Voltage Nodes with Phasors

EE 206 Circuits I

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Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

Voltage Nodes:

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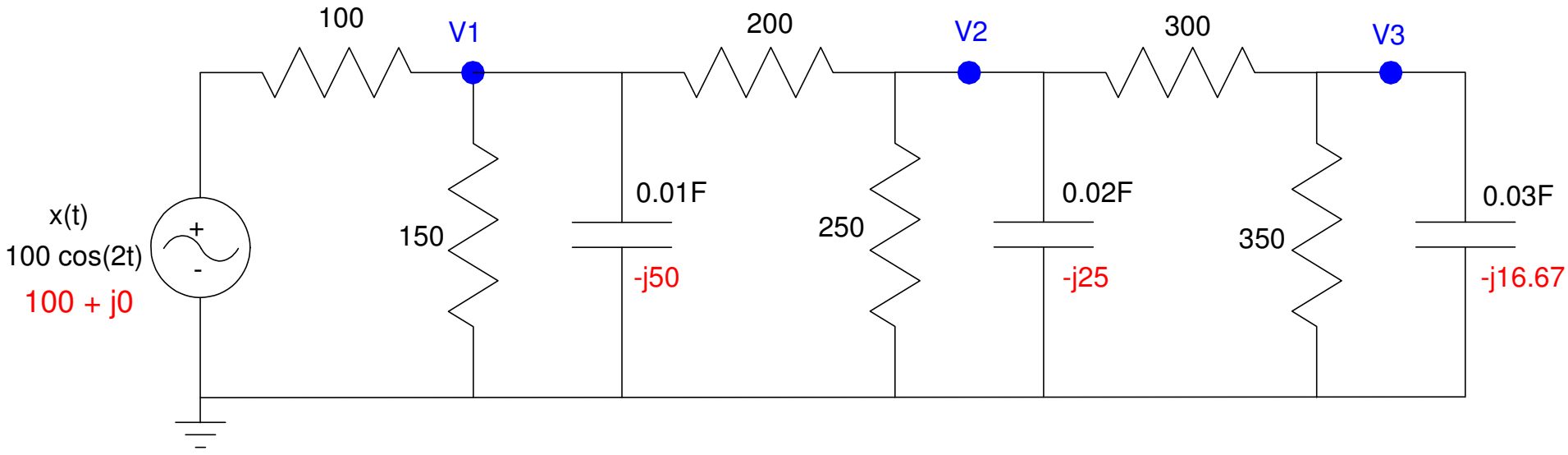
Example 2: 3-Stage RC Circuit

Find the voltages for the following circuit when the input is

$$x(t) = 100 \cos(2t)$$



Step 1: Express in Phasor form

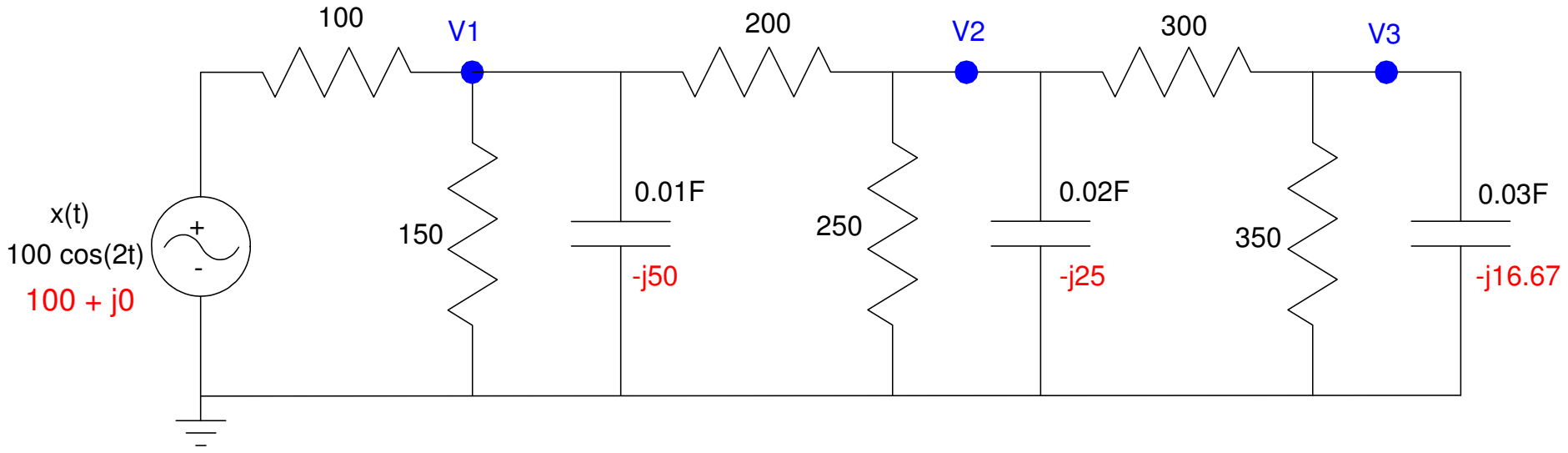
- (shown in red)

