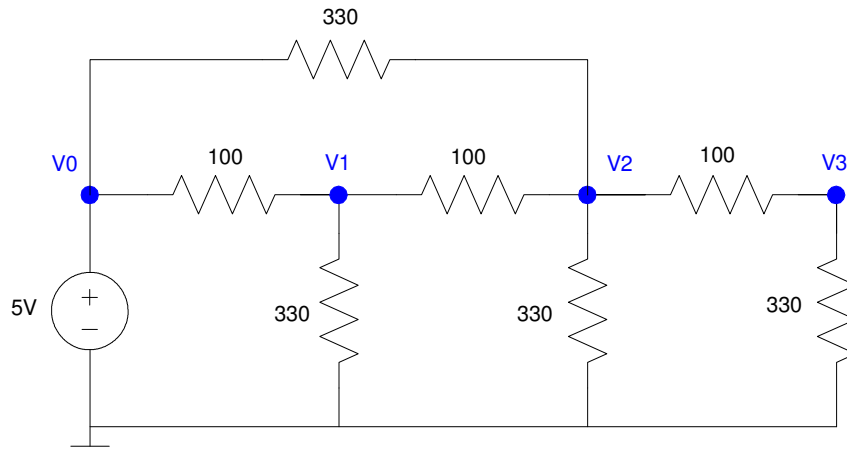


ECE 211: Homework #3

Voltage Nodes, Current Loops.

Voltage Nodes

1) Write the voltage node equations for the following circuit



There are four voltage nodes.

- 4 equations are needed to solve for 4 unknowns

Start with the easy one

$$V_0 - 0 = 5V$$

We still need three more equations. Write the voltage node equation at node V1:

$$\left(\frac{V_1 - V_0}{100} \right) + \left(\frac{V_1}{330} \right) + \left(\frac{V_1 - V_2}{100} \right) = 0$$

Node V2

$$\left(\frac{V_2 - V_0}{330} \right) + \left(\frac{V_2 - V_1}{100} \right) - \left(\frac{V_2}{330} \right) + \left(\frac{V_2 - V_3}{100} \right) = 0$$

Node V3

$$\left(\frac{V_3 - V_2}{100} \right) + \left(\frac{V_3}{330} \right) = 0$$

That gives four equations for four unknowns.

Comment: There's a pattern

- At node V1, all the V1 terms are positive, all other terms are negative
- At node V2, all the V2 terms are positive, all the other terms are negative
- Units match up. Volts - Volts = Volts (eq 1). Amps + Amps + Amps = 0 (eq 2).

2) Solve for {V1, V2, V3}

Start with the four equations

$$V_0 = 5$$

$$\left(\frac{V_1 - V_0}{100}\right) + \left(\frac{V_1}{330}\right) + \left(\frac{V_1 - V_2}{100}\right) = 0$$

$$\left(\frac{V_2 - V_0}{330}\right) + \left(\frac{V_2 - V_1}{100}\right) + \left(\frac{V_2}{330}\right) + \left(\frac{V_2 - V_3}{100}\right) = 0$$

$$\left(\frac{V_3 - V_2}{100}\right) + \left(\frac{V_3}{330}\right) = 0$$

Group terms

$$V_0 = 5$$

$$-\left(\frac{1}{100}\right)V_0 + \left(\frac{1}{100} + \frac{1}{330} + \frac{1}{100}\right)V_1 - \left(\frac{1}{100}\right)V_2 = 0$$

$$-\left(\frac{1}{330}\right)V_0 - \left(\frac{1}{100}\right)V_1 + \left(\frac{1}{330} + \frac{1}{100} + \frac{1}{330} + \frac{1}{100}\right)V_2 - \left(\frac{1}{100}\right)V_3 = 0$$

$$-\left(\frac{1}{100}\right)V_2 + \left(\frac{1}{100} + \frac{1}{330}\right)V_3 = 0$$

