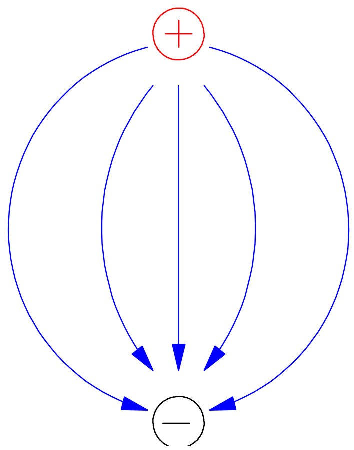
Mutual Inductance

EE 206 Circuits I

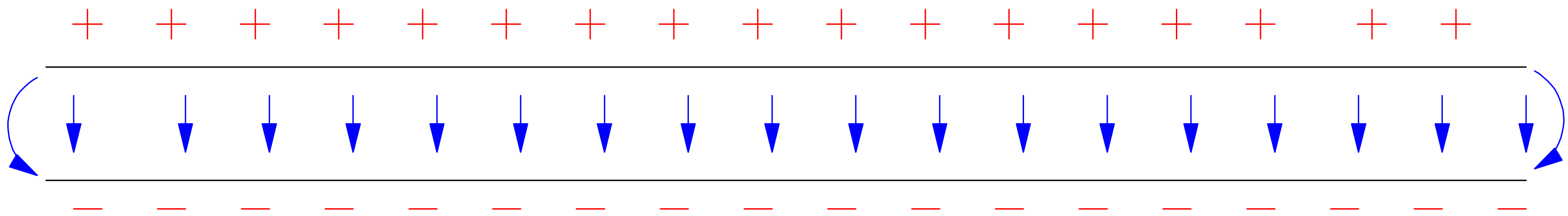
Jake Glower - Lecture #19

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Capacitors

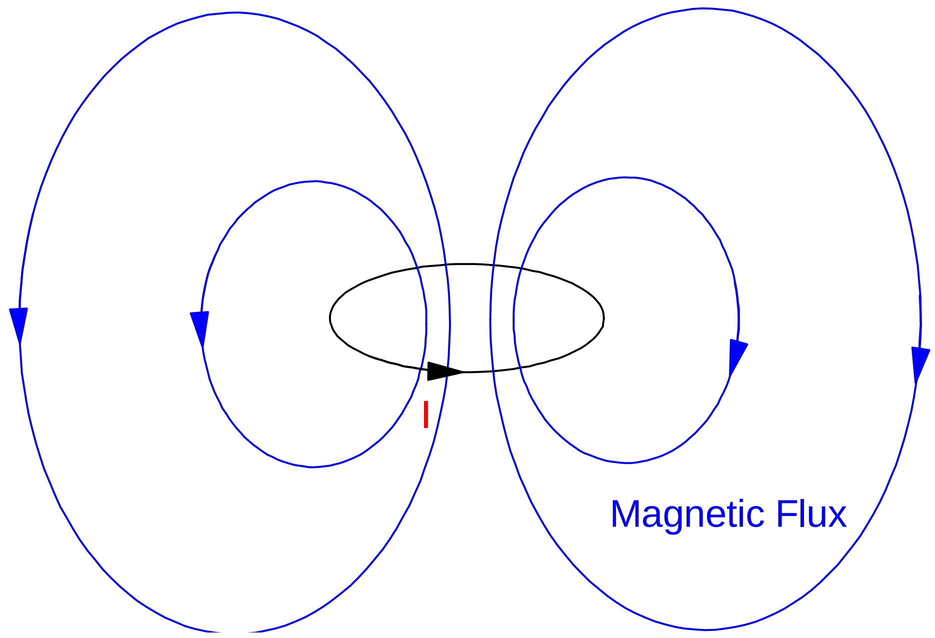


- Electric field lines have a starting point and an ending point.
- In a parallel plate capacitor, some of the field lines will extend beyond the plates of the capacitor. If these field lines overlap with another capacitor's field lines, there will be coupling. Since the spacing is so small, this coupling is effectively zero.



