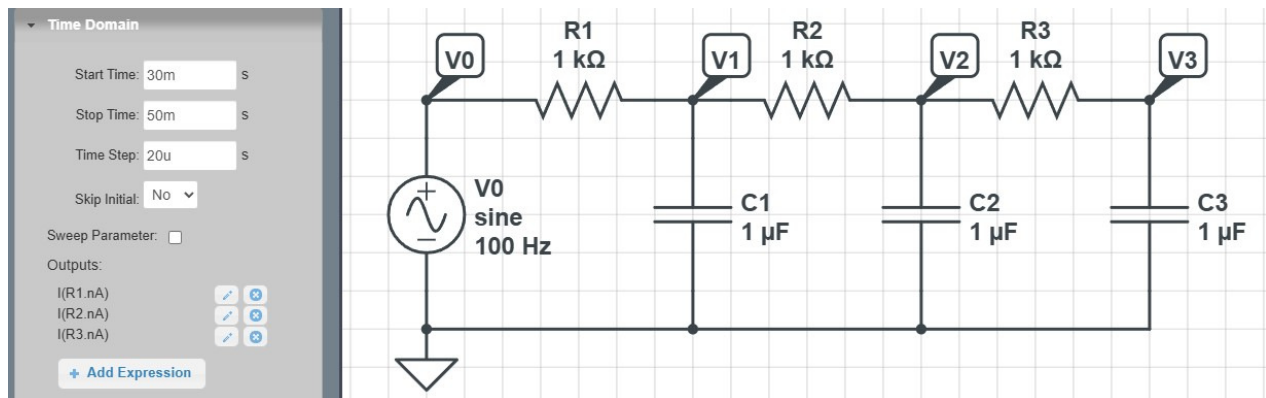
I1    0.9336    mA
I2    0.5407    mA
I3    0.2579    mA

