Complex Numbers

EE 206 Circuits I

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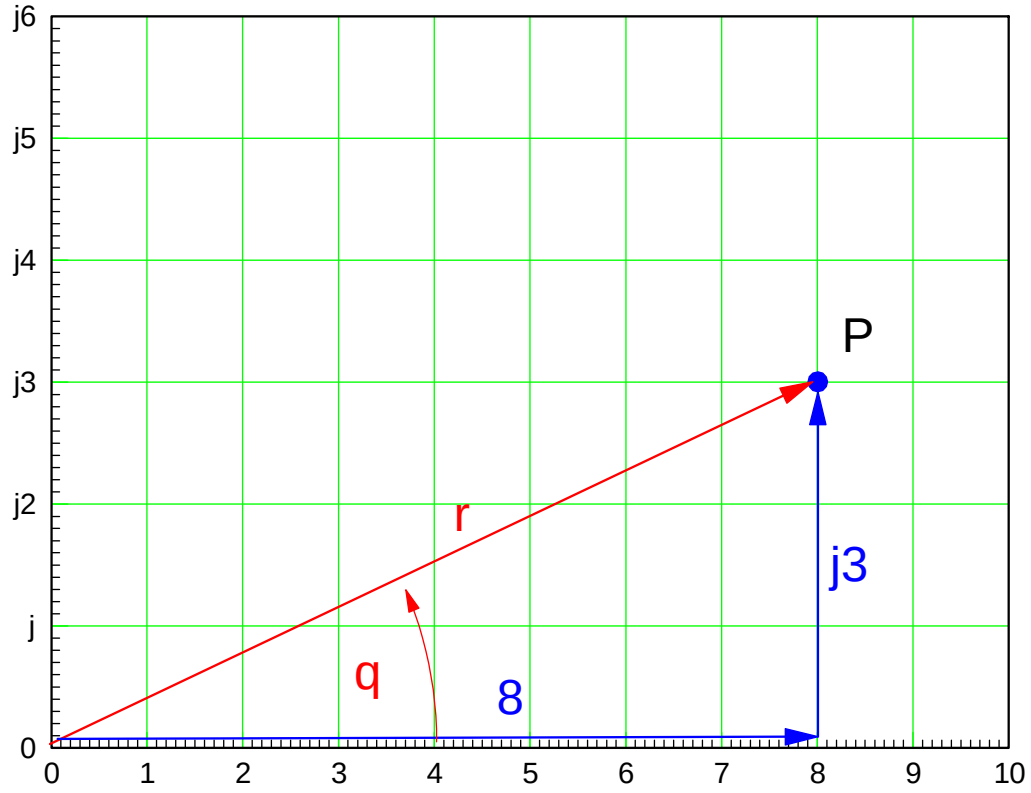
03/18/20

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

Objective:

- Become familiar with using complex numbers for addition, subtraction, multiplication, and division



The number Zero:

- Zero is an odd concept: something that represents nothing
- Zero isn't needed: the Romans had an extensive economy without the number zero.
- Without zero, addition becomes difficult.
- Without zero, multiplication becomes difficult

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    MXXIII                 MXXIII
+      CVI             *     CVI
=        ?             =      ?
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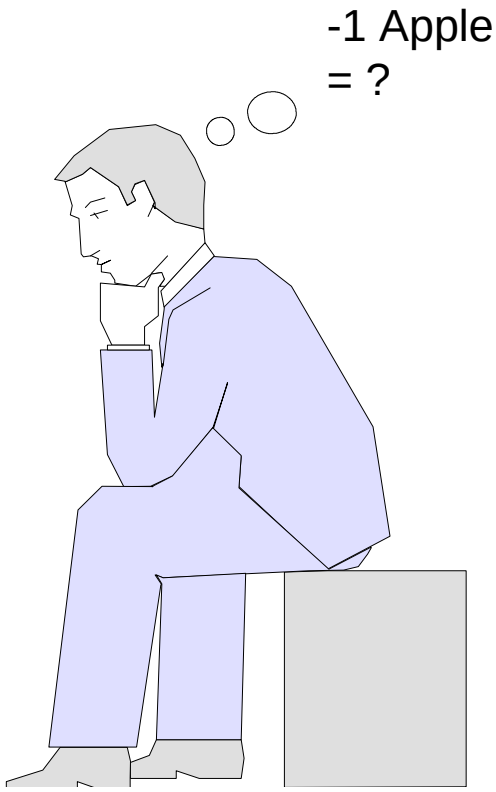
Negative Numbers:

- Negative numbers are even more starange
- Their invention allowed Holland to become a world power

By keep tracking of credits (+) and debits (-), the double-entry book-keeping system allowed Dutch merchants to understand what ventures were profitable and which were not.



Polar and Rectangular Form:

A complex number can be represented in rectangular or polar form

$$x + jy \qquad\qquad r \cdot e^{j\theta} = r\angle\theta$$

The relationship is

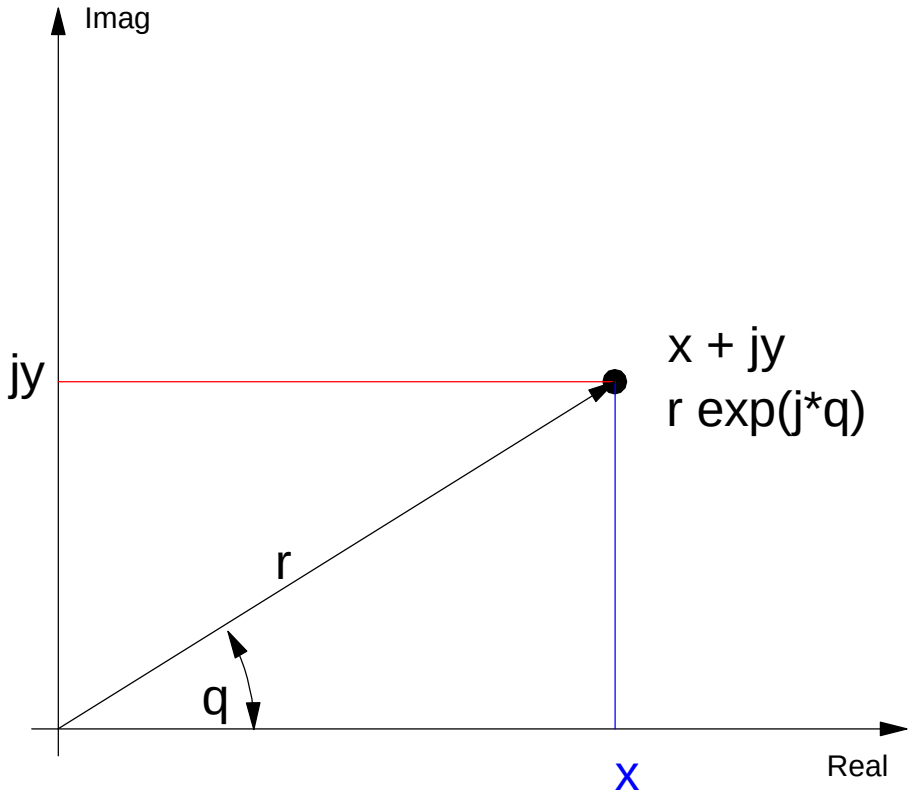
$$r = \sqrt{x^2 + y^2}$$

$$\tan(\theta) = \frac{y}{x}$$

or

$$x = r \cdot \cos(\theta)$$

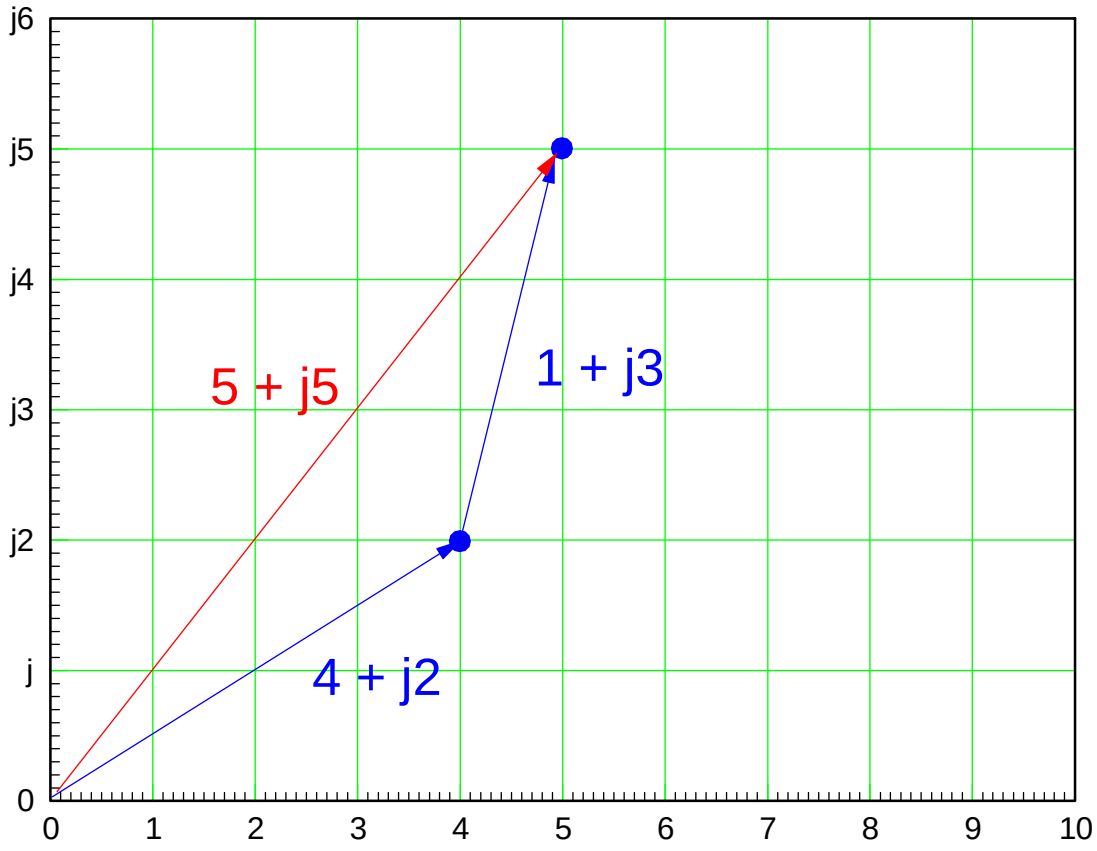
$$y = r \cdot \sin(\theta)$$



Addition

Add real to real, complex to complex

$$\begin{aligned} & 4 + j2 \\ + \quad & 1 + j3 \\ = \quad & 5 + j5 \end{aligned}$$