Place in matrix form (some of the terms simplify)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{100}\right) & \left(\frac{2}{100} + \frac{1}{330}\right) & \left(\frac{-1}{100}\right) & 0 \\ \left(\frac{-1}{330}\right) & \left(\frac{-1}{100}\right) & \left(\frac{2}{330} + \frac{2}{100}\right) & \left(\frac{-1}{100}\right) \\ 0 & 0 & \left(\frac{-1}{100}\right) & \left(\frac{1}{100} + \frac{1}{330}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> a1 = [1, 0, 0, 0];
>> a2 = [-1/100, 2/100+1/330, -1/100, 0];
>> a3 = [-1/330, -1/100, 2/330+2/100, -1/100];
>> a4 = [0, 0, -1/100, 1/100+1/330];
>> A = [a1;a2;a3;a4]
```

```
1.0000    0    0    0
-0.0100    0.0230 -0.0100    0
-0.0030 -0.0100    0.0261 -0.0100
0    0 -0.0100    0.0130
```

```
>> B = [5;0;0;0]
```

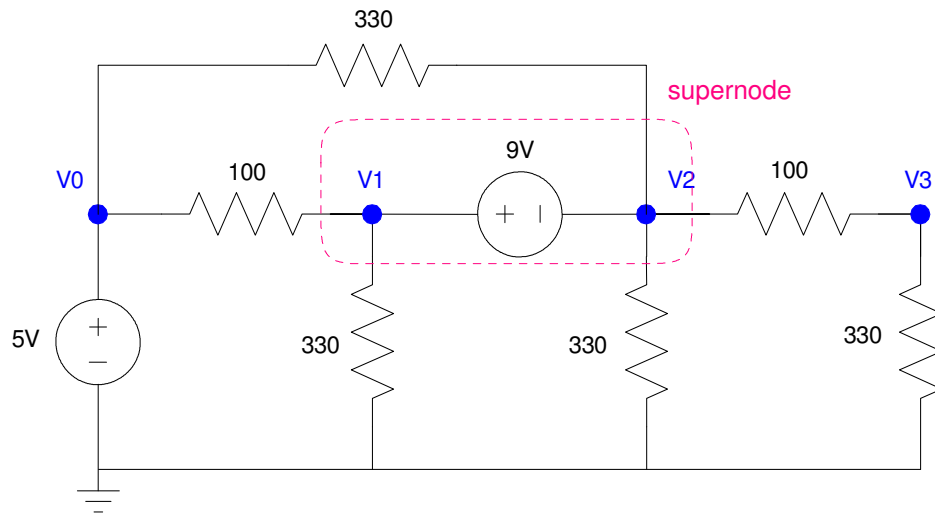
```
5  
0  
0  
0
```

