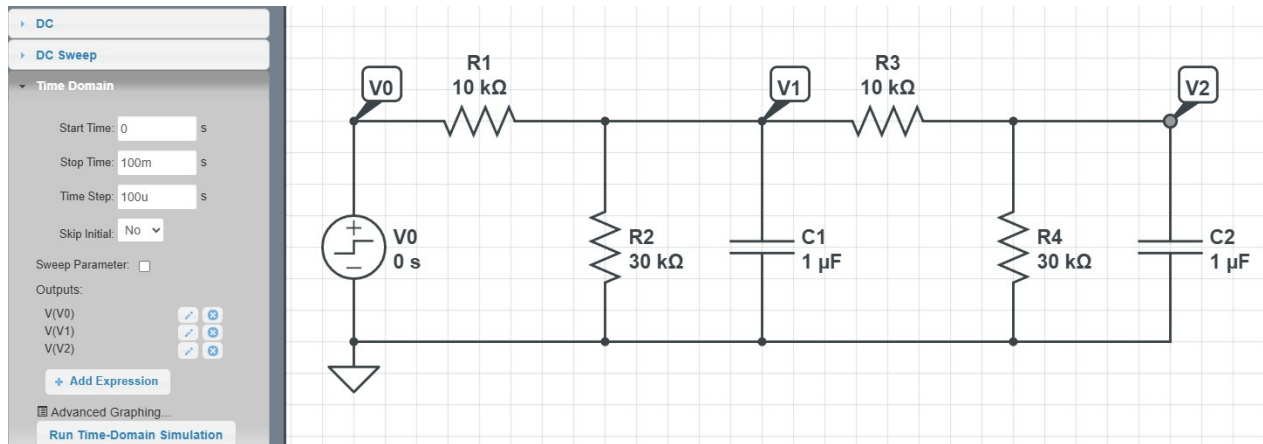


ECE 211: Homework #6

Capacitors, Inductors, Complex Numbers, Phasors

Capacitors

1) Find the differential equations which describes the following circuit



Problem 1-4

First capacitor

$$I_1 = CV'_1 = \left(\frac{V_0 - V_1}{10k} \right) + \left(\frac{0 - V_1}{30k} \right) + \left(\frac{V_2 - V_1}{10k} \right)$$

Second capacitor

$$I_2 = CV'_2 = \left(\frac{V_1 - V_2}{10k} \right) + \left(\frac{0 - V_2}{30k} \right)$$

Simplifying

$$V'_1 = 100V_0 - 233.33V_1 + 100V_2$$

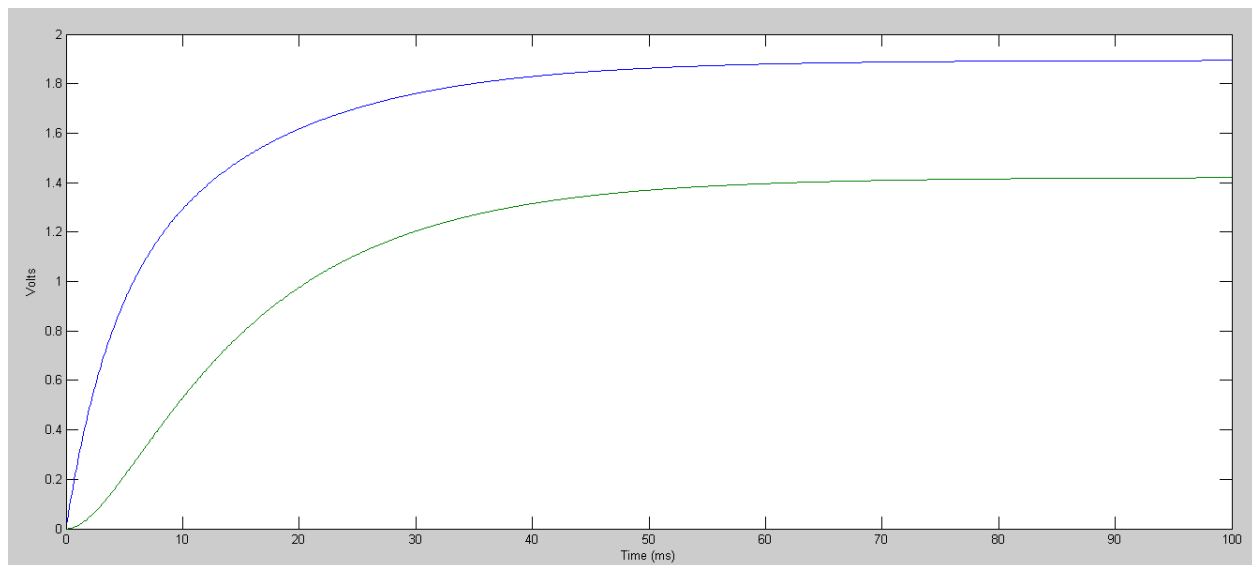
$$V'_2 = 100V_1 - 133.33V_2$$

2) Using Matlab and numerical integration, determine the voltages for $0 < t < 100\text{ms}$