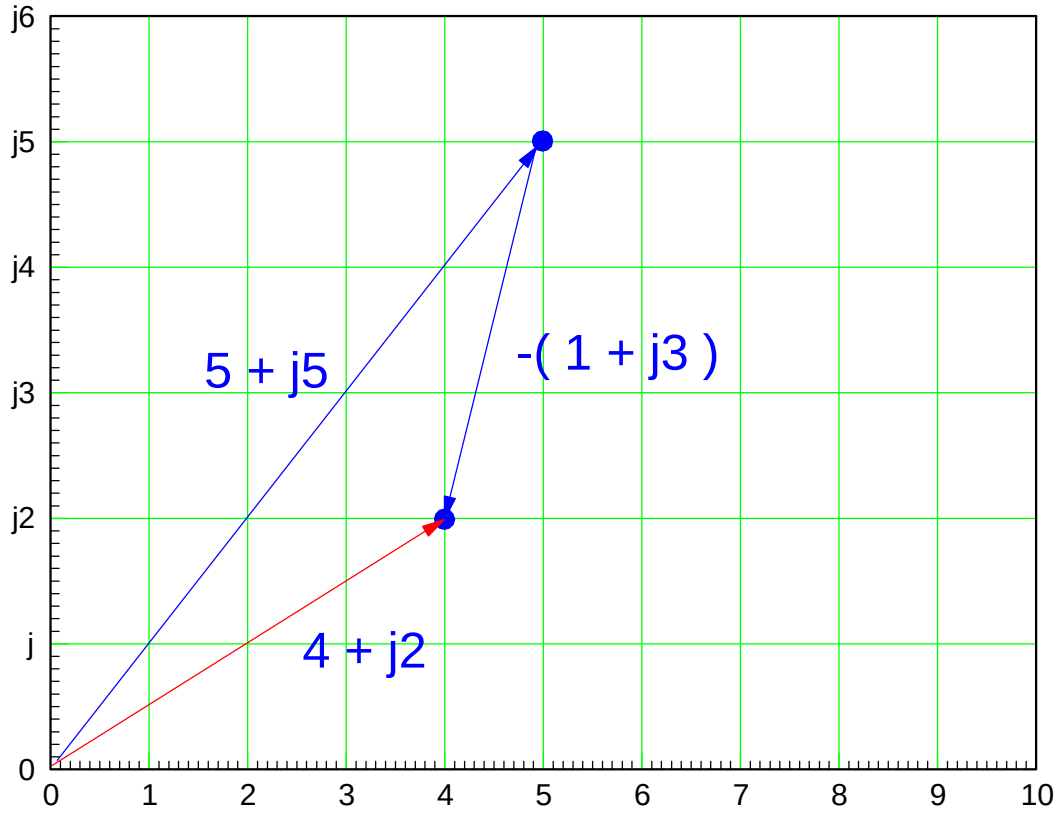
Subtraction:

Subtract real from real, complex from complex

$$5 + j5$$

$$- \quad 1 + j3$$

$$= \quad 4 + j2$$



Multiplication

Rectangular Form:

$$(2+j3)(4+j5) = (2\cdot 4)+(2\cdot j5)+(j3\cdot 4)+(j3\cdot j5)$$

$$= (8)+(j10)+(j12)+(j^2 15)$$

Note that j2 = -1:

$$= (8-15)+j(10+12)$$

$$= -7+j22$$

Multiplication is easier in polar form:

$$(a\angle\theta)(b\angle\phi) = ab\angle(\theta+\phi)$$

$$(a\cdot e^{j\theta})(b\cdot e^{j\phi}) = ab\cdot e^{j(\theta+\phi)}$$

Complex Conjugates:

The complex conjugate (symbol *) is

$$(x + jy)^* = x - jy$$

A number multiplied by its complex conjugate is

- The real squared, plus
- The imaginary squared

$$(x + jy)(x - jy) = (x^2 + jxy - jxy - j^2 y)$$
$$= x^2 + y^2$$

Division

Polar Form

$$\left|\frac{a\angle\theta}{b\angle\phi}\right| = \left|\frac{a}{b}\right| \angle(\theta-\phi)$$

Rectangular Form

$$\left|\frac{a+jb}{c+jd}\right| = \left|\frac{a+jb}{c+jd}\right| \left|\frac{c-jd}{c-jd}\right|$$

$$= \left|\frac{ac-bd+jbc-jad}{c^2+d^2}\right|$$

$$= \left|\frac{ac-bd}{c^2+d^2}\right| + j\left|\frac{bc-ad}{c^2+d^2}\right|$$

HP Calculators

I ***strongly*** recommend getting an HP calculator

- HP35s: $52 on Amazon
- Free42: free app for the HP42 (great calculator)

You will be using complex numbers extensively in Electrical and Computer engineering.

Get a calculator that does complex numbers

I've found that HP calculators are worth about 10 points on midterms (they breeze through complex math)