```
>> V = inv(A)*B
```

```
V0    5.0000  
V1    3.3107  
V2    2.6247  
V3    2.0143
```

3) Write the voltage node equations for the following circuit

- you don't need to solve



This circuit has four voltage nodes (plus ground)

- four equations are needed to solve for four unknowns

Start with the easy ones: the voltage sources

$$V_0 = 5$$

$$V_1 - V_2 = 9$$

Two more equations are needed. Write the voltage node equation at V3

$$\left(\frac{V_3 - V_2}{100} \right) + \left(\frac{V_3}{330} \right) = 0$$

One more equation is needed. Write the voltage node equation for the supernode (other supernodes are also valid)

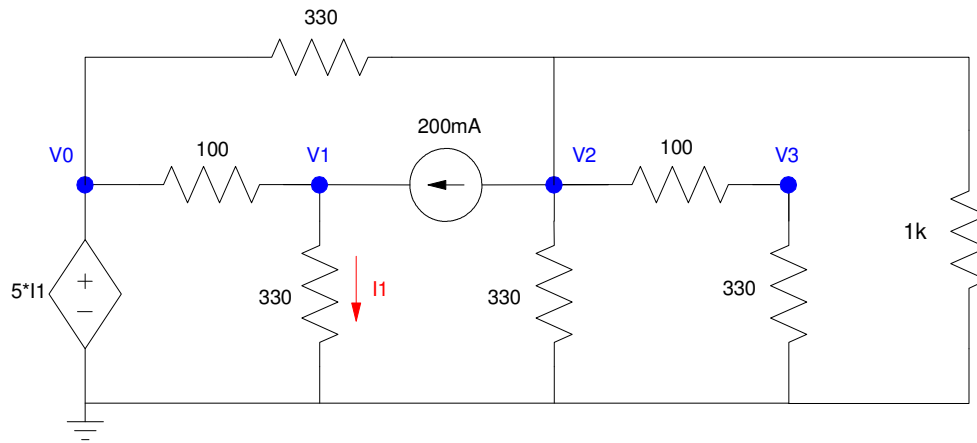
$$\left(\frac{V_1 - V_0}{100} \right) + \left(\frac{V_1}{330} \right) + \left(\frac{V_2}{330} \right) + \left(\frac{V_2 - V_3}{100} \right) + \left(\frac{V_2 - V_1}{330} \right) = 0$$

That gives four equations and four unknowns. At this point, you could solve for the voltages

- same procedure as the last problem

4) Write the voltage node equations for the following circuit

- you don't need to solve



This circuit has four voltage nodes (plus ground) and one dependent source

- Five equations are needed to solve for five unknowns

Start with the easy ones

$$I_1 = \left(\frac{V_1}{330} \right)$$

$$V_0 = 5I_1$$

Write the voltage node equation at V1

$$\left(\frac{V_1 - V_0}{100} \right) + I_1 - 200mA = 0$$

Voltage node equation at V2

$$\left(\frac{V_2 - V_0}{330} \right) + 200mA + \left(\frac{V_2}{330} \right) + \left(\frac{V_2 - V_3}{100} \right) + \left(\frac{V_2}{1k} \right) = 0$$

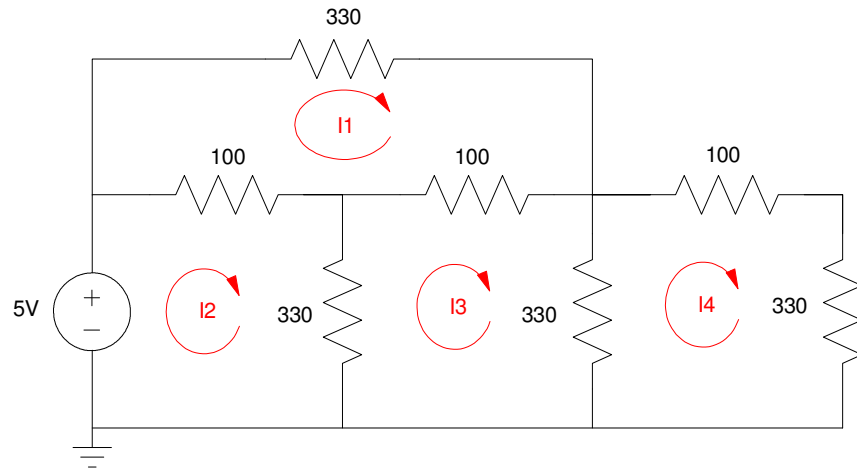
Voltage node equation at V3

$$\left(\frac{V_3 - V_2}{100} \right) + \left(\frac{V_3}{330} \right) = 0$$

That gives five equations for five unknowns

Current Loops

5) Write the current loop equations for the following circuit



There are four loops. Four equations are needed for four unknowns

Loop I1

$$330I_1 + 100(I_1 - I_3) + 100(I_1 - I_2) = 0$$

Loop I2

$$-5 + 100(I_2 - I_1) + 330(I_2 - I_3) = 0$$

Loop I3

$$330(I_3 - I_2) + 100(I_3 - I_1) + 330(I_3 - I_4) = 0$$

Loop I4

$$330(I_4 - I_3) + 100(I_4) + 330(I_4) = 0$$

That gives four equations for four unknowns

- Other sets of equations are valid as well

Comment - There's a pattern:

- Around loop I1, all I1 terms are positive. All other terms are negative.
- Around loop I2, all I2 terms are positive. All other terms are negative.
- Units match up
 - Volts + Volts + Volts = 0 (1st equation)
 - Volts + Volts + Volts = 0 (2nd equation)

6) Solve for $\{I_1, I_2, I_3, I_4\}$

Start with four equations for four unknowns (problem #5)

$$330I_1 + 100(I_1 - I_3) + 100(I_1 - I_2) = 0$$

$$-5 + 100(I_2 - I_1) + 330(I_2 - I_3) = 0$$

$$330(I_3 - I_2) + 100(I_3 - I_1) + 330(I_3 - I_4) = 0$$

$$330(I_4 - I_3) + 100(I_4) + 330(I_4) = 0$$

Group terms

$$530I_1 - 100I_2 - 100I_3 = 0$$

$$-100I_1 + 430I_2 - 330I_3 = 5$$

$$-100I_1 - 330I_2 + 760I_3 - 330I_4 = 0$$

$$-330I_3 + 760I_4 = 0$$

Place in matrix form

$$\begin{bmatrix} 530 & -100 & -100 & 0 \\ -100 & 430 & -330 & 0 \\ -100 & -330 & 760 & -330 \\ 0 & 0 & -330 & 760 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> A = [530,-100,-100,0 ; -100,430,-330,0 ; -100,-330,760,-330 ; 0,0,-330,760]
```

```
530    -100    -100         0
-100     430    -330         0
-100    -330     760    -330
  0         0    -330     760
```

```
>> B = [0;5;0;0]
```

```
0
5
0
0
```

