SuperNodes:

EE 206 Circuits I

Jake Glower - Lecture #6

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

SuperNodes:

- A closed path the encloses 2+ nodes
- The current coming out of any closed path must sum to zero

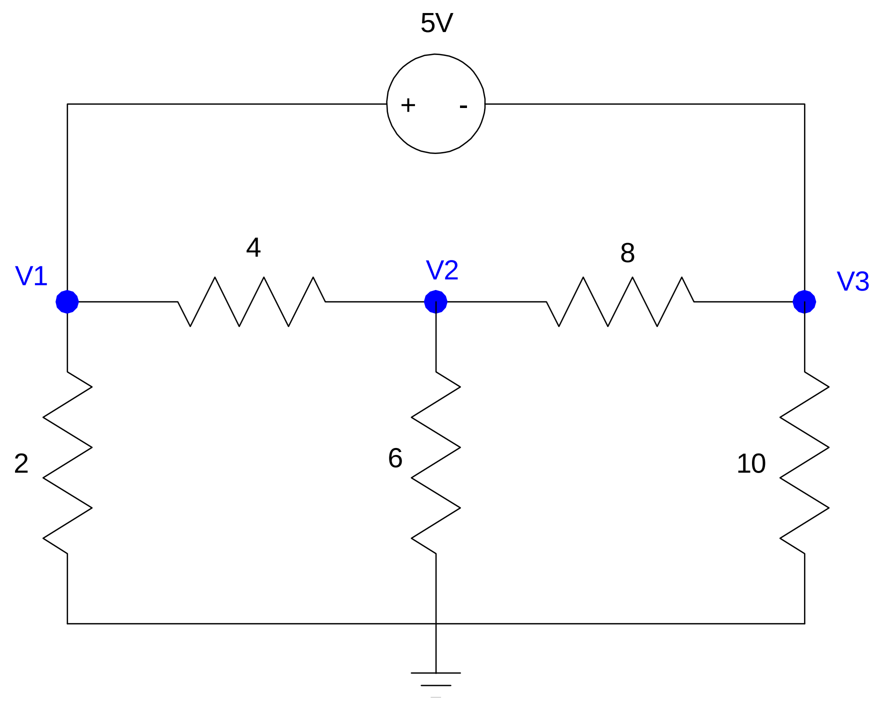
Why?

- Sometimes you can't sum the current to zero at each node
  - A voltage source is connected to the node
  - It supplies whatever current is needed to maintain the voltage
- In that case, use a SuperNode

SuperNode Example:

Find V1, V2, and V3

- Need 3 equations for 3 unknowns



Voltage Source

$$V_1 - V_3 = 5 \qquad (1)$$

Node V2

$$\frac{V_2 - V_1}{4} + \frac{V_2}{6} + \frac{V_2 - V_3}{8} = 0 \qquad (2)$$

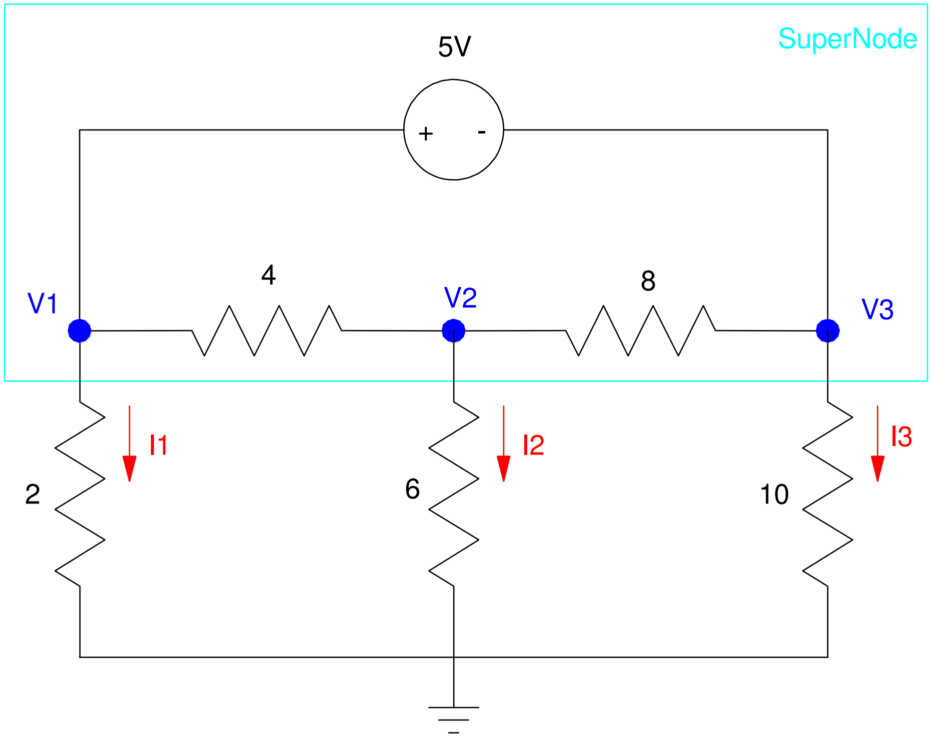
Now we're stuck.

- We can't sum the current to zero at node V1 since we don't know the current to the 5V source
- We can's sum the current to zero at node V3 since we don't know the current into the - terminal of the 5V soruce

SuperNode:

- Draw a path that encloses the 5V source and only has resistors attached to it

$$\frac{V_1}{2} + \frac{V_2}{6} + \frac{V_3}{10} = 0 \qquad (3)$$



SuperNode

- Another perfectly valid 3rd equation

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_1 - V_2}{4}\right) + \left(\frac{V_3}{10}\right) + \left(\frac{V_3 - V_2}{8}\right) = 0$$

Solve:

- Group terms:

$$V_1 - V_3 = 5$$

$$-\left(\frac{1}{4}\right)V_1 + \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right)V_2 - \left(\frac{1}{8}\right)V_3 = 0$$

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_2}{6}\right) + \left(\frac{V_3}{10}\right) = 0$$

Place in matrix form:

$$\begin{bmatrix} 1 & 0 & -1 \\ -0.25 & 0.5417 & -0.125 \\ 0.5 & 0.1666 & 0.1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
A = [1,0,-1 ; -0.25,0.5417,-0.125 ; 0.5,0.16666,0.1]

    1.0000         0   -1.0000
   -0.2500    0.5417   -0.1250
    0.5000    0.1667    0.1000

B = [5;0;0]

