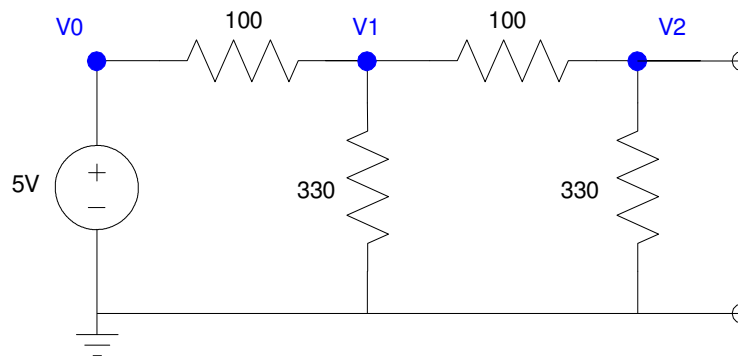


ECE 211: Homework #4

Thevenin, Superposition, Op-Amps.

Thevenin Equivalents

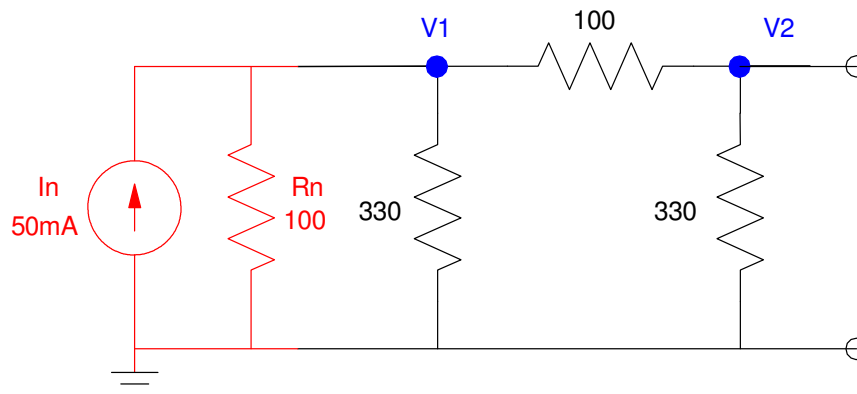
1) Find the Thevenin equivalent for the following circuit by transforming between Thevenin and Norton equivalents:



Convert (5V, 100Ohms) to its Norton equivalent

$$I_N = \left(\frac{5V}{100\Omega} \right) = 50mA$$

$$R_N = R_{th} = 100\Omega$$



Combine the 100 Ohm resistor and 330 Ohm resistor in parallel

$$R_N \Rightarrow \left(\frac{1}{100} + \frac{1}{330} \right)^{-1} = 76.7442\Omega$$

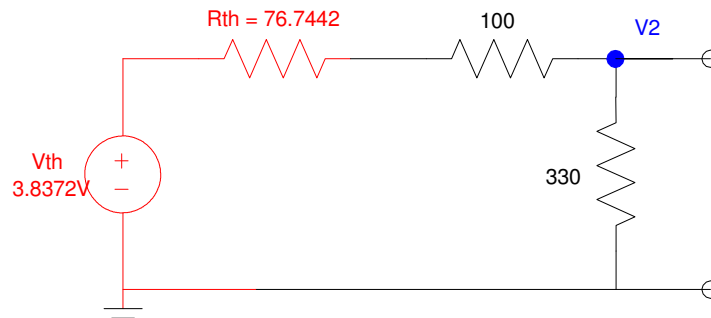
So now

$$I_N = 50mA \quad R_N = 76.7442\Omega$$

Convert to a Thevenin equivalent

$$V_{th} = I_N \cdot R_N = 3.8372V$$

$$R_{th} = R_N = 76.7442\Omega$$



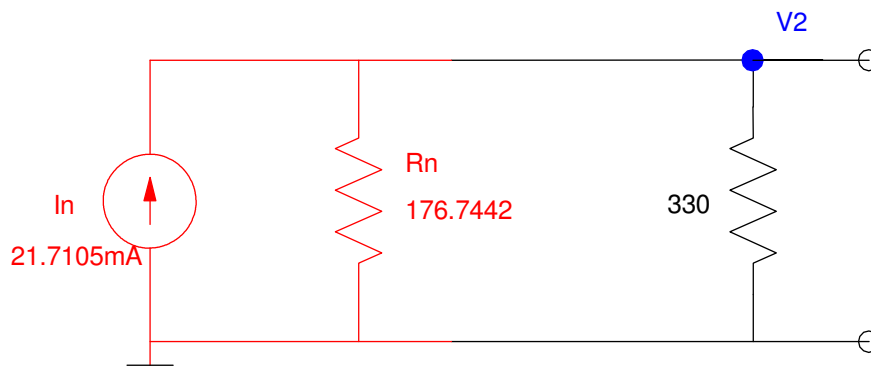
Add the 100 Ohms in series

$$R_{th} \Rightarrow 176.7442\Omega$$

Convert to a Norton equivalent

$$I_N = \left(\frac{V_{th}}{R_{th}} \right) = \left(\frac{3.8372V}{176.7442\Omega} \right) = 21.7105mA$$