Inductors

- Magnetic field do *not* have a start or an end (magnetic monopoles do not exist)
- Magnetic fields are difficult to contain
- If the field lines of two inductors overlap, the two inductors affect each other
- This is called *utua nductanc*

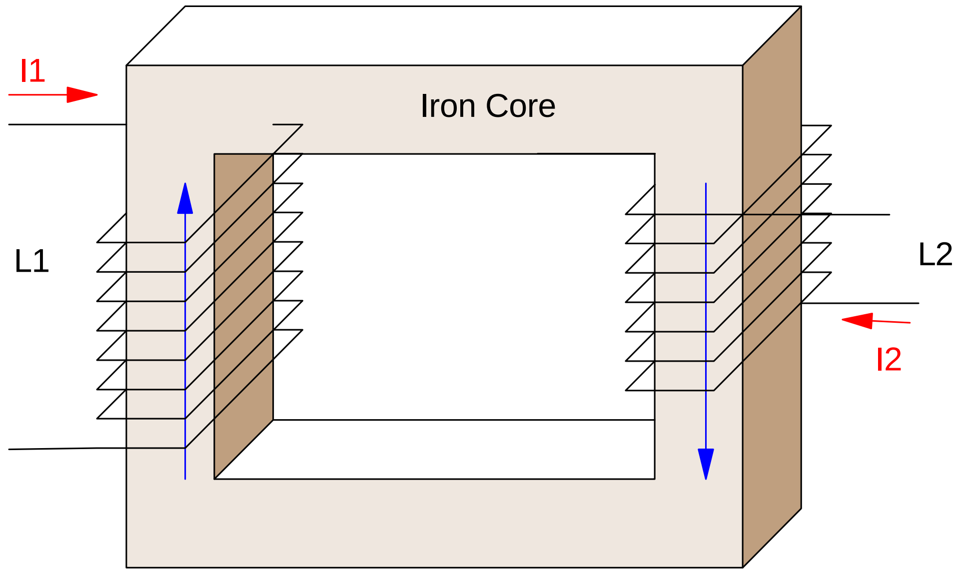


Mutual Inductance

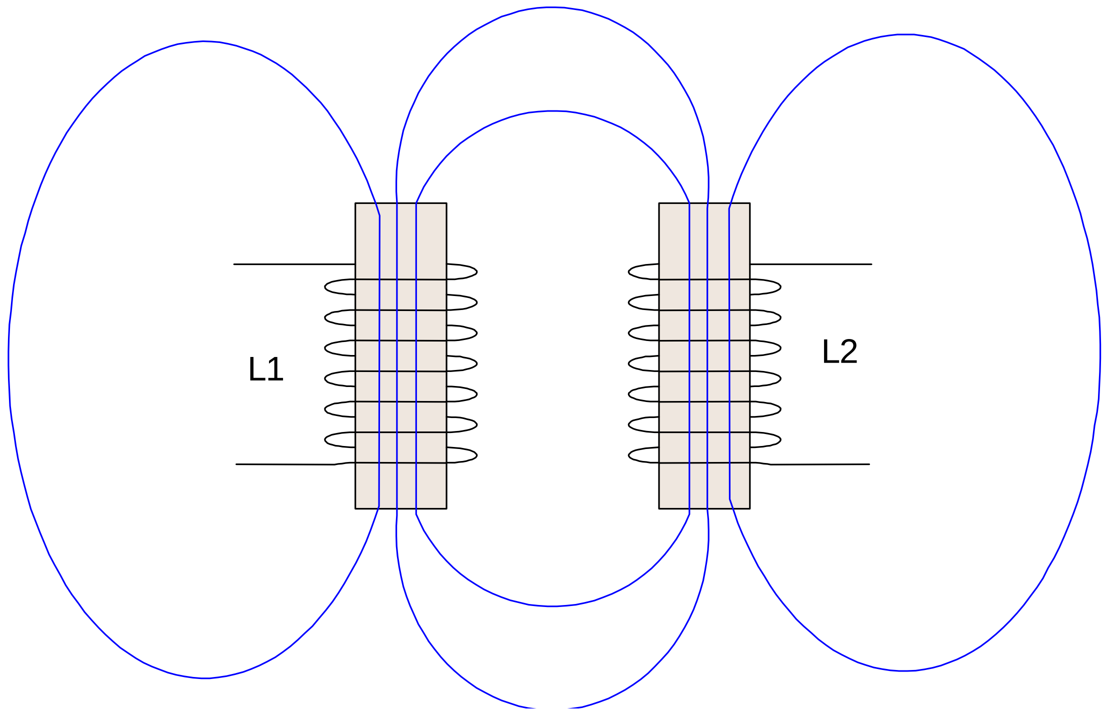
- Coupling between two inductors

Let 'k' be the percentage of the field lines shared by two inductors.