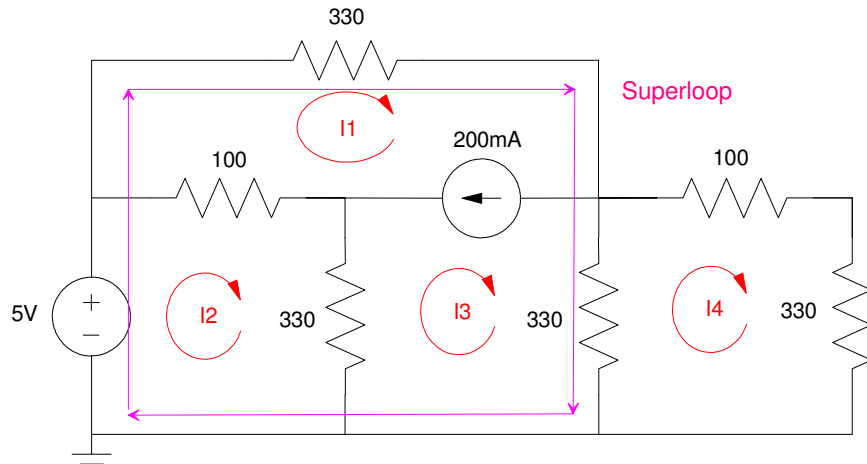
```
>> I = inv(A)*B;
```

```
>> mA = I*1000
```

```
I1      7.1978  mA
I2     24.0903  mA
I3     14.0578  mA
I4      6.1040  mA
```

7) Write the current loop equations for the following circuit

- You don't need to solve



There are four loops. Write four equations for four unknowns.

Start with the easy one (current source)

$$I_1 - I_3 = 200mA$$

Three to go. Go around loop I2

$$-5 + 100(I_2 - I_1) + 300(I_2 - I_3) = 0$$

Two to go. Go around loop I4

$$330(I_4 - I_3) + 100(I_4) + 330(I_4) = 0$$

One to go. Loop I1 and loop I3 don't work since the voltage drop across the 200mA source is unknowns. Use a superloop (shown in pink)