$$R_N = R_{th} = 176.7442\Omega$$



Combine R_N with 330 Ohms in parallel

$$R_N \Rightarrow 176.7442\Omega || 330\Omega$$

$$R_N = 115.0987\Omega$$

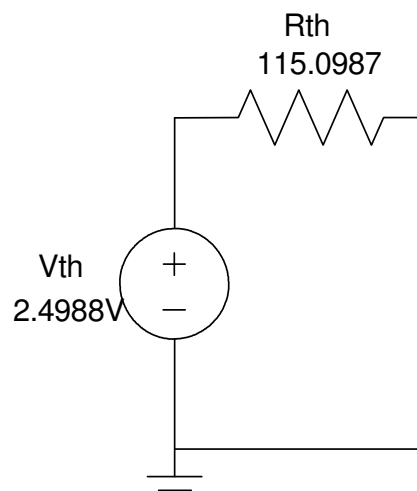
Convert back to a Thevenin equivalent

$$V_{th} = I_N R_N$$

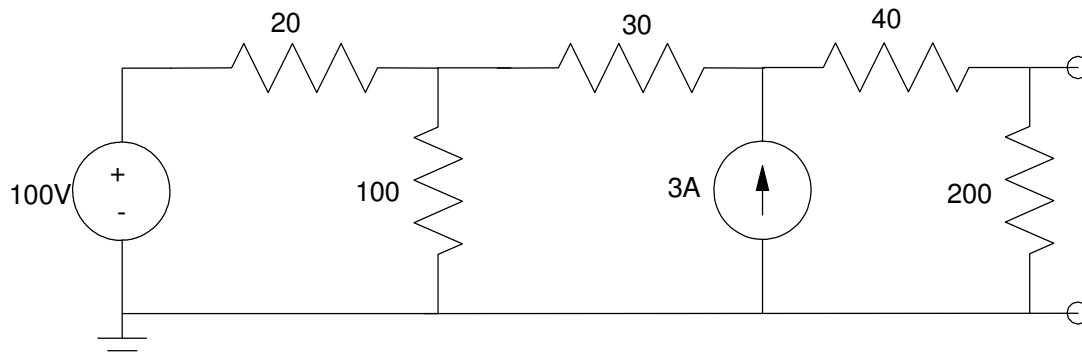
$$V_{th} = (21.7105mA)(115.0987\Omega)$$

$$V_{th} = 2.4988V$$

$$R_{th} = R_N = 115.0987\Omega$$



2) Find the Thevenin equivalent for the following circuit by transforming between Thevenin and Norton equivalents:



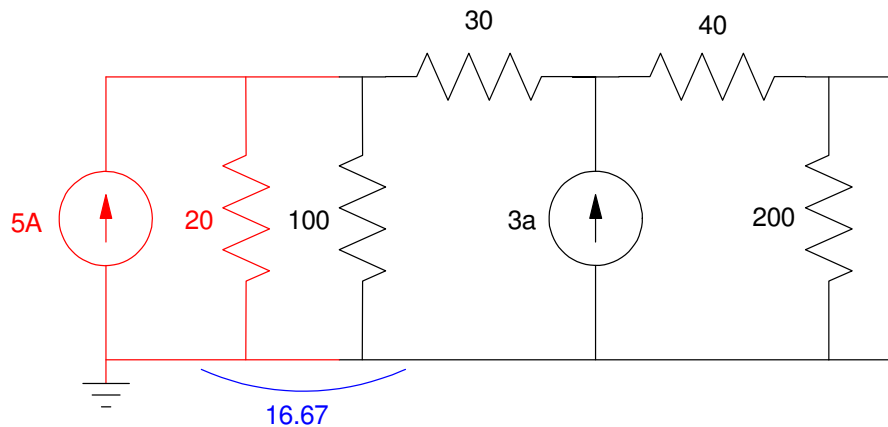
Convert (100V, 20 Ohms) to its Norton equivalent

$$I_N = \left(\frac{100V}{20\Omega} \right) = 5A$$

$$R_N = R_{th} = 20\Omega$$

Combine the two resistors that are in parallel

$$R_N \Rightarrow 20\Omega || 100\Omega = 16.667\Omega$$



Convert (5A, 16.667 Ohms) to its Thevenin equivalent

$$V_{th} = I_N \cdot R_N$$

$$V_{th} = (5A)(16.667\Omega) = 83.333V$$

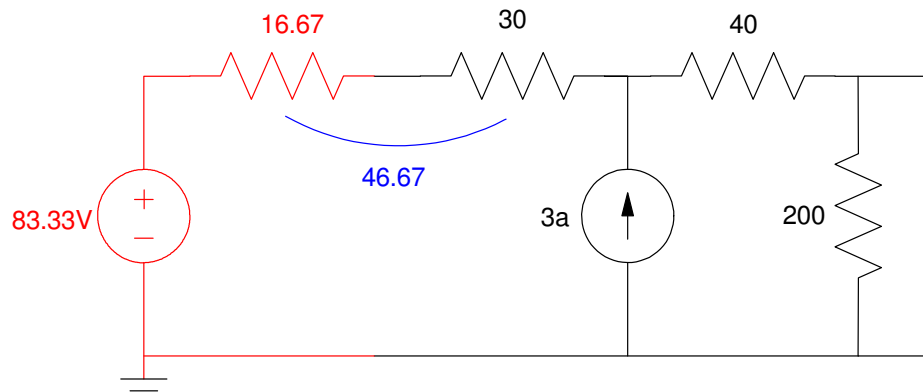
$$R_{th} = R_N = 16.667\Omega$$

Add the 30 Ohms in series

$$R_{th} \Rightarrow 16.667\Omega + 30\Omega$$

$$R_{th} = 46.667\Omega$$

$$V_{th} = 83.333V$$



Convert to Norton

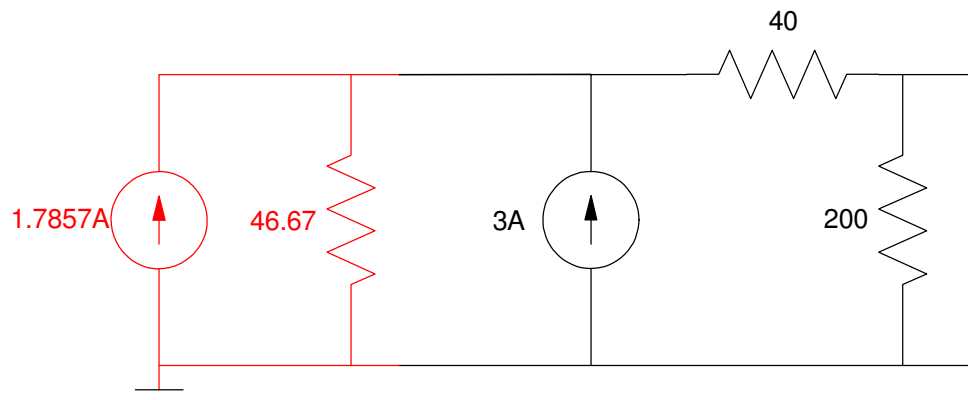
$$I_N = \left(\frac{83.333V}{46.667\Omega} \right) = 1.7857A$$