- k=1: all of the magnetic flux is shared (example: transformer)

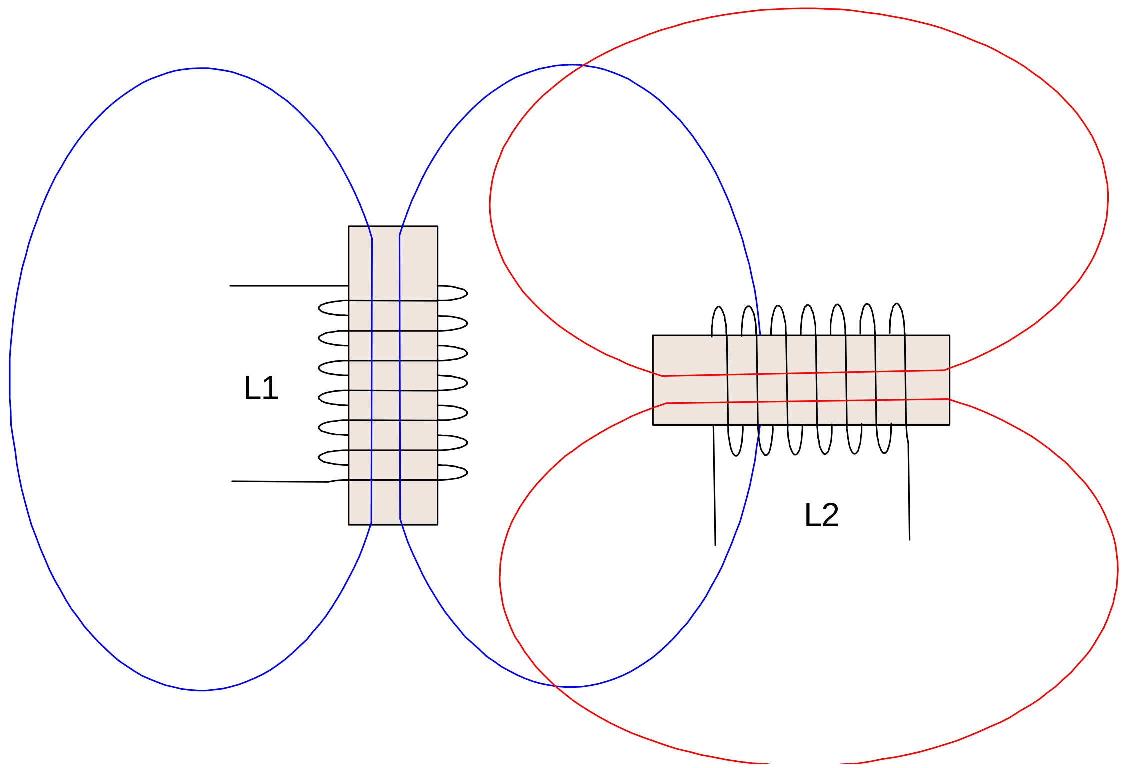


- 0 < k < 1: Two transformers are close to each other
  - Some of the magnetic flux lines will be shared
  - Magnetic field strength drops off as distance cubed



- k = 0: Two inductors are far apart or are perpendicular

This makes circuit board design with inductors somewhat annoying: the placement of the inductors can have a significant influence on how the circuit behaves.

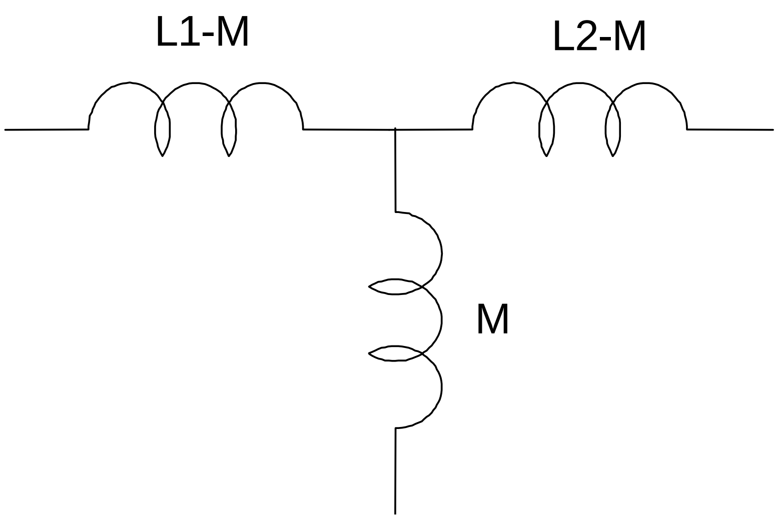


Electric Model for Mutual Inductance

If 'k' is the percentage of field lines that are common between two inductors, the mutual inductance, M, is

