Superposition (take 2)

Superposition allows you to analyze circuits with multiple sinusoidal inputs. If this is the case

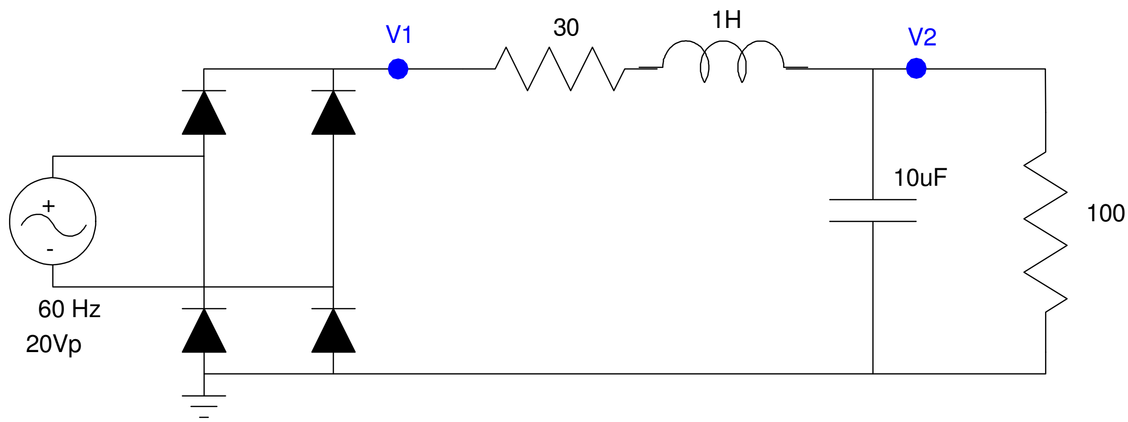
- Treat the problem as N separate problems, each with a single sinusoidal input.
- Solve each of the N problems separately using phasor analysis
- Add up all of the answers to get the total output.

Suppose your circuit has an input that *isn't* a sum of sinusoids. A typical engineering solution is to change the problem so that the inputs *are* sinusoids. The trick is you want to change the problem so that

- It is solvable (a big plus), and
- It keeps the flavor of the original problem.

Example 1: AC to DC Converter

The following circuit is an AC to DC converter that we'll cover in ECE 320 Electronics I. Determine the voltage at V2:



AC to DC Converter covered in ECE 320 Electronics I

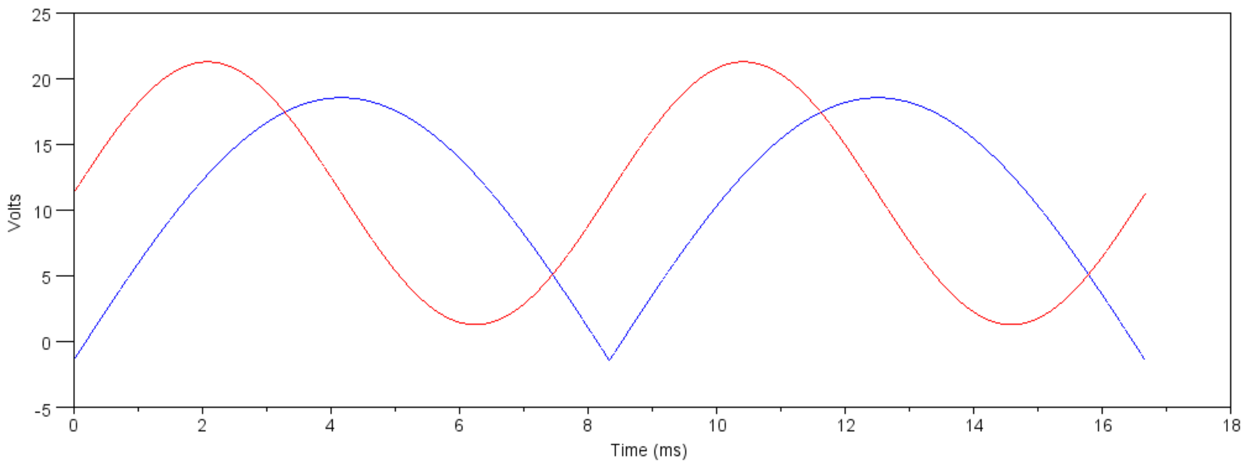
This is actually a difficult problem. From PartSim, the voltage at V1 (on the left) looks like a full-wave rectified sine wave (see figure below). We don't have tools to analyze a circuit with such an input. So, change the problem. Assume instead that the input

- Is a DC signal 11.31V on average (the same as the average of a 20V sine wave which has been shifted down by 1.4V)
- The frequency of AC signal is 120Hz (same as the actual voltage)
- The amplitude of the AC signal is 20Vpp (same as the actual voltage).