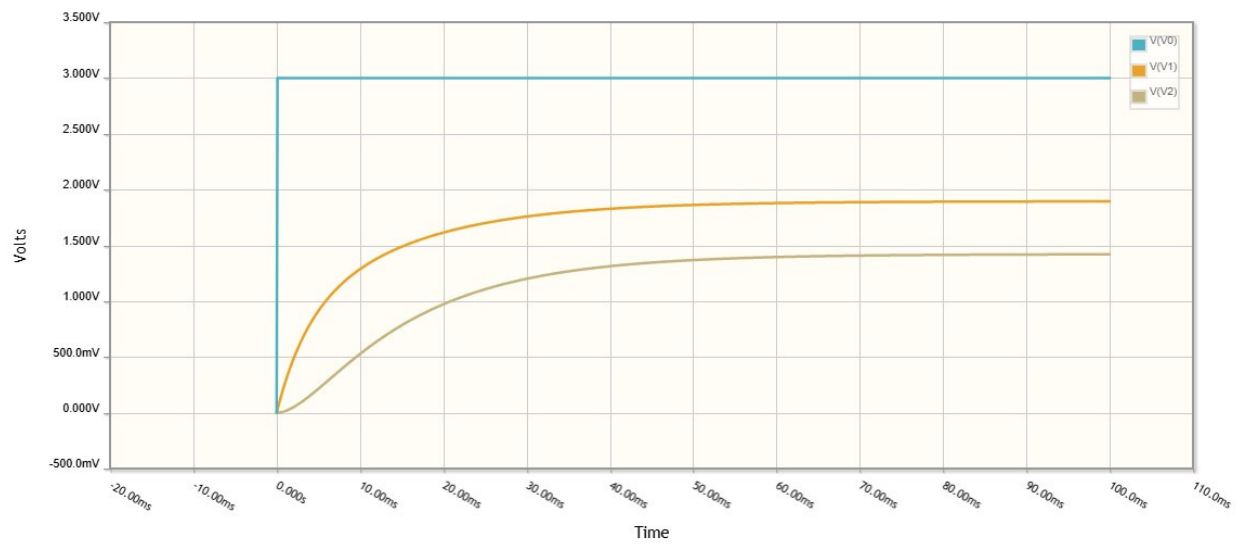
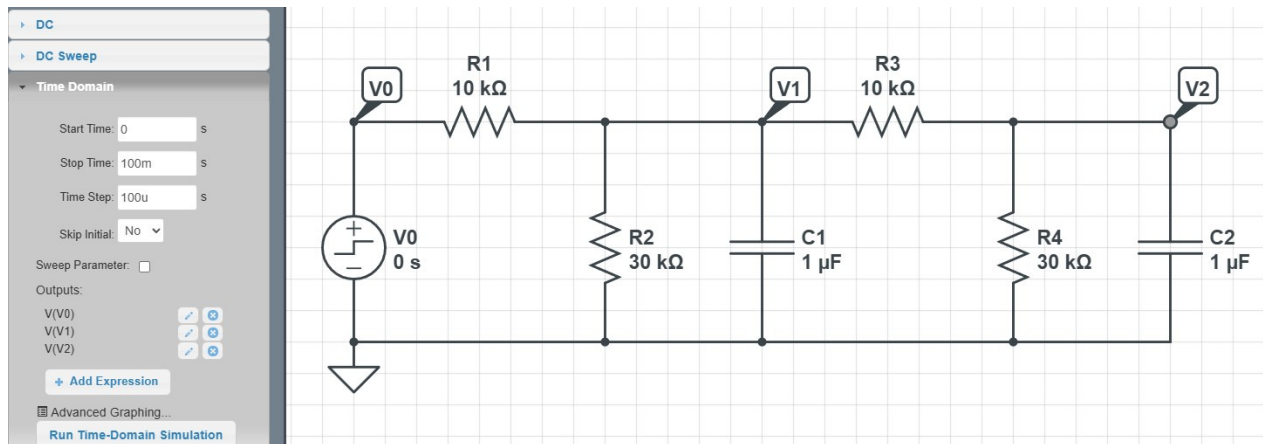
- Set the simulation step size to $100\mu\text{s}$ (1000x smaller than the simulation time)