$$R_N = R_{th} = 46.667\Omega$$

Add 3A in parallel

$$I_N \Rightarrow 3A + 1.7857A = 4.7857A$$

$$R_N = 46.667\Omega$$

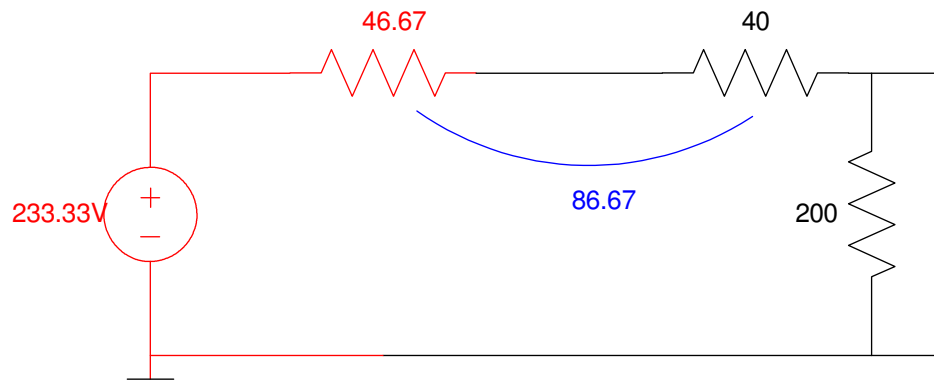


Convert to a Thevenin equivalent

$$V_{th} = (4.7857A)(46.667\Omega)$$

$$V_{th} = 223.333V$$

$$R_{th} = R_N = 46.667\Omega$$



Add 40 Ohms in series

$$V_{th} = 233.3333V$$

$$R_{th} \Rightarrow 40\Omega + 46.667\Omega = 86.667\Omega$$

Convert to Norton

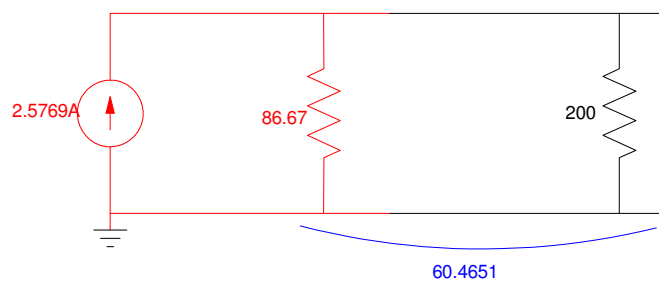
$$I_N = \left(\frac{233.333V}{86.6667\Omega} \right) = 2.5769A$$

$$R_N = R_{th} = 86.6667\Omega$$

Add 200 Ohms in parallel

$$R_N \Rightarrow 200\Omega || 86.666 = 60.4651\Omega$$

$$I_N = 2.5769A$$

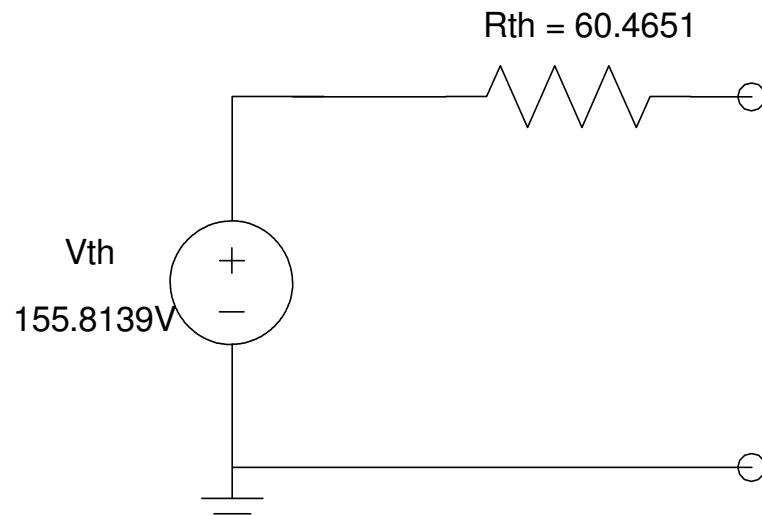


Convert to Thevenin

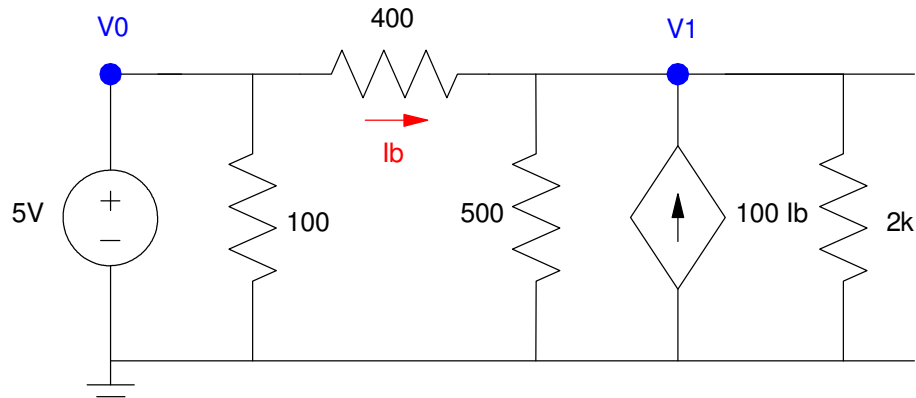
$$V_{th} = (2.5769A)(60.4561\Omega)$$

$$V_{th} = 155.8139V$$

$$R_{th} = R_N = 60.4651\Omega$$



3) Find the Thevenin equivalent for the following circuit:



Vth: Find the open-circuit voltage

Write the voltage node equation at V1

$$\left(\frac{V_1 - 5}{400}\right) + \left(\frac{V_1}{500}\right) + 100\left(\frac{V_1 - 5}{400}\right) + \left(\frac{V_1}{2k}\right) = 0$$

$$V_1 = 4.9510V$$

Ish: Short V1 to ground. Find the current

$$I_{sh} = \left(\frac{5V}{400\Omega}\right) + 100\left(\frac{5V}{400\Omega}\right)$$

$$I_{sh} = 1.2625A$$

Rth (option 1)

$$R_{th} = \left(\frac{V_{open}}{I_{short}}\right)$$

$$R_{th} = \left(\frac{4.9510V}{1.2625A}\right) = 3.9216\Omega$$

Rth (option 2)

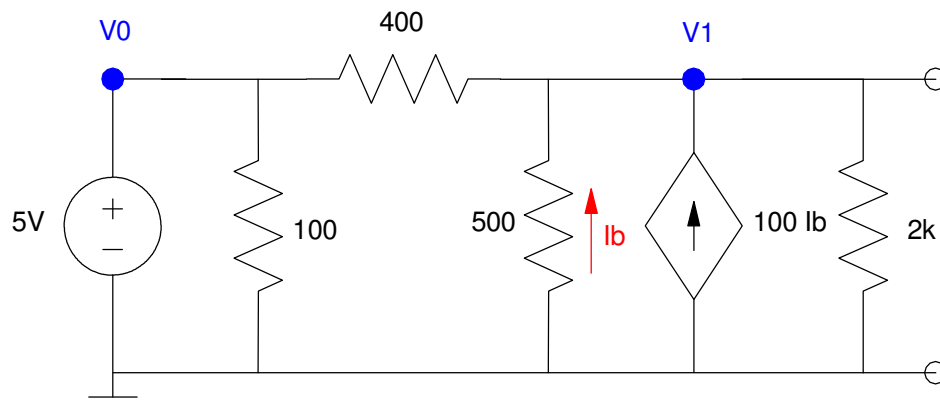
- Turn off sources (5V becomes 0V)
- Apply a 1V test voltage to V1
- Compute the current draw

$$I_{in} = \left(\frac{1V}{400\Omega}\right) + \left(\frac{1V}{500\Omega}\right) + 100\left(\frac{1V}{400\Omega}\right) + \left(\frac{1V}{2k\Omega}\right)$$

$$I_{in} = 255.0mA$$

$$R_{th} = \left(\frac{1V}{255.0mA}\right) = 3.9216\Omega$$

4) Find the Thevenin equivalent for the following circuit:



Vth: Find the open-circuit voltage

Write the node equation at V1

$$\left(\frac{V_1 - 5}{400}\right) + \left(\frac{V_1}{500}\right) + 100\left(\frac{V_1}{500}\right) + \left(\frac{V_1}{2000}\right) = 0$$

$$V_1 = 61.0mV$$

Ish: Short V1 to ground. Find the current draw

$$I_{sh} = \left(\frac{5V}{400\Omega}\right)$$

$$I_{sh} = 12.5mA$$

Rth: (Option 1)

$$R_{th} = \left(\frac{V_{open}}{I_{short}}\right) = \left(\frac{61.0mV}{12.5mA}\right)$$

$$R_{th} = 4.8780\Omega$$

Rth (option 2)

- Turn off sources (5V becomes 0V)
- Apply a 1V test voltage at V1
- Compute the current draw

$$I = \left(\frac{1V}{400\Omega}\right) + \left(\frac{1V}{500\Omega}\right) + 100\left(\frac{1V}{500\Omega}\right) + \left(\frac{1V}{2k}\right)$$

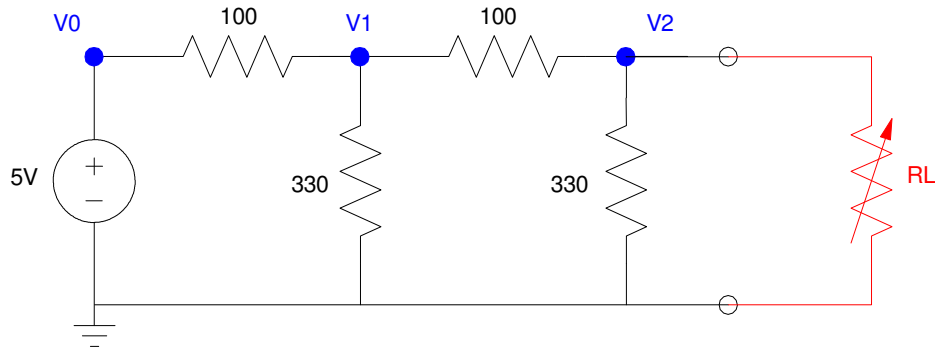
$$I = 205.0mA$$

$$R_{th} = \left(\frac{1V}{205.0mA}\right) = 4.8780\Omega$$

Maximum Power Transfer

5) Determine R_L so that the maximum power is delivered to the load (R_L)

- What is the voltage at V_2 for max power transfer?
- What is the maximum power you can deliver to R_L ?