Step 2: Write N equations for N unknowns

$$\left(\frac{V_1 - X}{100}\right) + \left(\frac{V_1}{150}\right) + \left(\frac{V_1}{-j50}\right) + \left(\frac{V_1 - V_2}{200}\right) = 0$$

$$\left(\frac{V_2 - V_1}{200}\right) + \left(\frac{V_2}{250}\right) + \left(\frac{V_2}{-j25}\right) + \left(\frac{V_2 - V_3}{300}\right) = 0$$

$$\left(\frac{V_3 - V_2}{300}\right) + \left(\frac{V_3}{350}\right) + \left(\frac{V_3}{-j16.67}\right) = 0$$



Step 3: Solve.

First, group terms

$$\left(\frac{1}{100}+\frac{1}{150}+\frac{1}{-j50}+\frac{1}{200}\right)V_1+\left(\frac{-1}{200}\right)V_2=\left(\frac{1}{100}\right)X$$

$$\left(\frac{-1}{200}\right)V_1+\left(\frac{1}{200}+\frac{1}{250}+\frac{1}{-j25}+\frac{1}{300}\right)V_2+\left(\frac{-1}{300}\right)V_3=0$$

$$\left(\frac{-1}{300}\right)V_2+\left(\frac{1}{300}+\frac{1}{350}+\frac{1}{-j16.67}\right)V_3=0$$

Place in matrix form

$$\begin{bmatrix} \left(\frac{1}{100}+\frac{1}{150}+\frac{1}{-j50}+\frac{1}{200}\right) & \left(\frac{-1}{200}\right) & 0 \\ \left(\frac{-1}{200}\right) & \left(\frac{1}{200}+\frac{1}{250}+\frac{1}{-j25}+\frac{1}{300}\right) & \left(\frac{-1}{300}\right) \\ 0 & \left(\frac{-1}{300}\right) & \left(\frac{1}{300}+\frac{1}{350}+\frac{1}{-j16.67}\right) \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} \left(\frac{1}{100}\right) \\ 0 \\ 0 \end{bmatrix}X$$

Solve in Matlab

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V = inv(A)*B*X

   24.2853 -23.2416i
   -1.7971 - 3.5726i
   -0.2066 + 0.0785i
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$$v_1(t) = 24.28\cos(2t) + 23.24\sin(2t)$$