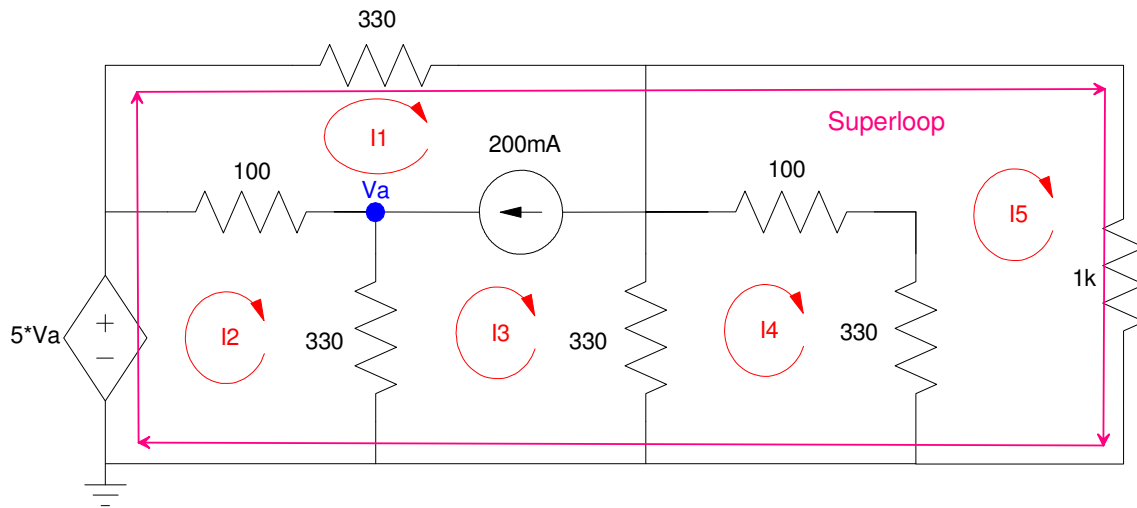
- Other superloops are valid

$$-5 + 330(I_1) + 330(I_3 - I_4) = 0$$

This gives four equations for four unknowns

8) Write the current loop equations for the following circuit

- You don't need to solve



Five loops and a dependent source means six equations are needed. Start with the easy one (200mA source)

$$I_1 - I_3 = 200mA$$

Define V_a

$$V_a = 330(I_2 - I_3)$$

Four to go. Write the loop equation around L2

$$-5V_a + 100(I_2 - I_1) + 330(I_2 - I_3) = 0$$

Loop I4

$$330(I_4 - I_3) + 100(I_4 - I_5) + 330(I_4 - I_5) = 0$$

Loop I5

$$330(I_5 - I_4) + 100(I_5 - I_4) + 1000(I_5) = 0$$

One to go. Loop I1 and loop I3 don't work since we don't know the voltage drop across the 200mA source. Use a superloop (shown in pink)