Matlab Code

```
V1 = 0;  
V2 = 0;  
t = 0;  
dt = 100e-6;  
V0 = 3;  
Data = [];  
  
while(t < 100e-3)  
    dV1 = 100*V0 - 233.33*V1 + 100*V2;  
    dV2 = 100*V1 - 133.33*V2;  
  
    V1 = V1 + dV1*dt;  
    V2 = V2 + dV2*dt;  
  
    t = t + dt;  
    Data = [Data ; V1, V2];  
end  
t = [1:length(Data)]' * dt;  
plot(t*1000,Data);  
xlabel('Time (ms)');  
ylabel('Volts')
```



3) Using CircuitLab, determine the voltages for $0 < t < 100\text{ms}$



Lab (part 2)

4) Build this circuit with your lab kit

Apply a 3V square wave to V0

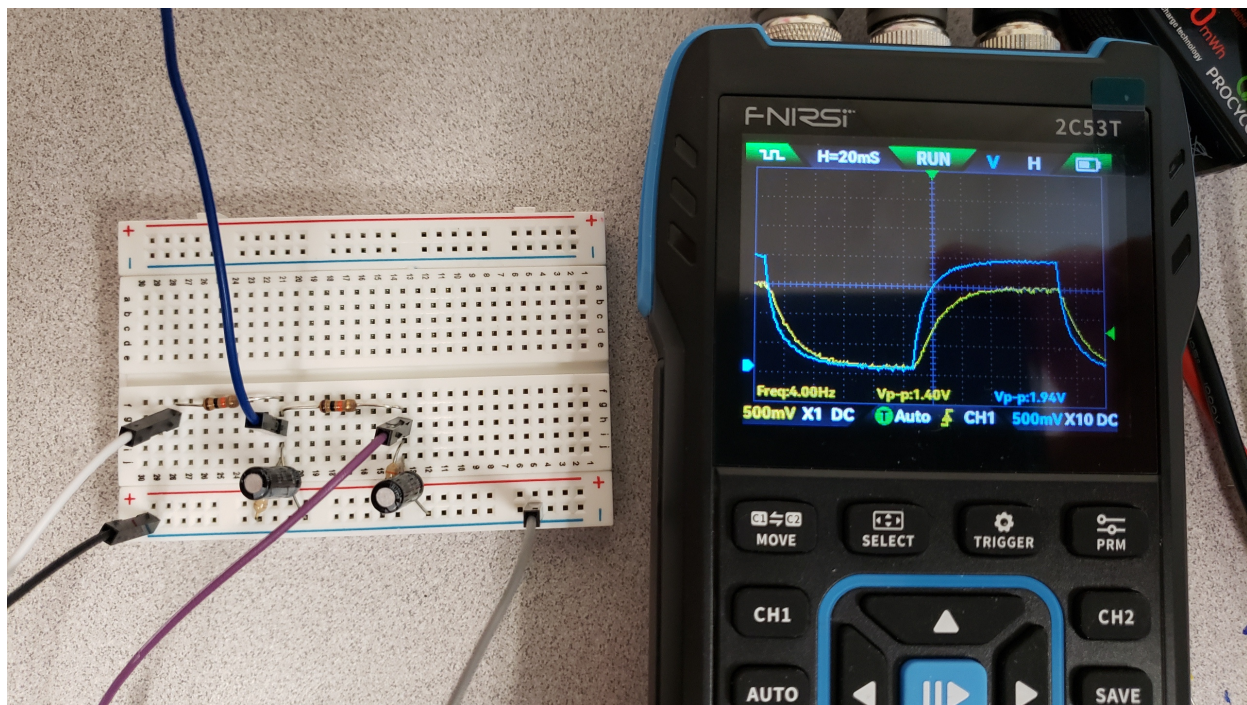
- Use your FNRSI meter as a signal generator
- Output a 3V square wave with frequency of 5Hz

Measure the voltage at V1 and V2

- Use your FNRSI meter as an oscilloscope
- Include a photo of your FNRSI meter's display in your homework set