Find the Thevenin equivalent

$V(\text{open})$: Use voltage division to find V_1

$$R_1 = 330 \parallel (100 + 330)$$

$$R_1 = 186.7105 \Omega$$

$$V_1 = \left(\frac{R_1}{R_1 + 100} \right) V_0$$

$$V_1 = 3.2561 V$$

Use voltage division to find V_2

$$V_2 = \left(\frac{330}{330 + 100} \right) V_1$$

$$V_2 = 2.4989 V$$

R_{th} : Turn off the sources (5V becomes 0V). Find the resistance looking in

```
>> R1 = 1 / (1/100 + 1/330)
R1 = 76.7442
```

```
>> R2 = R1 + 100
R2 = 176.7442
```

```
>> R3 = 1 / (1/R2 + 1/330)
R3 = 115.0987
```

So, the Thevenin equivalent is

- $V_{th} = 2.4989 V$
- $R_{th} = 115.0987 \text{ Ohms}$

Max power is when $R_L = R_{th}$

- $R_L = 115.0987 \text{ Ohms}$

The max power to the load is then

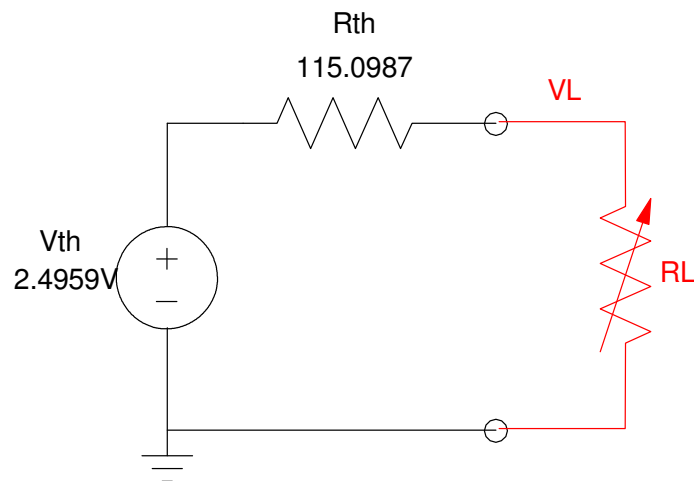
$$V_L = \frac{1}{2} V_{th}$$

$$V_L = 1.2495V$$

$$P_L = \left(\frac{V_L^2}{R_L} \right)$$

$$P_L = \left(\frac{(1.2495V)^2}{115.0987\Omega} \right)$$

$$P_L = 13.56mW$$

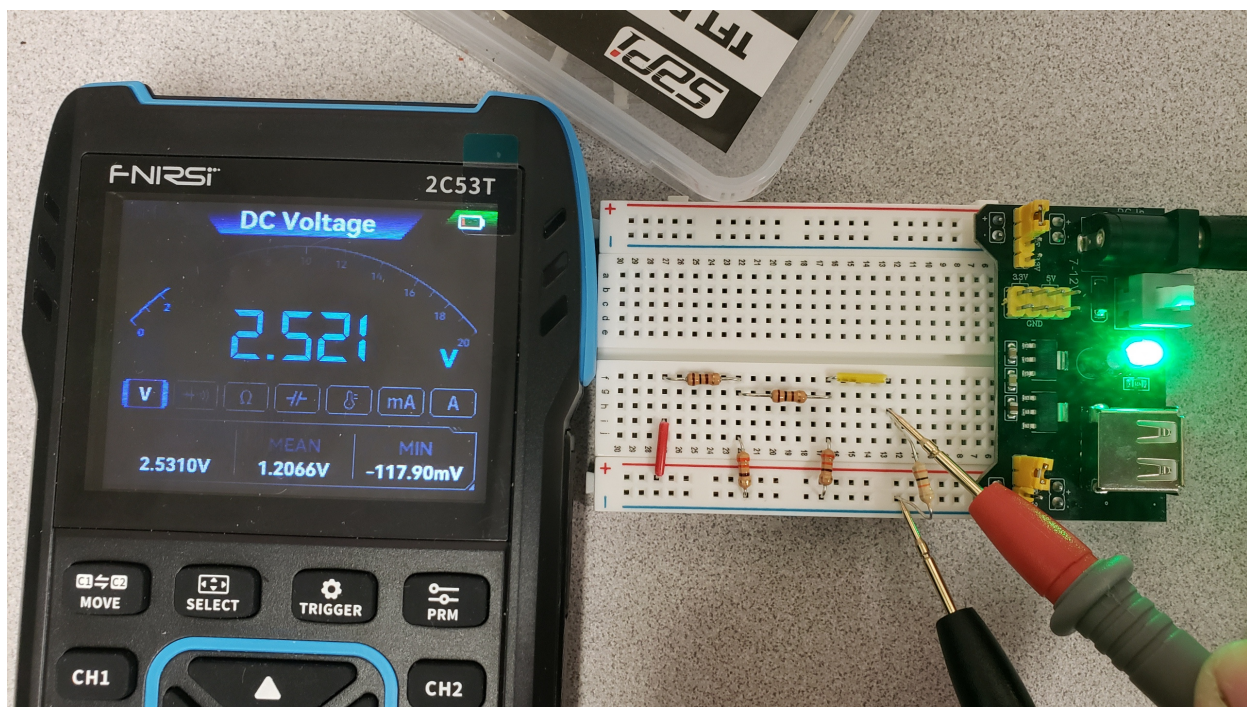


Lab

6) For the circuit of problem #5, measure the voltage and current to the load (R_L) as R_L varies from 0 Ohms to infinite Ohms.