>> angle(mA)*180/pi

I1   -61.4844 degrees
I2   -88.0088 degrees
I3  -105.4447 degrees
```

9) Simulate this circuit using CircuitLab. Find the currents {I1, I2, I3}

- Both magnitude and phase



From the graph

I1:

Peak = 932.2uA @ 1.78ms

$$\theta_1 = -\left(\frac{1.78ms}{10ms}\right) 360^\circ = -64.08^\circ$$

$$I_1 = 932.2 \angle -64.08^\circ \mu A$$

Calculations

$$I_1 = 933.6 \angle -61.48^\circ$$

I2: peak = 538.3uA @ 2.50ms

$$\theta_2 = -\left(\frac{2.50ms}{10ms}\right) 360^0 = -90^0$$

$$I_2 = 538.3 \angle -90^0 \text{ uA}$$

calculation

$$I_2 = 540.7 \angle -88.0^0 \text{ uA}$$

I3: peak = 256.4uA @ 2.96ms

$$\theta_3 = -\left(\frac{2.96ms}{10ms}\right) 360^0 = -106.56^0$$

$$I_3 = 256.4 \angle -106.56^0 \text{ uA}$$

calculation

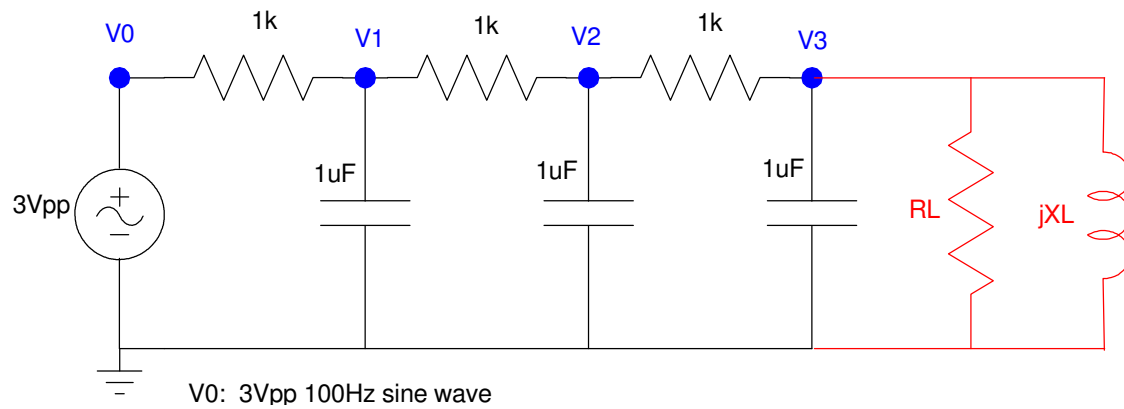
$$I_3 = 257.9 \angle -105.4^0 \text{ uA}$$

Calculations match CircuitLab

Max Power Transfer

10) Find R_L and jX_L to maximize the power to the load

- Also find the maximum power to the load



Problem 11

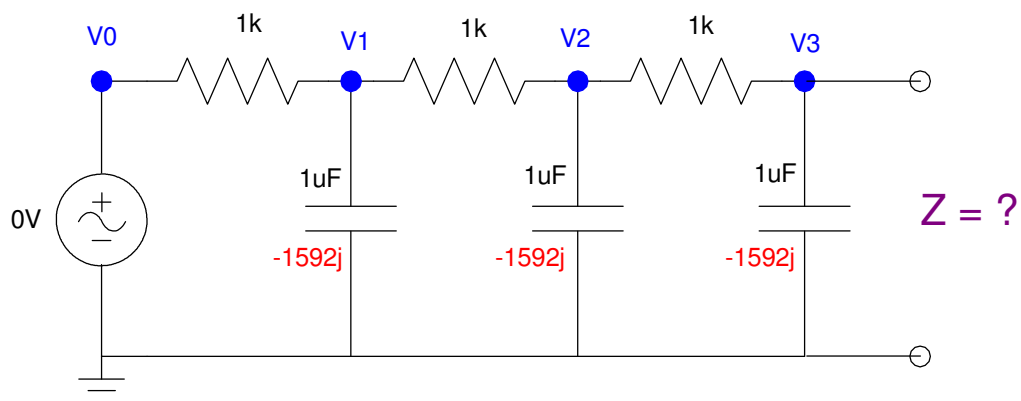
First, find the Thevenin equivalent

Find the open-circuit voltage (from problem #1). This is V_{th} .

$$V_3 = 0.4076 \angle -199.2^\circ$$

Find the impedance looking in:

- Turn off the source
- Compute the impedance looking in



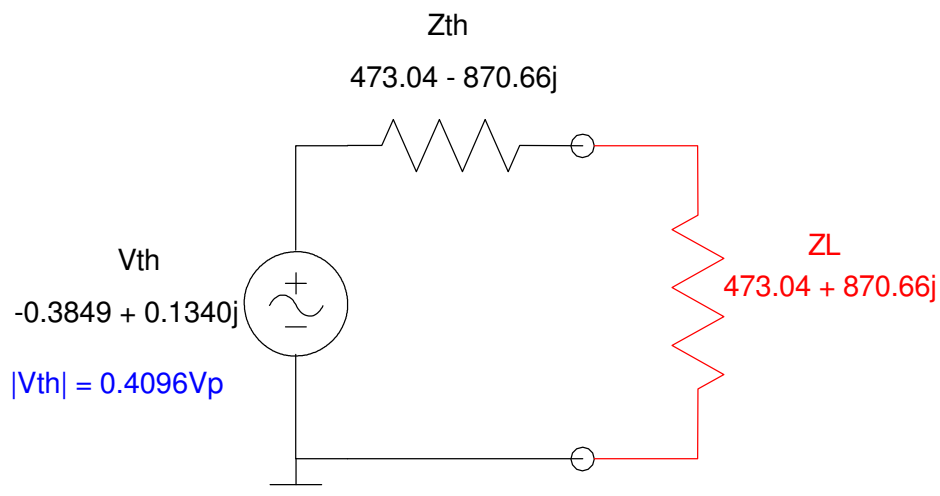
In Matlab

```
>> Z1 = 1 / (1/1000 + 1/(-1592j))  
  
Z1 = 7.1707e+002 -4.5042e+002i  
  
>> Z2 = Z1 + 1000;  
>> Z3 = 1 / (1/Z2 + 1/(-1592j))  
  
Z3 = 6.1123e+002 -8.6495e+002i  
  
>> Z4 = Z3 + 1000;  
>> Z5 = 1 / (1/Z4 + 1/(-1592j))  
  