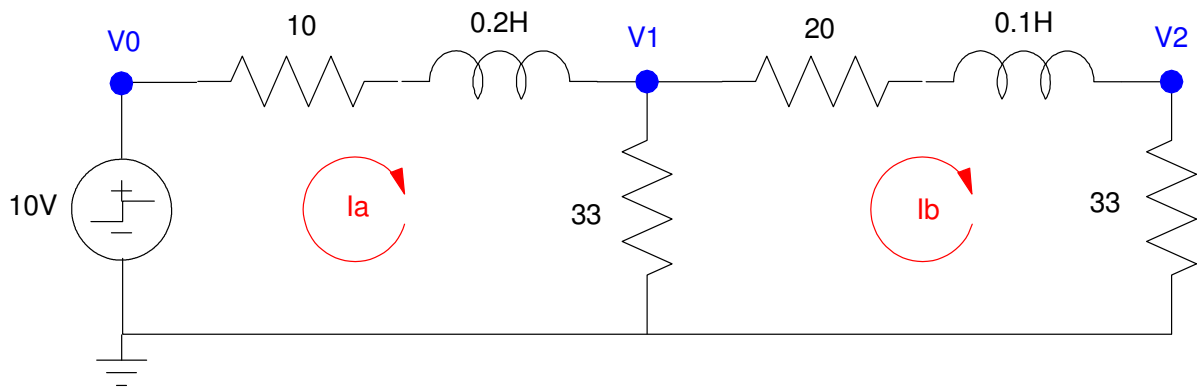
Note whether the signal at V2 matches your results from problems 5-7

- Waveforms are fairly similar
- Similar steady-state value
 - $V_1 = 2.00\text{V}$ (measured)
 - $V_2 = 1.50\text{V}$ (measured)
- Similar rise time (about 60ms)