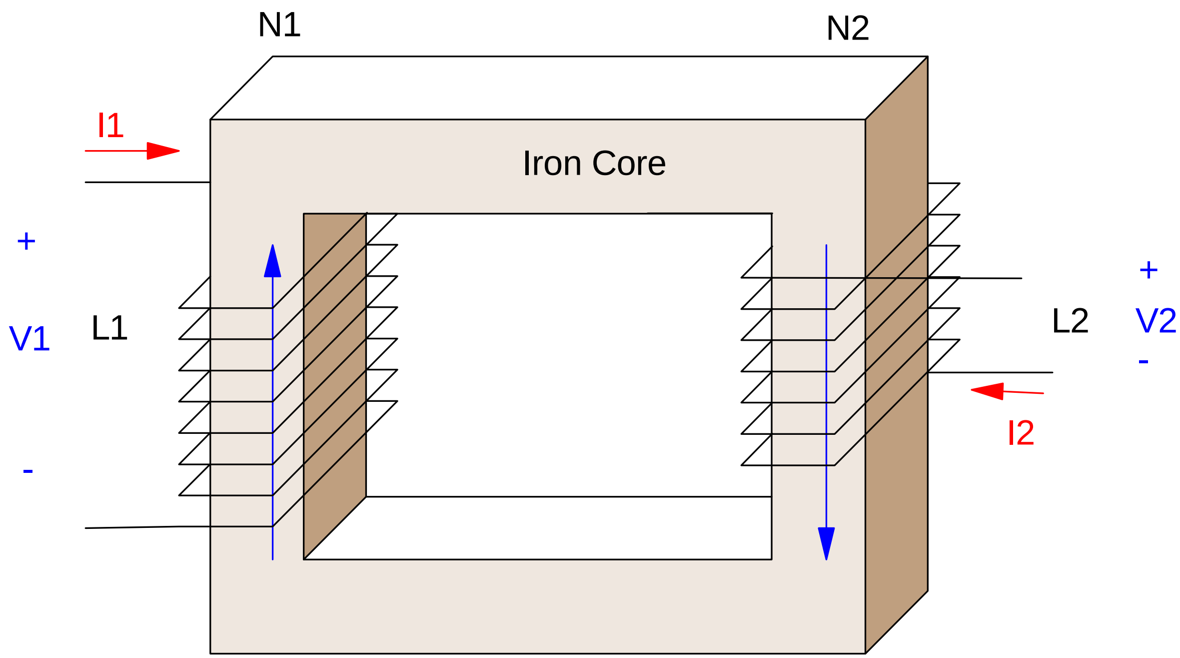
$$M = \sqrt{L_1 L_2}$$

The T-circuit model for two couples inductors is then



Usage of Mutual Inductance: Transformers

- Two inductors which share a common iron core
- Most of the magnetic flux from one inductor going through the other inductor.



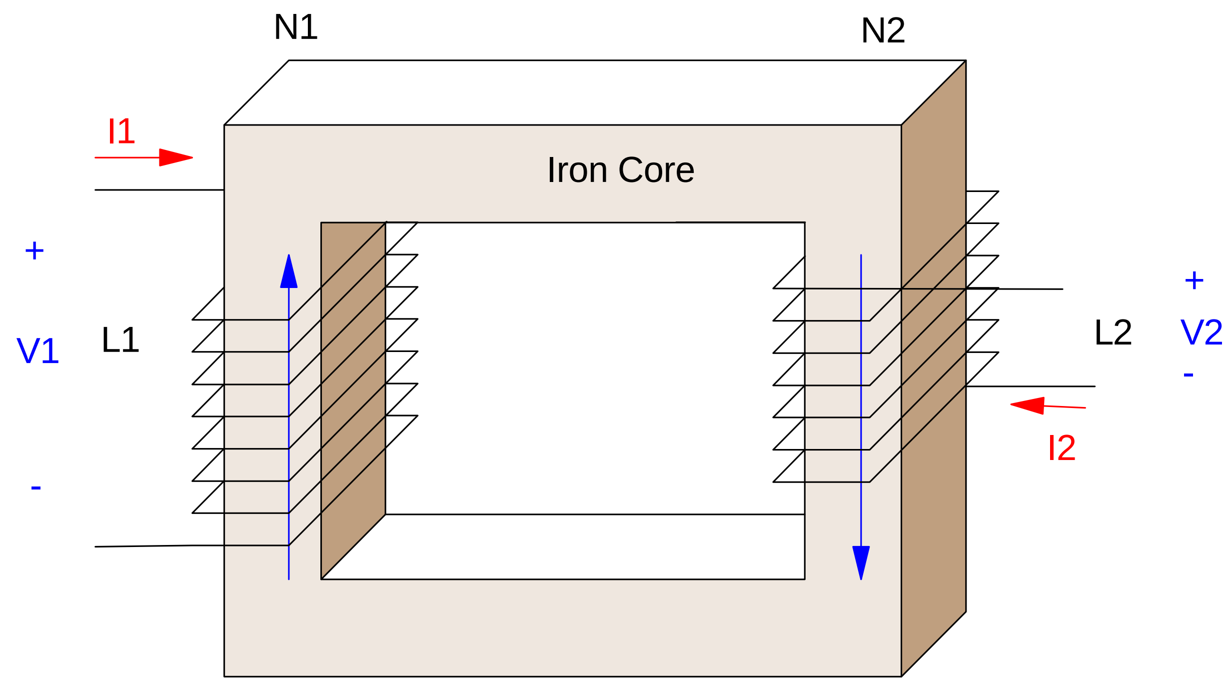
Transformer with N1 turns on inductor L1 and N2 turns on inductor L2