- Other superloop equations work

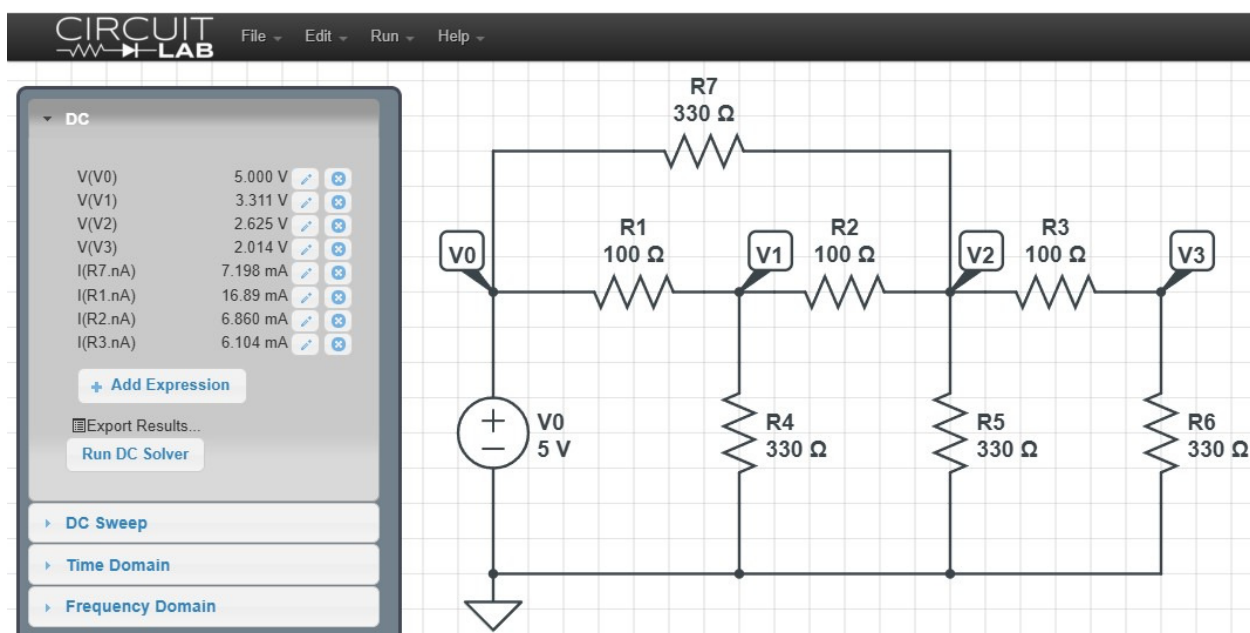
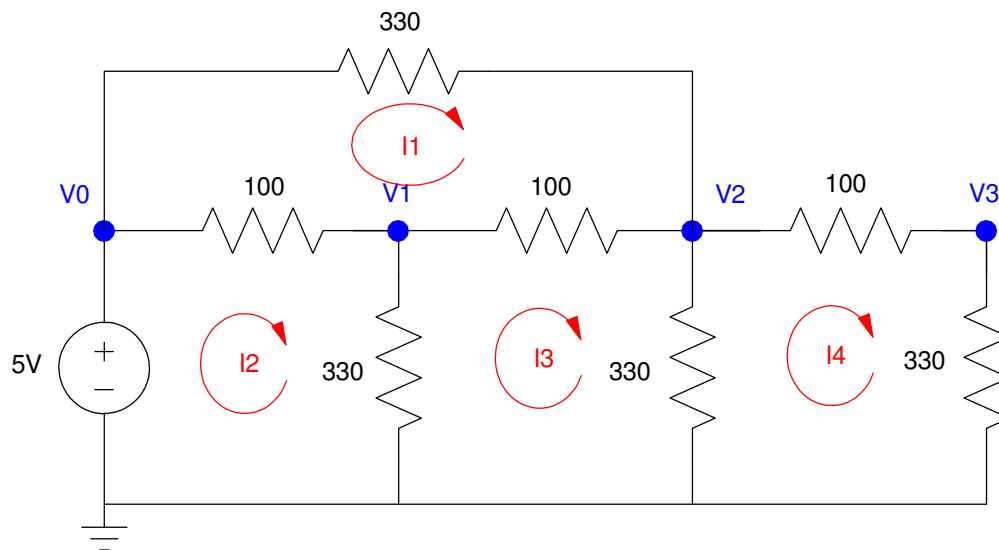
$$-5V_a + 330I_1 + 1000I_5 = 0$$

This gives six equations for six unknowns.

Lab

9) Find the voltages and currents for the following circuit using CircuitLab

- Include a screen shot of your circuit and CircuitLab's solution



Note the CircuitLab tells you the current through the resistors. This is a combination of the loop currents

- $I(R1) = I2 - I1$
- $I(R2) = I3 - I1$

Current	$I(x) = f(I_1, I_2, I_3, I_4)$	Calculated	CircuitLab
$I(R_7)$	I_1	7.1979 mA	7.198 mA
$I(R_1)$	$I_2 - I_1$	16.8926 mA	16.89 mA
$I(R_2)$	$I_3 - I_1$	6.8600 mA	6.860 mA
$I(R_3)$	I_4	6.1040 mA	6.104 mA