Inductors

5) Find the differential equation which describes the following circuit



Problem 5-7

Current loop equation for I_a

$$-10 + 10I_a + 0.2I'_a + 33(I_a - I_b) = 0$$

Current loop equation for I_b

$$33(I_b - I_a) + 20I_b + 0.1I'_b + 33I_b = 0$$

Group terms

$$0.2I'_a + 43I_a - 33I_b = 10$$

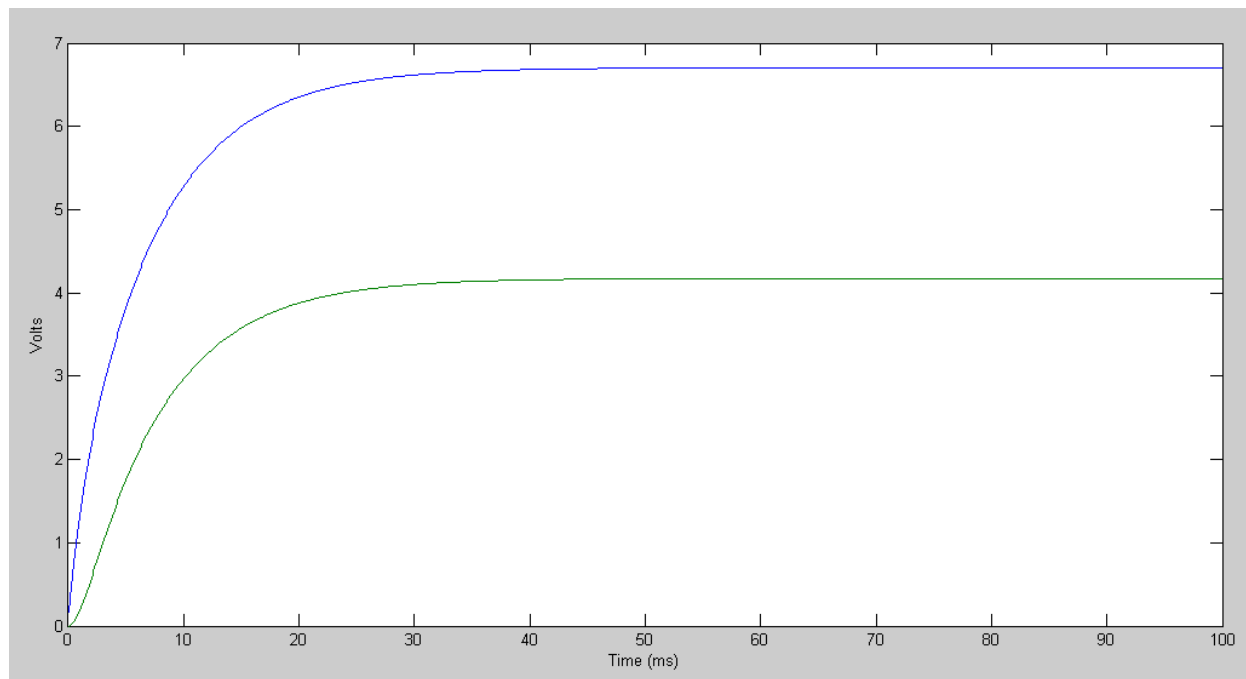
$$0.1I'_b - 33I_a + 86I_b = 0$$

Solve for the derivatives

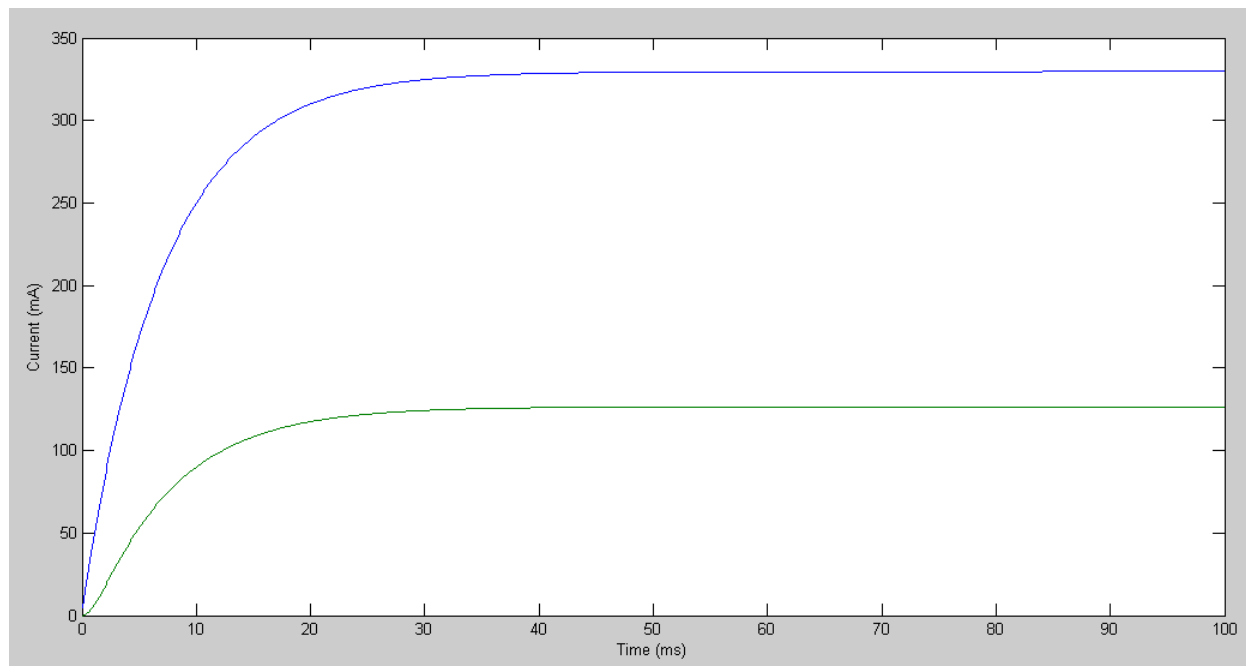
$$I'_a = -215I_a + 165I_b + 50$$

$$I'_b = 330I_a - 860I_b$$

6) Using Matlab and numerical integration, determine the currents $\{I_a, I_b\}$ for $0 < t < 100\text{ms}$



Voltages V1 (blue) and V2 (green)



Current Ia (blue) and Ib (green)