So, pretend instead that the voltage V1 is

$$V_1(t) = 11.31 + 10\cos(754t)$$

This isn't 100% correct, but it keeps the flavor of the problem (same DC signal, same frequency, same peak-to-peak value in voltage).



Actual Signal at V1 (blue) and its approximation (red)
Both signals have the same frequency, DC value, and peak-to-peak voltage.

With this approximate input, now calculate V2

DC Analysis: The capacitor is open and the inductor is a short. By voltage division

$$V_2 = \left(\frac{100}{100+30}\right) 11.31V$$

$$V_2 = 8.70V$$

AC Analysis: The capacitor and inductor become

$$L \rightarrow j\omega L = j754\Omega$$

$$C \rightarrow \frac{1}{j\omega C} = -j132.6\Omega$$

Adding the capacitor and 100 Ohm resistor in parallel

$$-j132.6\Omega \,||\, 100\Omega = 63.75 - j48.07$$

By voltage division, V2 is then

$$V_2 = \left(\frac{(63.75-j48.07)}{(63.75-j48.07)+(30+j754)}\right) \cdot (10 + j0)$$

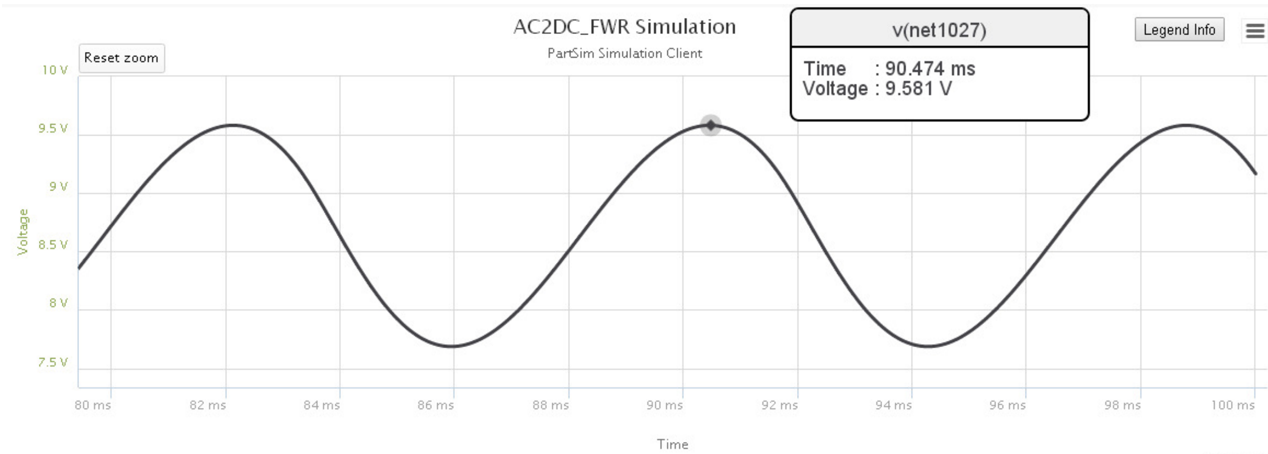
$$V_2 = -0.551 - j0.976$$

The peak-to-peak voltage at V2 will be twice the magnitude

$$V_{2pp} = 2 \cdot |-0.551 - j0.976|$$

$$V_{2pp} = 2.242 V_{pp}$$

The actual voltage at V2 (from PartSim) look like this:



	Calculated V2	PartSim V2
DC Value	8.70 V	8.634 V
AC Value	2.242 Vpp	1.895 Vpp

Note:

- The answers are fairly close. We kept the flavor of the problem
- The answers are a little off. This isn't surprising since the input isn't a pure sine wave like assumed.

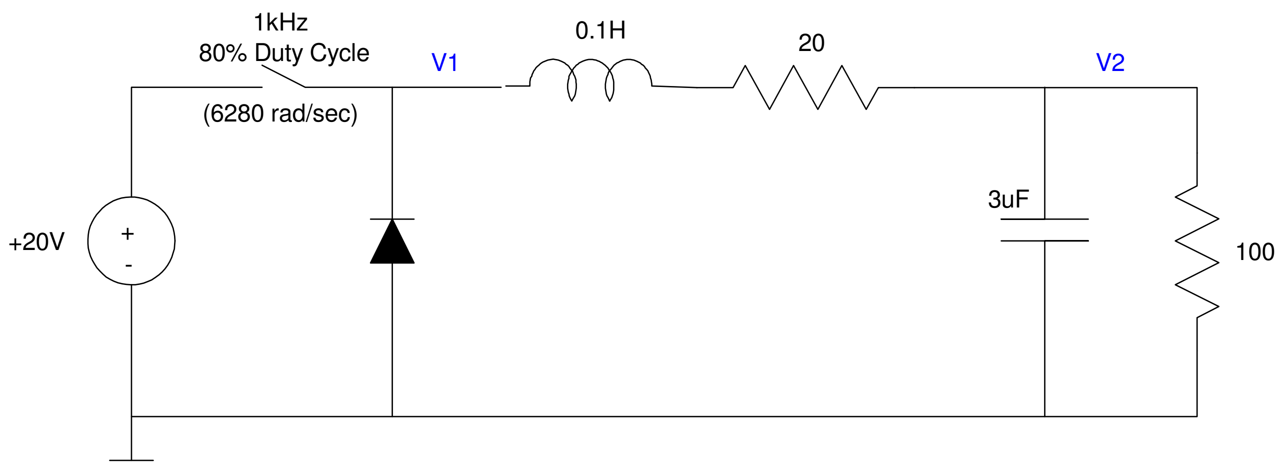
Also note that even though the input is *very* different from a sine wave, the output is almost a pure sine wave with a DC offset. This means that treating this as a superposition problem with two terms