- Calculate the power to the load (note: max power should match your answer in problem #5)

R(load)	V(load)	I(mA)	Power (mW)
10.0000	0.2017	20.1700	4.0683
33.0000	0.5694	17.2545	9.8247
100.0000	1.1833	11.8330	14.0020
330.0000	1.8798	5.6964	10.7080
1,000.0000	2.2680	2.2680	5.1438
3,300.0000	2.4450	0.7409	1.8115
10,000.0000	2.5020	0.2502	0.6260
30,000.0000	2.5210	0.0840	0.2118
100,000.0000	2.5280	0.0253	0.0639
infinite	2.5300	0.0000	0.0000



Measuring the voltage at the load when $R_L = 30k$ (orange - black - orange)

7) Plot the load line: V_L vs. I_L . From the load line, compute the Thevenin equivalent for this circuit

- This should match your calculations from problem #1

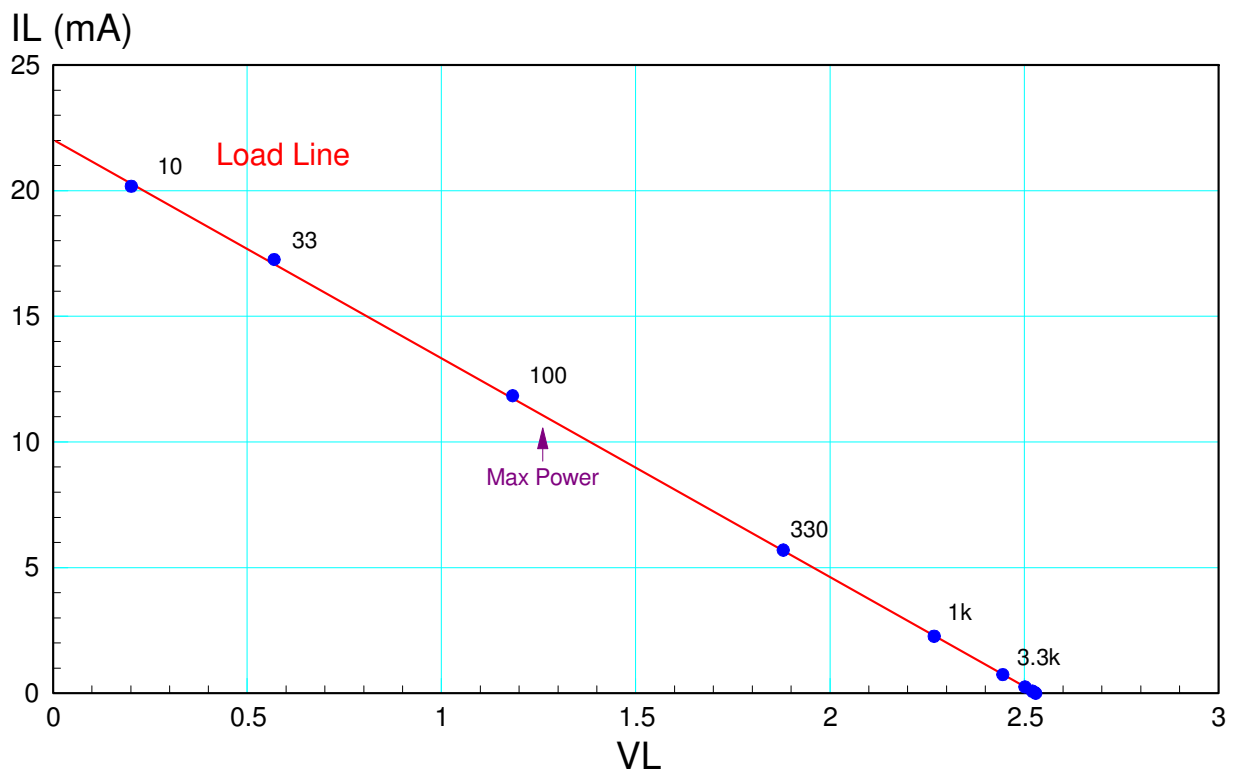
From the load line:

- $V(\text{open}) = V_{th} = 2.5300\text{V}$
- $I(\text{short}) = 22\text{mA}$

$$R_{th} = \left(\frac{V_{open}}{I_{short}} \right) = 115\Omega$$

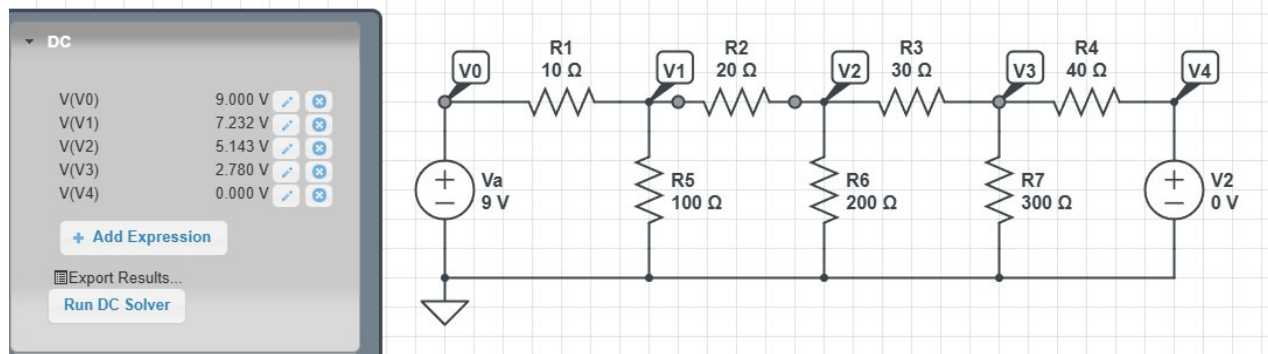
Calculations (problem 5)