Z5 = 4.7304e+002 -8.7066e+002i
```

The impedance looking in is

$$Z_{th} = 473.04 - 870.66j$$

The Thevenin equivalent is thus:



Max Power to the Load: To maximize power to the load, use the complex conjugate:

$$Z_L = Z_{th}^*$$

$$Z_L = 473.04 + 870.66j$$

This can be implemented using R&L in series or parallel.

The series model is pretty straight-forward

```
>> ZL = conj(Z5)

ZL = 4.7304e+002 +8.7066e+002i

>> Rs = real(ZL)

Rs = 473.0403

>> Xs = imag(ZL)

Xs = 870.6648
```

The parallel model is a little more tricky. To convert to a parallel model, take the inverse:

$$\frac{1}{R_L} + \frac{1}{jX_L} = \frac{1}{Z_L}$$

The real part is the inverse of ZL. The imaginary part is the inverse of -ZL

```
>> ZLi = 1/ZL

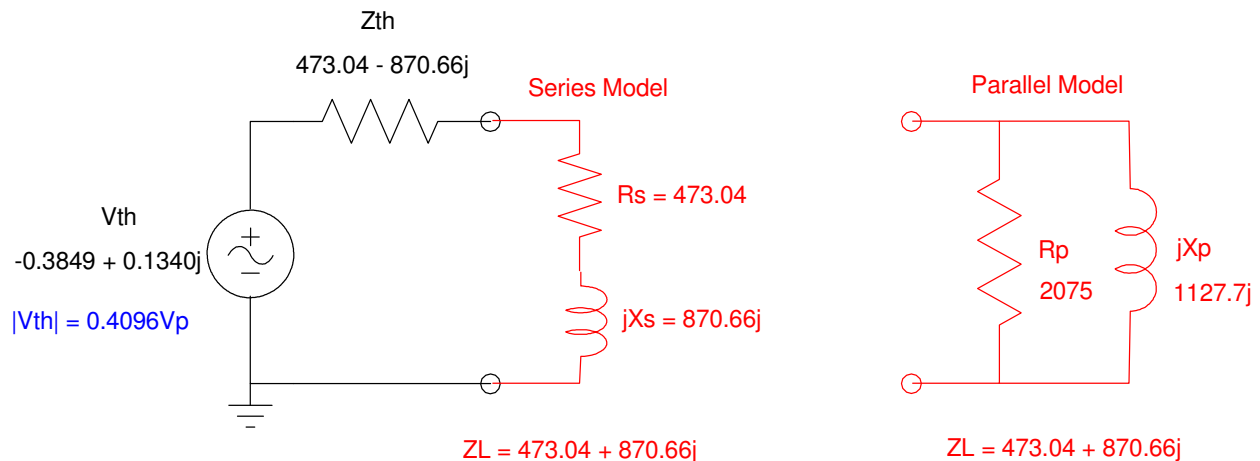
ZLi = 4.8180e-004 -8.8678e-004i

>> Rp = 1 / real(ZLi)

Rp = 2.0756e+003

>> Xp = -1 / imag(ZLi)

Xp = 1.1277e+003
```



The load can be implemented using R and L in series (left) or parallel (right)

Power to the Load: You can use either the series or parallel model - makes no difference. Let's use the series model to compute the power to the load. In Matlab

```
>> Vth = -0.3958 + 0.1094i
Vth = -0.3958 + 0.1094i
>> Rth = 4.7304e+002 -8.7066e+002i;
>> RL = conj(Rth)
RL = 4.7304e+002 +8.7066e+002i
>> I = Vth / (Rth + RL)
I = -4.1836e-004 +1.1564e-004i
```

Once you know the loop current (in peak units), you can compute the power to the load as

$$P = \frac{1}{2} \cdot V_p \cdot I_p^* = \frac{1}{2} |I_p|^2 \cdot Z$$

```
>> PL = 0.5 * abs(I)^2 * RL
PL = 4.4559e-005 +8.2014e-005i
```

The real part is the power to the load

$$P(\text{max}) = 44.559 \mu W$$

The complex part is the reactive power (power that bounces back and forth between the source and the load.) Utilities hate reactive power: it costs them money (current is bouncing back and forth costing the utility money through I^2R losses) that the customer doesn't pay for (no net power goes to the customer). Otherwise, you generally ignore the reactive power.