$$v_2(t) = -1.79\cos(2t) + 3.57\sin(2t)$$

$$v_1(t) = 0.21\cos(2t) + 0.08\sin(2t)$$

$$v_1(t) = 33.61\cos(2t - 43.7^0)$$

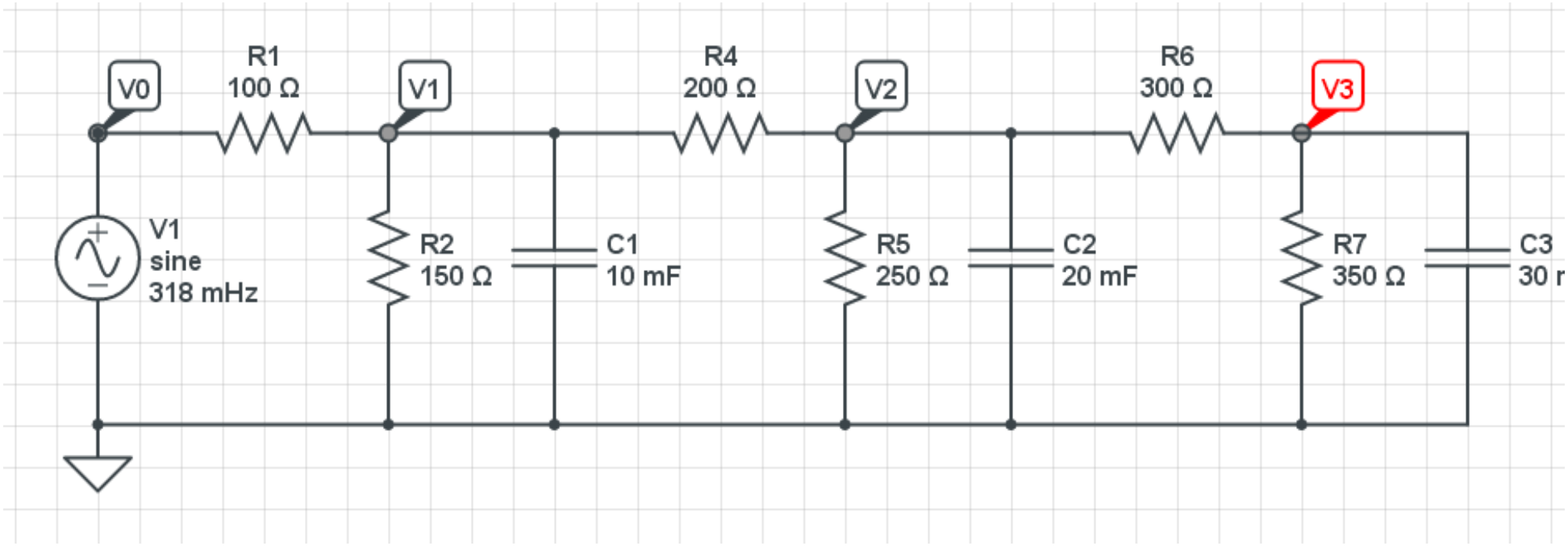
$$v_2(t) = 3.999\cos(2t - 116.7^0)$$

$$v_3(t) = 0.22\cos(2t + 159.2^0)$$

CircuitLab Simulation

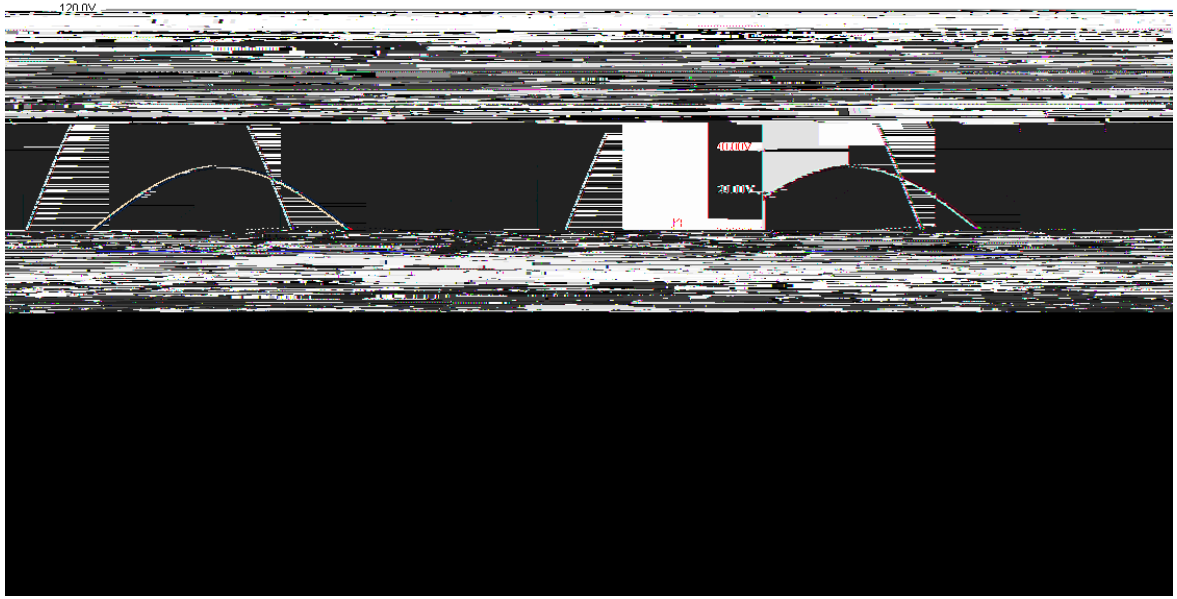
Build the circuit

- Note: 2 rad/sec = 0.318 Hz



Run a transient simulation

- max time = 20 seconds (6 cycles)
- time step = 20ms (1000 points on the plot)



This matches our calculations:

$$v_1(t) = 33.61\cos(2t - 43.7^0)$$

$$v_2(t) = 3.999\cos(2t - 116.7^0)$$