- A DC term, and
- A 120Hz term

was a pretty good assumption.

Example 2: Buck Converter

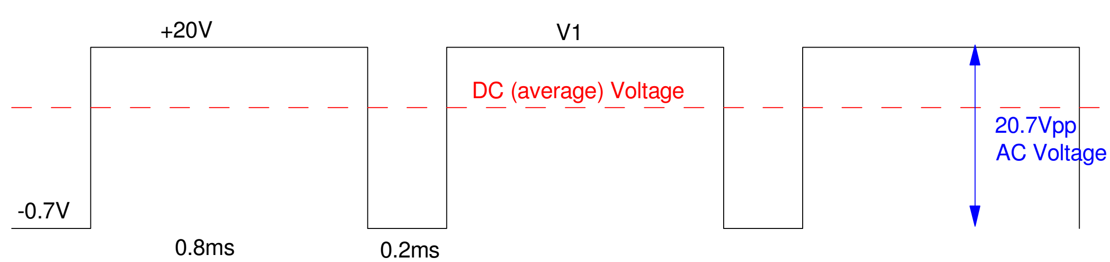
Another circuit we cover in Electronics I is a Buck Converter. This circuit converts a DC signal (such as a 20.0V DC signal from a battery) to a different voltage, such as 5V to drive your electronics. To do this, an electronic switch chatters on and off at 1kHz so that the average voltage at V1 is what you need to make the output 5.00V. The inductor and capacitor filter the signal to remove the chatter at the switching frequency (1kHz in this case)



Example 2: A Buck Converter (covered in ECE 320 Electronics I)

Determine the signal at V2.

Again, this is a difficult problem. The signal at V1 looks like the following:



Voltage at V1 with a Buck Converter

This isn't a sine wave, so change the problem so that

- V1 is a DC term plus a sine wave, while
- Keeping the flavor of the problem.

The DC voltage is the average of V1:

$$V_1 = 0.8 \cdot 20V + 0.2 \cdot (-0.7V)$$

$$V_1 = 15.86V$$

The AC voltage at V1 is

- 1KHz
- 20.7Vpp

So, assume

$$V_1 \approx 15.86 + \frac{20.7}{2}\cos(6280t)$$

DC Analysis:

$$V_1 = 15.86$$

The inductor is a short and the capacitor is an open circuit.

$$V_2 = \left(\frac{100}{100+20}\right) V_1$$

$$V_2 = 13.21V$$

AC Analysis

$$V_1 = 10.35\cos(6280t) \Rightarrow 10.35 + j0$$

$$\omega = 6280 \qquad (\text{ 1kHz })$$

$$L \rightarrow j\omega L = j628\Omega$$

$$C \rightarrow \frac{1}{j\omega C} = -j53.1\Omega$$

Adding the 100 Ohm resistor and capacitor in parallel

$$100\Omega \,||\, -j53.1\Omega = 21.98 - j41.41$$

From voltage division

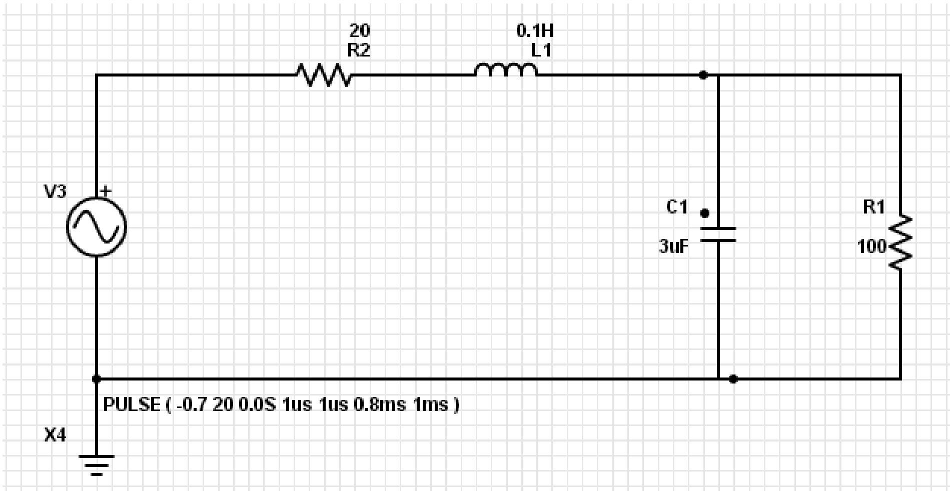
$$V_2 = \left(\frac{(21.98-j41.41)}{(21.98-j41.41)+(20+j628)}\right)(10.35 + j0)$$