- $V_{th} = 2.4989\text{ V}$
- $R_{th} = 115.0987\text{ Ohms}$
-

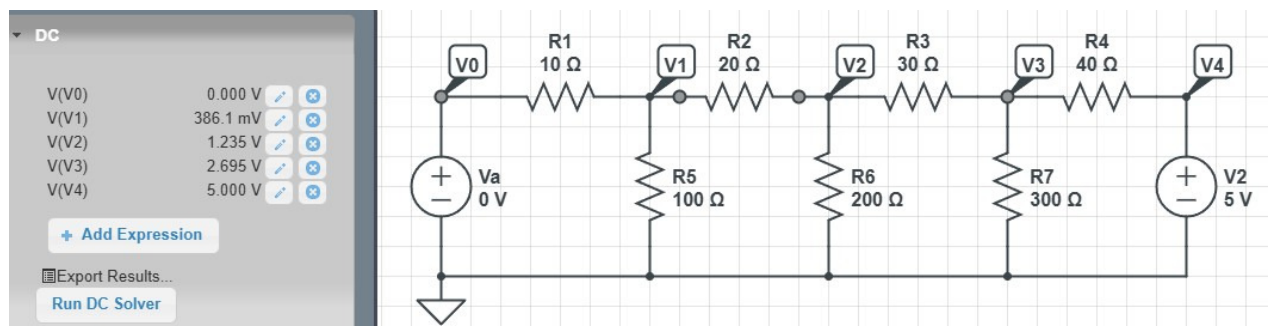


Superposition

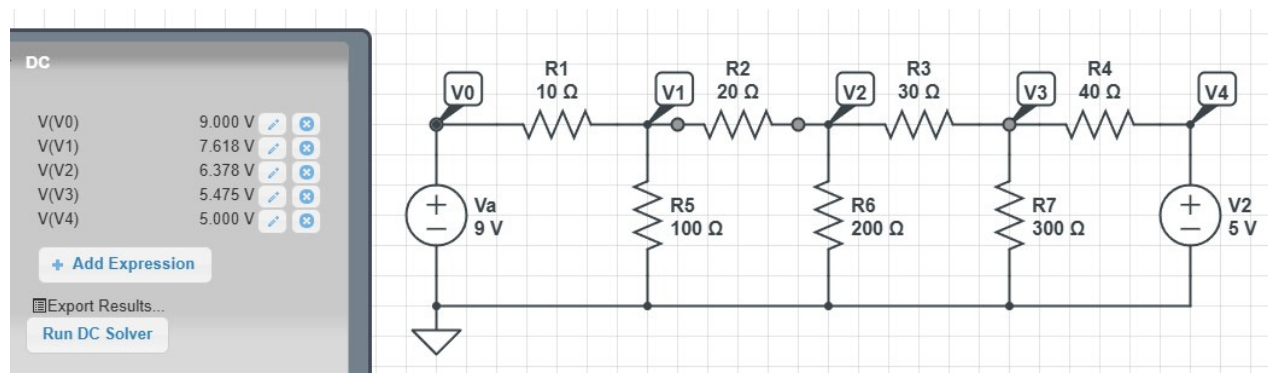
8) Use Circuitlab to determine the voltage at Y assuming $V_a = 9V$, $V_b = 0V$.



9) Use Circuitlab to determine the voltage at Y assuming $V_a = 0V$, $V_b = 5V$.



10) Use Circuitlab to determine the voltage at Y assuming $V_1 = 9V$, $V_b = 5V$



Checking:

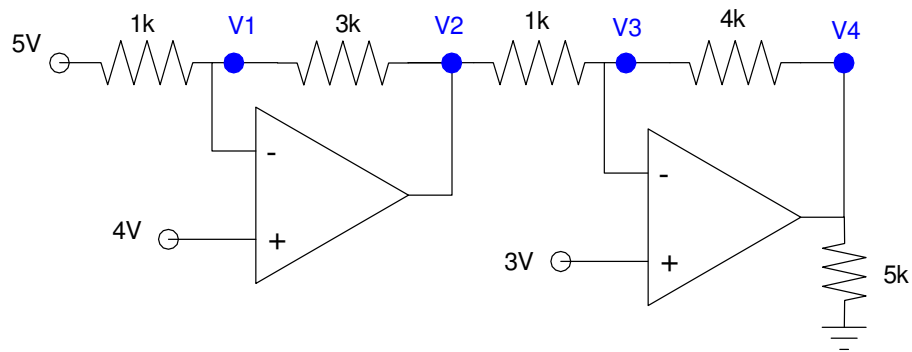
- Problem 8 + Problem 9 = Problem 10
- Superposition holds

	Va	V1	V2	V3	Vb
Va = 9V Vb = 0V (problem 8)	9.0000	7.2320	5.1430	2.7800	0.0000
Va = 0V Vb = 5V (problem 9)	0.0000	0.3861	1.2350	2.6950	5.0000
Sum p8 + p9	9.0000	7.6181	6.3780	5.4750	5.0000
Va = 9V Vb = 5V (problem 10)	9.0000	7.6180	6.3780	5.4750	5.0000