     5
     0
     0

V = inv(A)*B
```

```
    V1      0.9677
    V2     -0.4838
    V3     -4.0323
```

Same as CircuitLab

Voltage Nodes with Dependent Sources

- Same as voltage nodes
- Plus one equation for each dependent source

Example: Find V1, V2, V3, Vx

- 4 equations for 4 unknowns

Easy ones:

$$V_x = V_3 - V_2$$

$$V_1 = 12$$

$$V_3 = 4V_x$$

Node equation at V2

$$\left(\frac{V_2 - V_1}{2}\right) + \left(\frac{V_2}{4}\right) + \left(\frac{V_2 - V_3}{6}\right) = 0$$

Solve: Group terms

$$V_x - V_3 + V_2 = 0$$

$$V_1 = 12$$

$$V_3 - 4V_x = 0$$

$$\left(\frac{-1}{2}\right)V_1 + \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)V_2 + \left(\frac{-1}{6}\right)V_3 = 0$$

Placing in matrix form

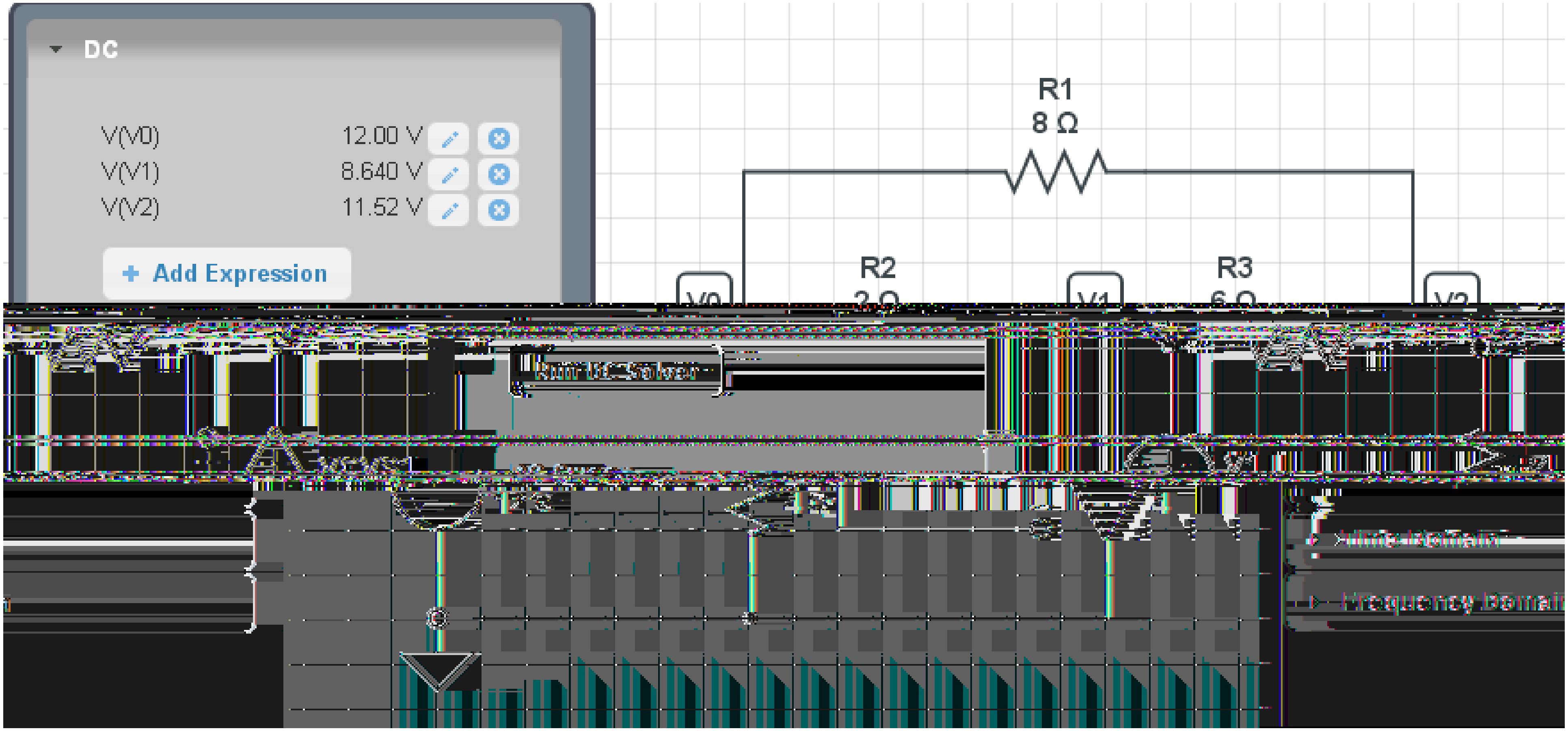
$$\begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 \\ -0.5 & 0.9167 & -0.1666 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_x \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
A = [0,1,-1,1 ; 1,0,0,0 ; 0,0,1,-4 ; -0.5,0.9167,-0.1666,0]

         0    1.0000   -1.0000    1.0000
    1.0000         0         0         0
         0         0    1.0000   -4.0000
   -0.5000    0.9167   -0.1666         0

B = [0;12;0;0]

     0
    12
     0
     0


V = inv(A)*B
```