Matlab Code

```
Ia = 0;
Ib = 0;
t = 0;
dt = 100e-6;
Data = [];

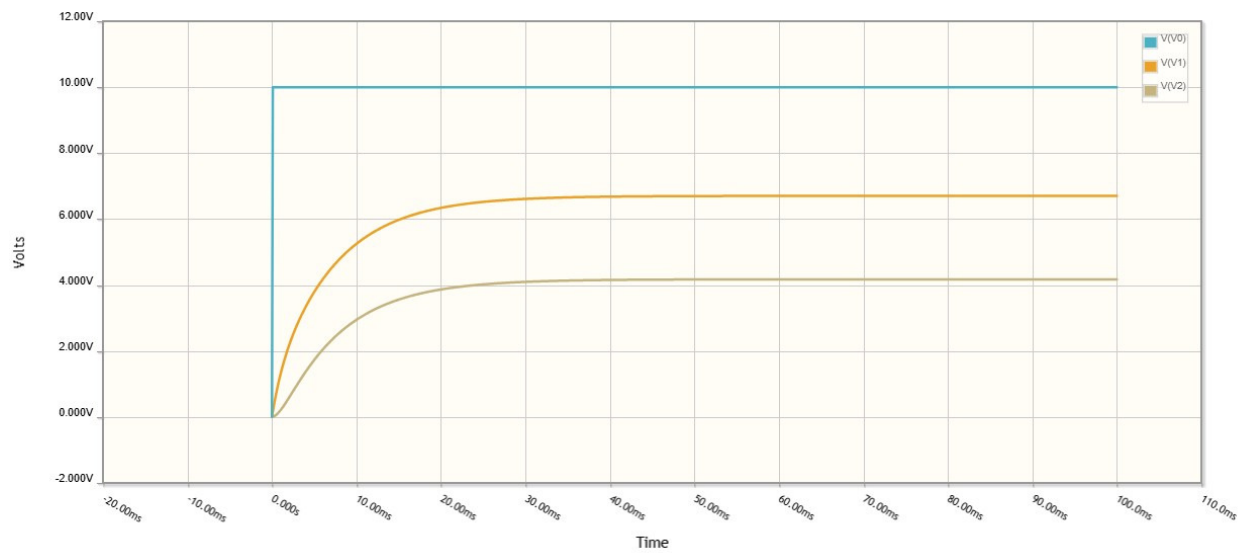
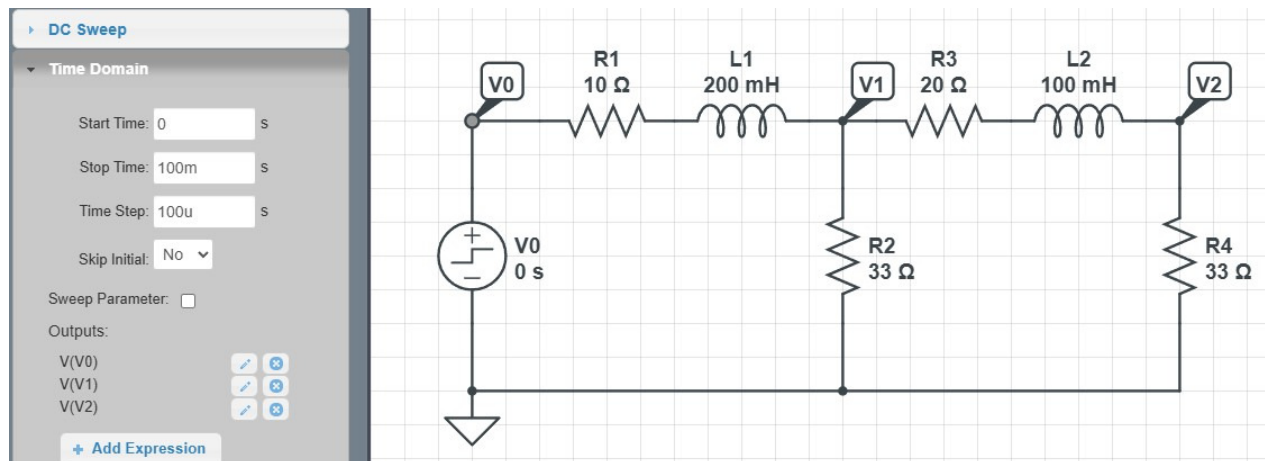
while(t < 100e-3)
    dIa = -215*Ia + 165*Ib + 50;
    dIb = 330*Ia - 660*Ib;

    Ia = Ia + dIa*dt;
    Ib = Ib + dIb*dt;

    V1 = 33*(Ia - Ib);
    V2 = 33*Ib;

    t = t + dt;
    Data = [Data ; Ia, Ib, V1, V2];
end
t = [1:length(Data)]' * dt;
plot(t*1000,Data(:,1:2)*1000);
xlabel('Time (ms)');
ylabel('mA')
```

7) Using CircuitLab, determine the currents $\{I_a, I_b\}$ for $0 < t < 100\text{ms}$



This matches up with Matlab

- Settling time is about 30ms for both
- V1 ends up at 6.70V for both
- V2 ends up at 4.17V for both

Complex Numbers

8) Find y

$$Y = \left(\frac{(2+3j)(6-5j)}{(8+2j)} \right)$$

Using Matlab

```
>> Y = (2+3j)*(6-5j) / (8+2j)
```

```
Y =
```

```
3.4118 + 0.1471i
```

Using the Free42 app

```
2  
enter  
3  
complex  
6  
enter  
-5  
complex  
*  
8  
enter  
2  
complex  
/  

```



9) Find the partial fraction expansion

$$\left(\frac{10(x+2)(x+3j)(x-3j)}{x(x+1)(x+2+j)(x+2-j)} \right) = \left(\frac{a}{x} \right) + \left(\frac{b}{x+1} \right) + \left(\frac{c}{x+2+j} \right) + \left(\frac{d}{x+2-j} \right)$$