Since the magnetic field is the same for both inductors

$$\phi_1 = \phi_2$$

$$_1 I_1 = \ _2 I_2$$

$$I_2 = \left(\frac{1}{2}\right) I_1$$



Power must balance

$$_1 = \ _2$$

$$_1 I_1 = \ _2 I_2$$

This results in

$$_2 = \left(\frac{2}{1}\right) \ _1$$

Transformers are a convenient way to change voltages and currents.

Transformer Uses: Power Grid

- Transformers allow you to transmit power over large distances
- Transformers only work at AC (they don't work at DC)
- This is why the power grid operates at 60Hz

Problem: Transmit power from western North Dakota to Minneapolis (1000km)

- R = 213 Ohms   (1000km of 1cm dia copper wire)

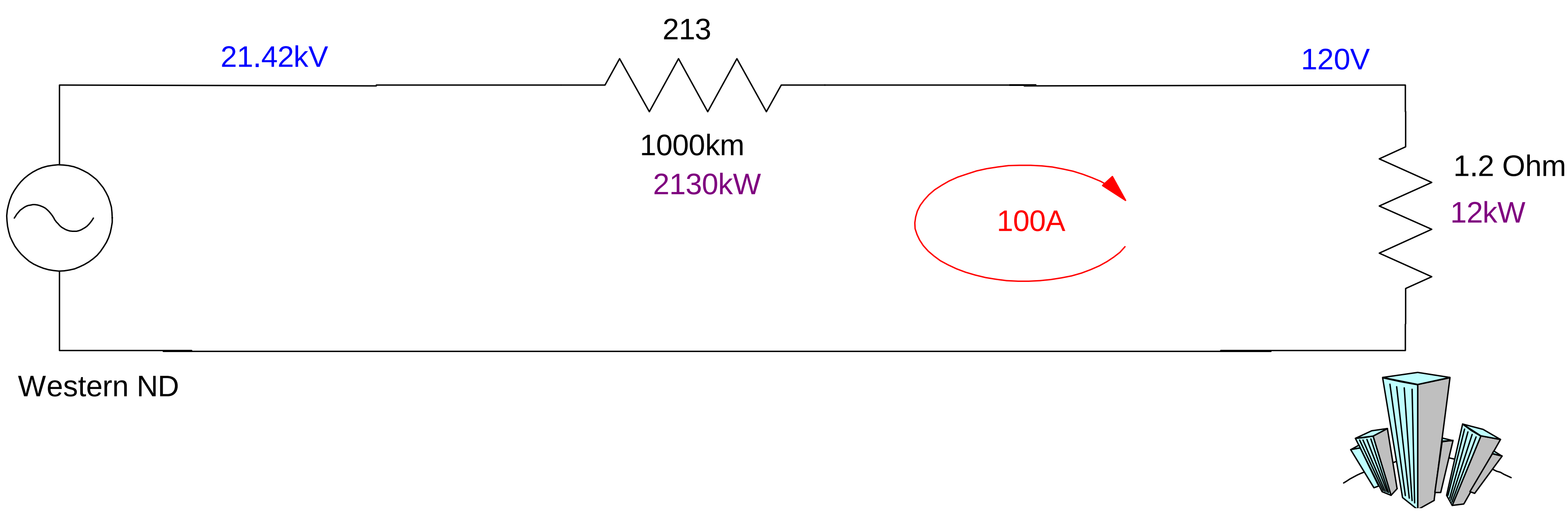
DC Solution: Transmit 12kW @ 120V (I = 100A)