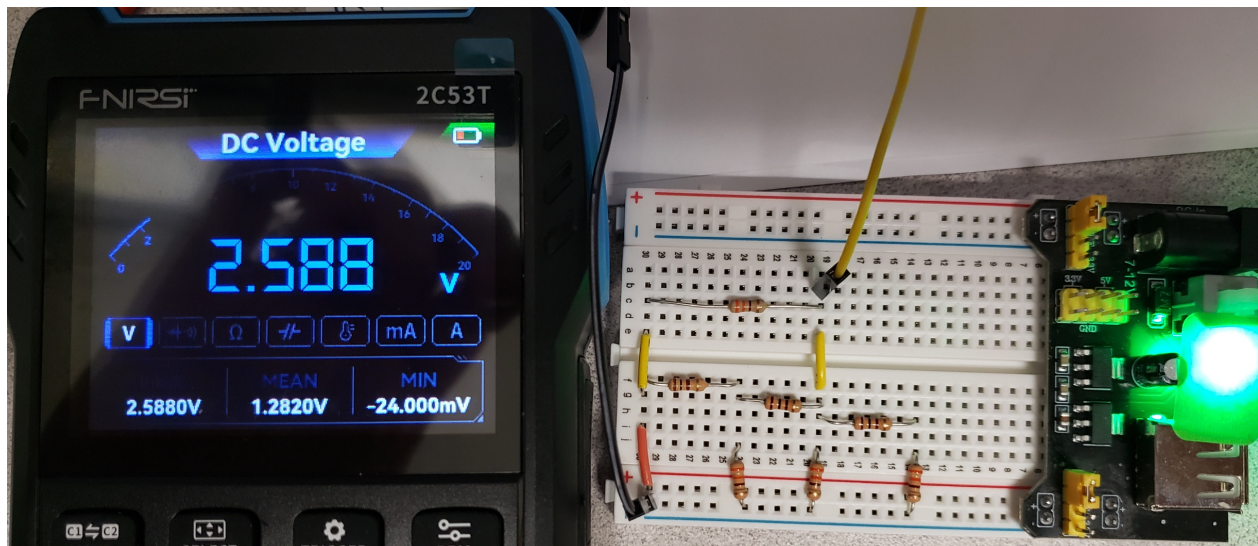
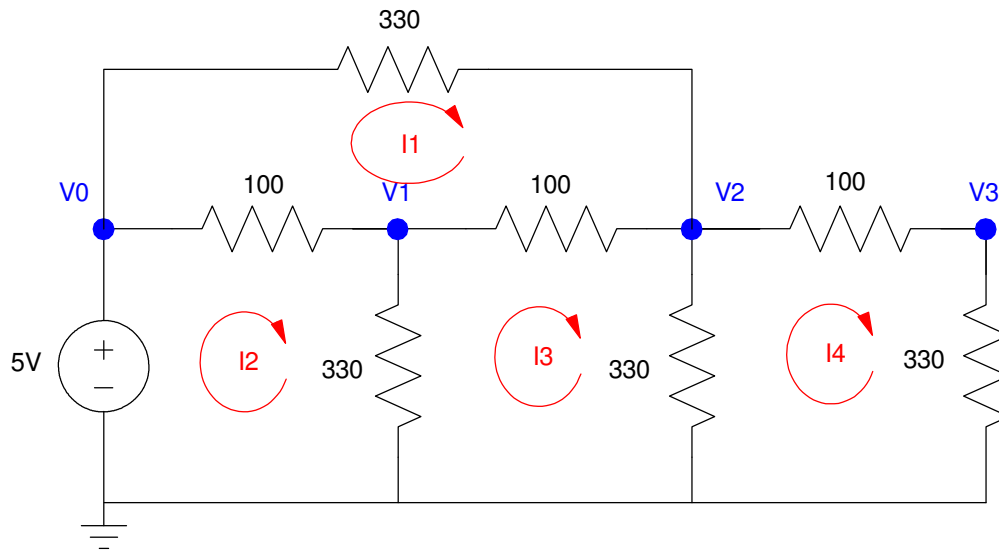
The currents match up

10) Build this circuit on your breadboard.

- Measure the voltages {V0, V1, V2, V3}
- Measure the currents {I1, I2, I3, I4}

note: To measure current, measure the voltage across a resistor and compute the current from $V = IR$

- Include a photo of your breadboard in your solutions



Voltages

- Voltages match up

	V0	V1	V2	V3
Calculated	5.0000 V	3.3107 V	2.6247 V	2.0143 V
CircuitLab	5.000 V	3.311 V	2.625 V	2.014 V
Measured	4.939 V	3.270 V	2.590 V	1.9867 V

Currents

- Currents are calculated from the voltages and resistances
- Currents match up

	I1 I(R7)	I2 - I1 I(R1)	I3 - I1 I(R2)	I4 I(R3)
Calculated	7.1979 mA	16.8926 mA	6.8600 mA	6.1040 mA
CircuitLab	7.198 mA	16.89 mA	6.860 mA	6.104 mA
Measured	7.1182 mA	16.69 mA	6.800 mA	6.0203 mA

Example:

$$I(R1) = \left(\frac{V_0 - V_1}{100} \right) = 16.69 \text{ mA}$$