Using Matlab and the cover-up method

$$a = \left(\frac{10(x+2)(x+3j)(x-3j)}{(x+1)(x+2+j)(x+2-j)} \right)_{x \rightarrow 0}$$

```
>> x = 0;
>> a = 10 * (x+2) * (x+3*j) * (x-3*j) / ( (x+1) * (x+2+j) * (x+2-j) )

a =      36
```

$$b = \left(\frac{10(x+2)(x+3j)(x-3j)}{x(x+2+j)(x+2-j)} \right)_{x \rightarrow -1}$$

```
>> x = -1;
>> b = 10 * (x+2) * (x+3*j) * (x-3*j) / ( (x) * (x+2+j) * (x+2-j) )

b =     -50
```

$$c = \left(\frac{10(x+2)(x+3j)(x-3j)}{x(x+1)(x+2-j)} \right)_{x \rightarrow -2-j}$$

```
>> x = -2-j;
>> c = 10 * (x+2) * (x+3*j) * (x-3*j) / ( (x) * (x+1) * (x+2-j) )

c =  12.0000 -16.0000i
```

$$d = \left(\frac{10(x+2)(x+3j)(x-3j)}{x(x+1)(x+2+j)} \right)_{x \rightarrow -2+j}$$

```
>> x = -2+j;
>> d = 10 * (x+2) * (x+3*j) * (x-3*j) / ( (x) * (x+1) * (x+2+j) )

d =  12.0000 +16.0000i
```

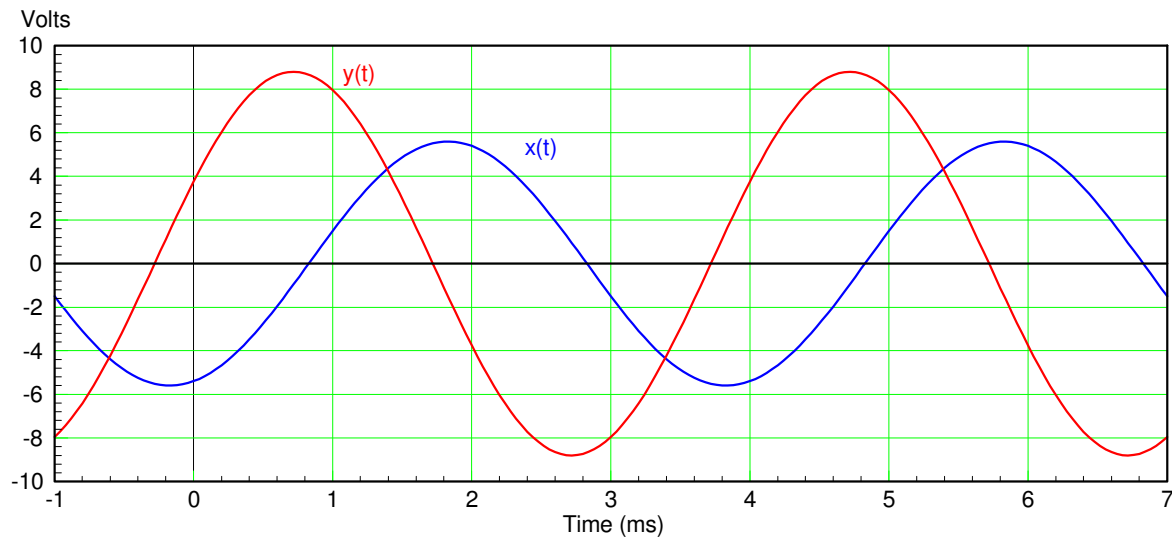
```
>>
```

$$\left(\frac{10(x+2)(x+3j)(x-3j)}{x(x+1)(x+2+j)(x+2-j)} \right) = \left(\frac{36}{x} \right) + \left(\frac{-50}{x+1} \right) + \left(\frac{12-16j}{x+2+j} \right) + \left(\frac{12+16j}{x+2-j} \right)$$

Phasors

10) Express $x(t)$ and $y(t)$ in phasor form

- If $x(t)$ is voltage and $y(t)$ is current in mA, what is the phasor impedance?