- The line losses would be

$$_{n} = I^2 \quad = (100A)^2 \cdot 213\Omega = 2130$$

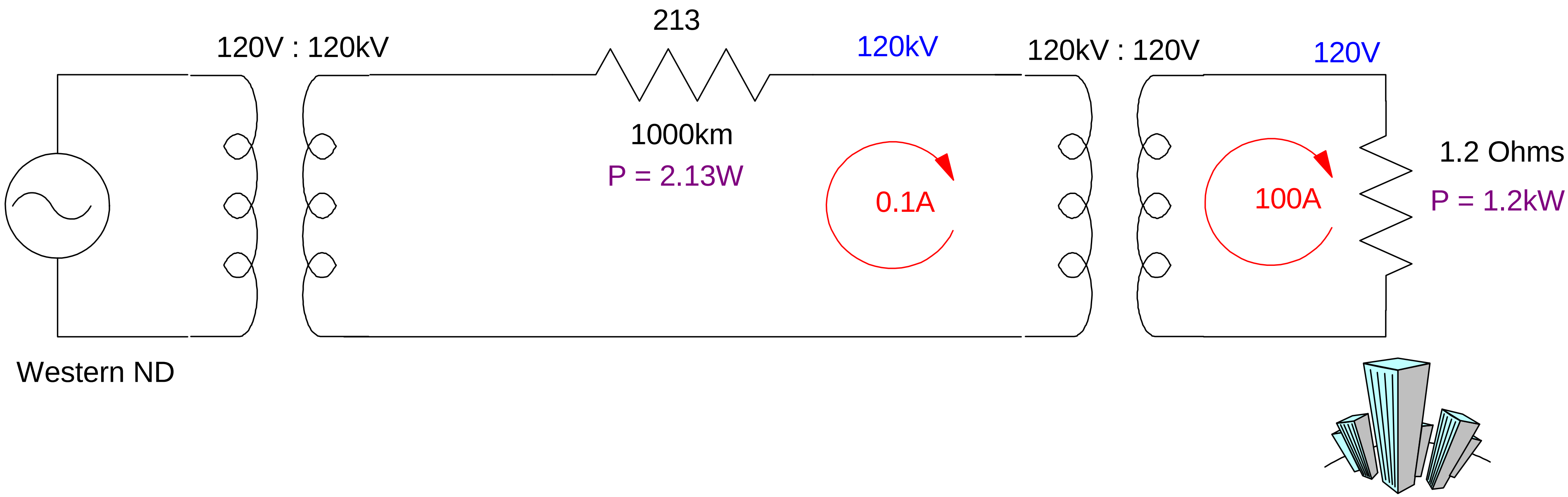
- The efficiency is 0.056%

$$\eta = \left(\frac{\text{power to load}}{\text{total power}}\right) = \left(\frac{12}{12 \quad +2130}\right) = 0.00056$$



AC Solution: Use transformers to bump up the voltage to 120kV

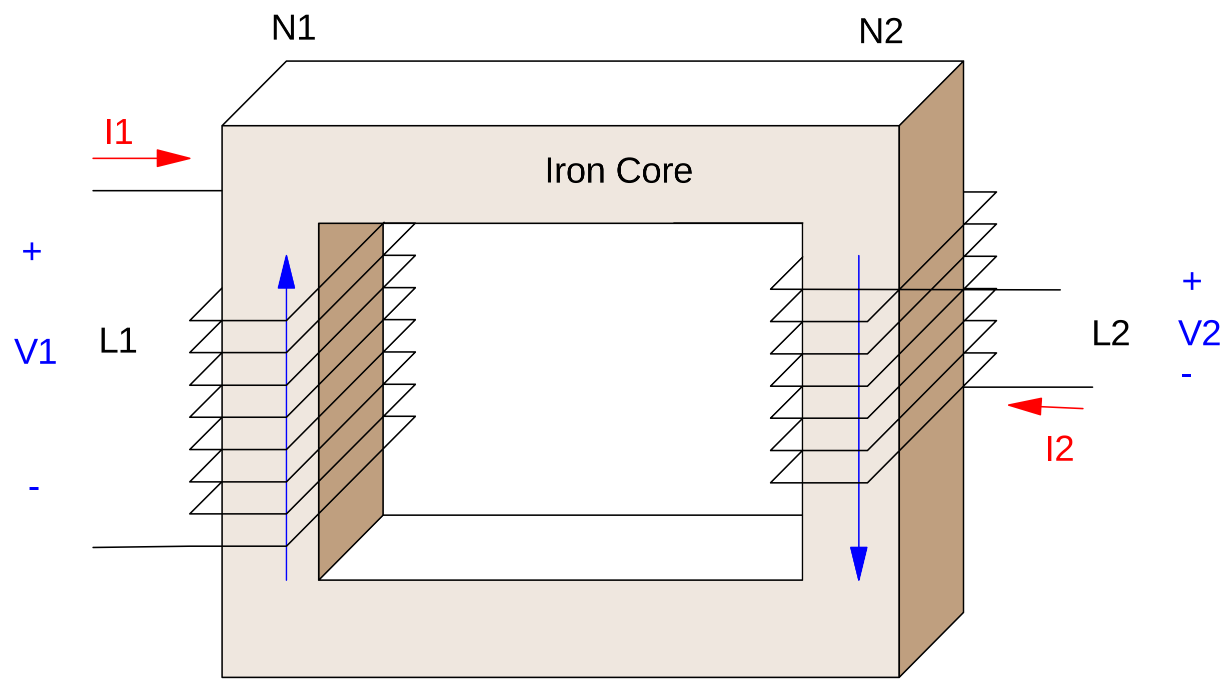
- Power must balance
- If voltage goes up 1000x, current goes down 1000x (P = VI = constant)
- Line losses drop by a factor of $(1000)^2$
- Efficiency increases to 99.8%