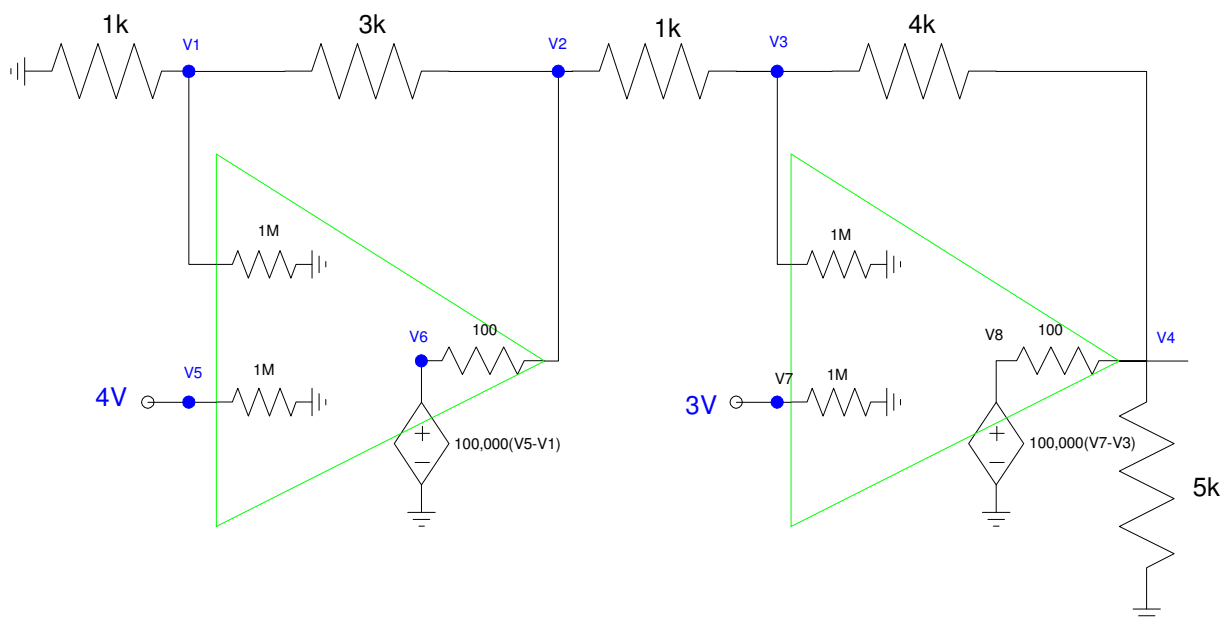
Op Amps

12) Write the voltage node equations for the following op-amp circuit. Assume non-ideal op-amps

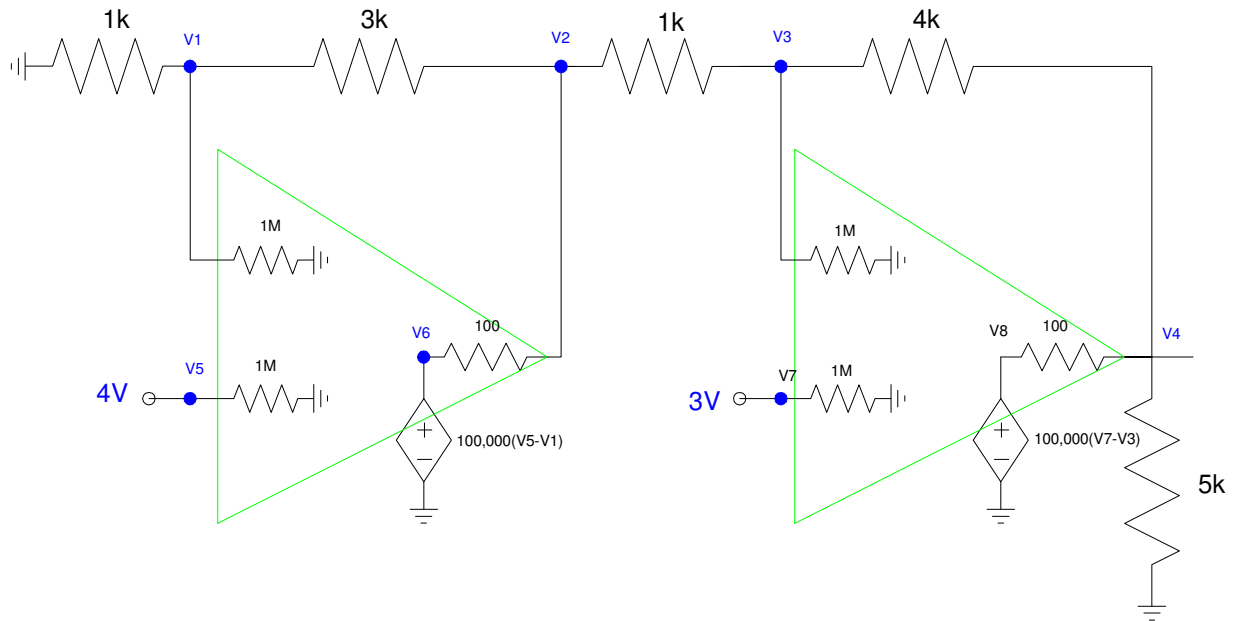
- $R_{in} = 1\text{M Ohms}$
- $R_{out} = 100\text{ Ohms}$
- $\text{Gain} = 100,000$



Replace the op-amps with their model



Now write the node equations (eight equations for eight unknowns)



$$V_5 = 4V$$

$$V_7 = 3V$$

$$V_6 = 100,000(V_5 - V_1)$$

$$V_8 = 100,000(V_7 - V_3)$$

$$\left(\frac{V_1}{1k}\right) + \left(\frac{V_1}{1M}\right) + \left(\frac{V_1 - V_2}{3k}\right) = 0$$

$$\left(\frac{V_2 - V_1}{3k}\right) + \left(\frac{V_2 - V_6}{100}\right) + \left(\frac{V_2 - V_3}{1k}\right) = 0$$

$$\left(\frac{V_3 - V_2}{1k}\right) + \left(\frac{V_3}{1M}\right) + \left(\frac{V_3 - V_4}{4k}\right) = 0$$

$$\left(\frac{V_4 - V_3}{4k}\right) + \left(\frac{V_4 - V_8}{100}\right) + \left(\frac{V_4}{5k}\right) = 0$$

note:

- This is way too painful. We need a better tool
- Ideal op-amp (next week's homework)