The period is about 4ms

$$\omega = \frac{2\pi}{T} = 1570.8$$

X:

- The peak is 5.8V (approx)
- The peak is delayed by 1.8 seconds

$$\theta_x = -\left(\frac{1.8ms}{4ms}\right) 360^\circ = -162^\circ$$

$$X = 5.8 \angle -162^\circ V$$

Y:

- The peak is 8.5mA
- The peak is delayed by 0.7ms

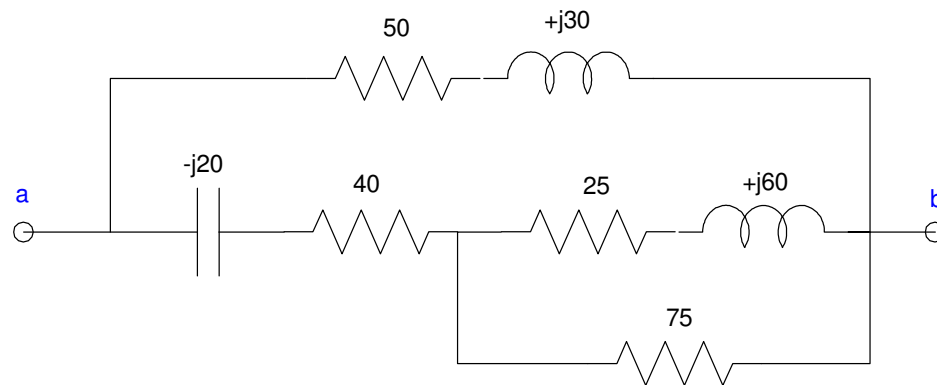
$$\theta_y = -\left(\frac{0.7ms}{4ms}\right) 360^\circ = -63^\circ$$

$$Y = 8.5 \angle -63^\circ mA$$

Impedance

$$Z = \left(\frac{V}{I}\right) = \left(\frac{5.8 \angle -162^\circ V}{8.5 \angle -63^\circ mA}\right) = 682.3 \angle -99^\circ$$

11) Find the impedance Z_{ab}



Problem 11

Using Matlab

```
>> z1 = 1 / (1/(25+60j) + 1/75)

z1 = 33.6397 +24.8162i

>> z2 = z1 + (40 - 20j)

z2 = 73.6397 + 4.8162i

>> z3 = 1 / (1/z2 + 1/(50+30j))

z3 = 31.6793 +10.8949i
```

Using the Free42 cell phone app

```
25
enter
60
complex

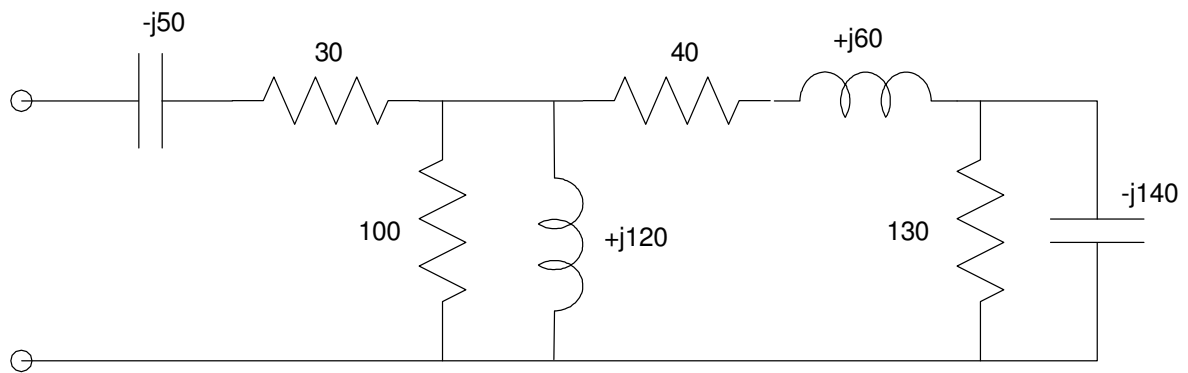
1/x
75
1/x
+
1/x

40
enter
-20
complex
+

1/x
50
enter
30
complex
1/x
+
1/x
```



12) Find the impedance Z_{ab}



Problem 12

Using Matlab

```
>> Z1 = 1 / (1/130 + 1/(-140j))

Z1 =    69.8082 -64.8219i

>> Z2 = Z1 + (40 + 60j)

Z2 =    1.0981e+002 -4.8219e+000i

>> Z3 = 1 / (1/100 + 1/120j + 1/Z2)

Z3 =    44.6688 +18.5660i

>> Zab = Z3 + (30 - 50j)

Zab =    74.6688 -31.4340i
```

Using the Free42 app on your cell phone

130	1/x
1/x	100
0	1/x
enter	+
-140	0
complex	enter
1/x	120
+	complex
1/x	1/x
	+
40	1/x
enter	
60	30
complex	enter
+	-50
	complex
	+