Transformers also change impedances

$$_2 = \left(\frac{_2}{_1}\right) {}_1$$

$$I_2 = \left(\frac{_1}{_2}\right) I_1$$



If you add a ressitance to V2

$$_2 = \frac{_2}{I_2}$$

As seen from size 1

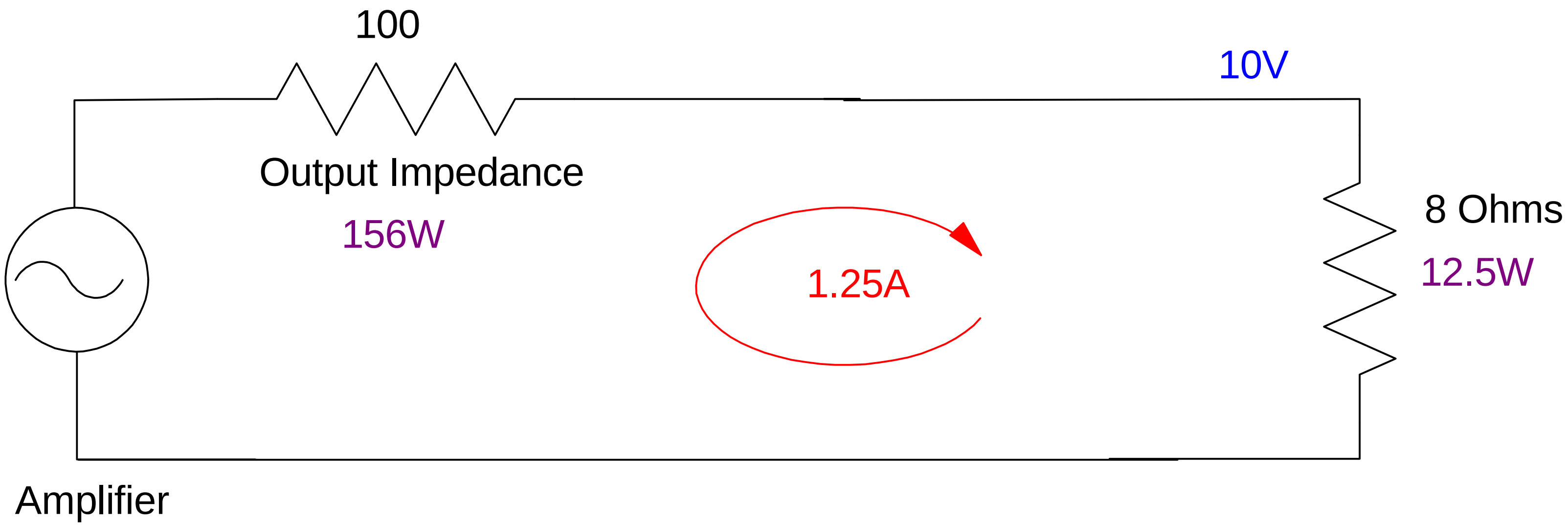
$$_2 = \frac{\left(\frac{_2}{_1}\right) {}_1}{\left(\frac{_1}{_2}\right) I_1}$$

$$_2 = \left(\frac{_2}{_1}\right)^2 {}_1$$

Transformers in Stereos:

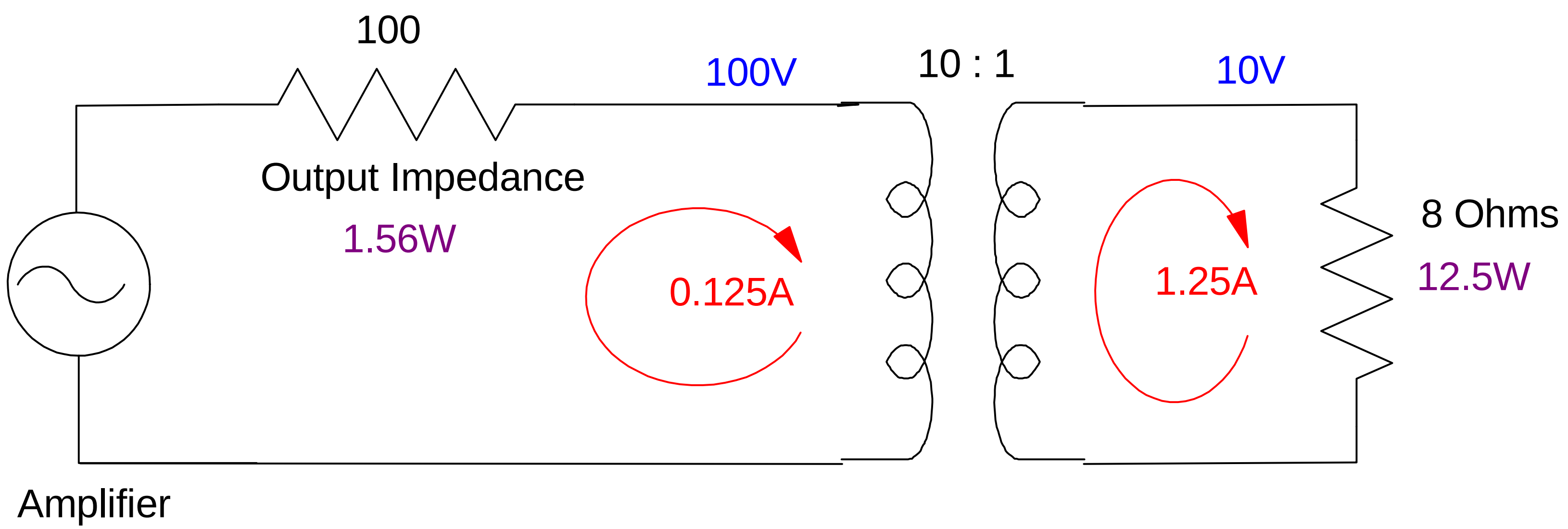
Problem: Drive an 8-Ohm speaker with a stereo with a 100 Ohm output impedance

Solution 1: Efficiency = 7%



Solution 2: Use a 10:1 transformer

- 8-Ohm speaker now looks like an 800 Ohm speaker
- Efficiency increases to 89%
- To get 10V at the speaker, the amplifier has to output 100V



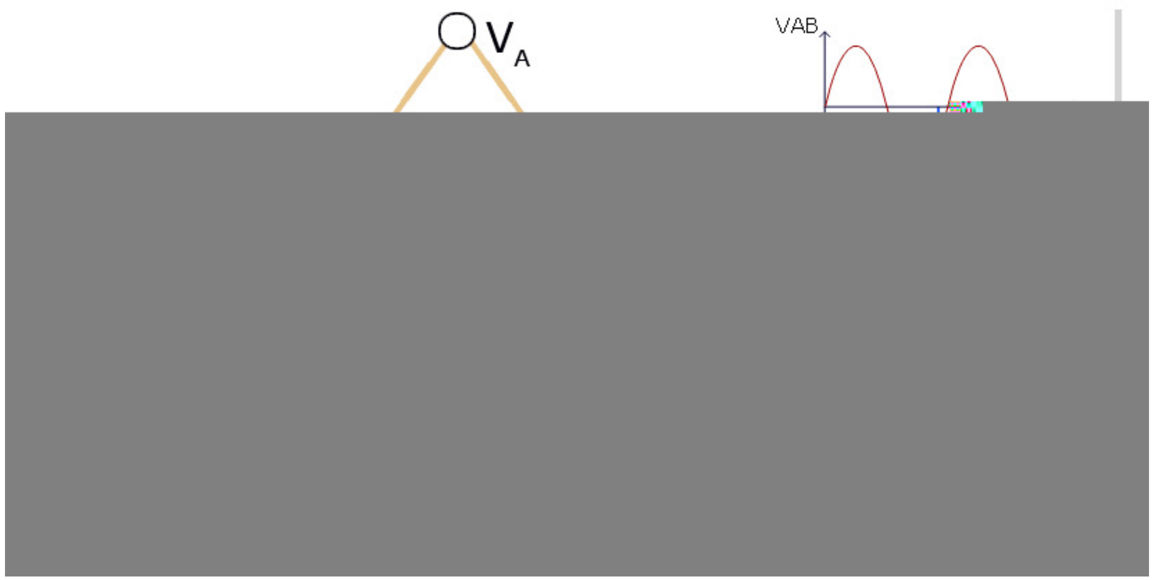
Transformers in Motors: AC Induction Motor

At the heart of an AC induction motor is a stator

- N : 1 transformer

A rotating field in the stator induces current in the rotor

- Which produces a magnetic field
- Which created torque and causes the motor to spin



http://www.animations.physics.unsw.edu.au/jw/electricmotors.html

Modeling an AC Induction Motor

- Essentially a N:1 transformer where the secondary side rotates
- The electrical model for an AC induction motor is almost identical to the electrical model for a transformer.
- This will be covered in more detail in ECE 331 Energy Conversion.