V1   12.0000
V2   8.6385
V3   11.5180
Vx   2.8795

Checking in Circuitlab - the voltages match



SuperNodes and Dependent Sources

- If needed, define a closed-path (i.e. a SuperNode) to give the rest of the N equtions needed

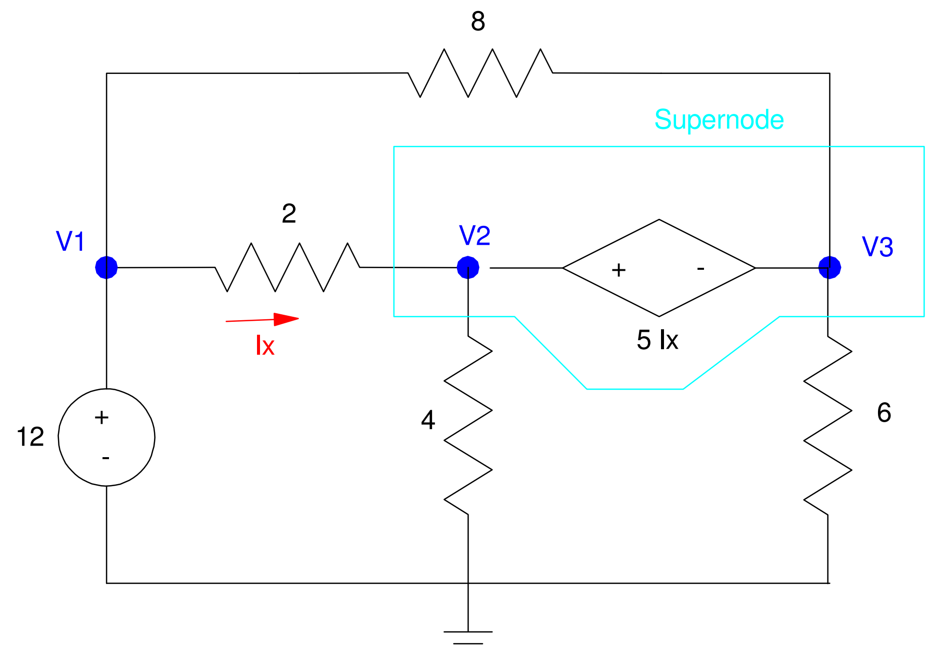
Example: Find { V1, V2, V3, Ix }

Easy Equations:

$$I_x = \left(\frac{V_1 - V_2}{2}\right)$$

$$V_1 = 12$$

$$V_2 - V_3 = 5I_x$$



Define a SuperNode

- Current out of the SuperNode = 0

$$\left(\frac{V_2 - V_1}{2}\right) + \left(\frac{V_2}{4}\right) + \left(\frac{V_3}{6}\right) + \left(\frac{V_3 - V_1}{8}\right) = 0$$

Group Terms

$$V_1 - V_2 - 2I_x = 0$$

$$V_1 = 12$$

$$V_2 - V_3 - 5I_x = 0$$

$$\left(\frac{-1}{2} + \frac{-1}{8}\right)V_1 + \left(\frac{1}{2} + \frac{1}{4}\right)V_2 + \left(\frac{1}{6} + \frac{1}{8}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & -1 & 0 & -2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -5 \\ -0.625 & 0.75 & 0.2917 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
A = [1,-1,0,-2 ; 1,0,0,0 ; 0,1,-1,-5 ; -0.625,0.75,0.2917,0]

    1.0000   -1.0000         0   -2.0000
    1.0000         0         0         0
         0    1.0000   -1.0000   -5.0000
   -0.6250    0.7500    0.2917         0

B = [0;12;0;0]

     0
    12
     0
     0

V = inv(A)*B
```

V1 12.0000
V2 9.1764
V3 2.1175
Ix 1.4118

Verify using Circuitlab