$$V_2 = -0.699 - j0.438 = 0.825\angle -148^0$$

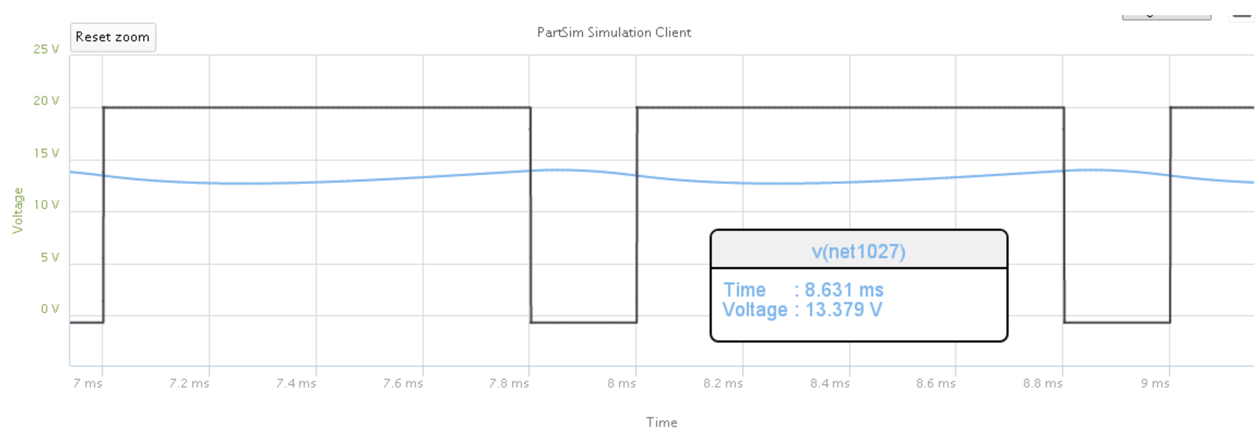
This tells you that

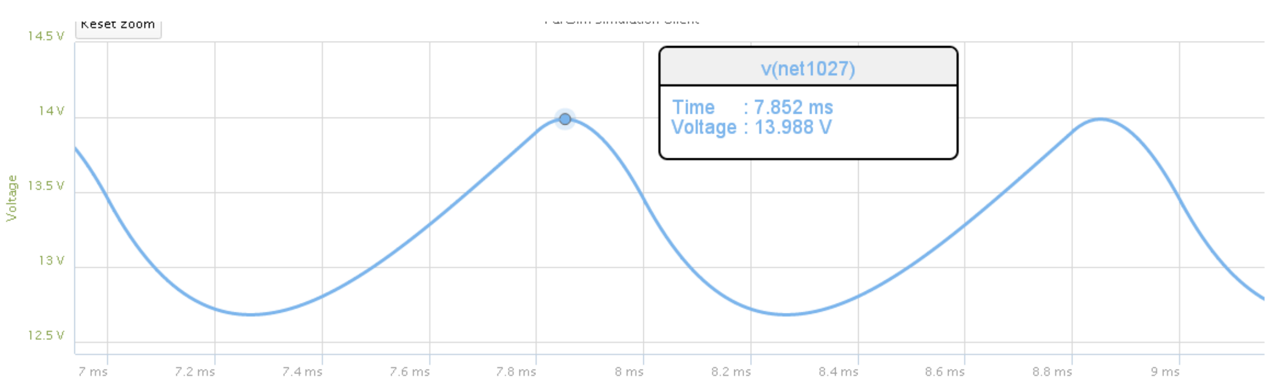
$$V_2 \approx 13.21 + 0.825\cos(6280t - 148^0)$$

From PartSim, the circuit is



The simulation results are





Comparing the simulated vs. calculated results

	Calculated V2	PartSim V2
DC Value	13.21 V	13.32 V
AC Value	1.65 Vpp	1.285 Vpp

Again, our results are close. By changing the problem

- We were able to solve for V2
- Without significantly changing the results (keeping the flavor of the problem)