Superposition with Phasors

Op-Amp circutis with RLC components are linear circuits. *Linear* means

$$f(ax+by) = af(x) + bf(y)$$

In English, this means that

- If the input has N terms,
- You can treat this as N separate problems.

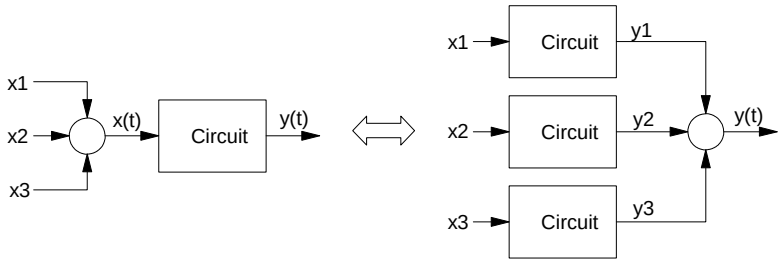
Solve the output for each term separately.

- To get the total input, you add all N input terms together.
- To get the total output, you add all N output terms together.

Pictorially, this means that

- If you have a single circuit with multiple inputs (which is how you'd build the circuit),
- You can treat this as if there were multiple circuits, each with a single input.

The total output will be the sum of each separate input.

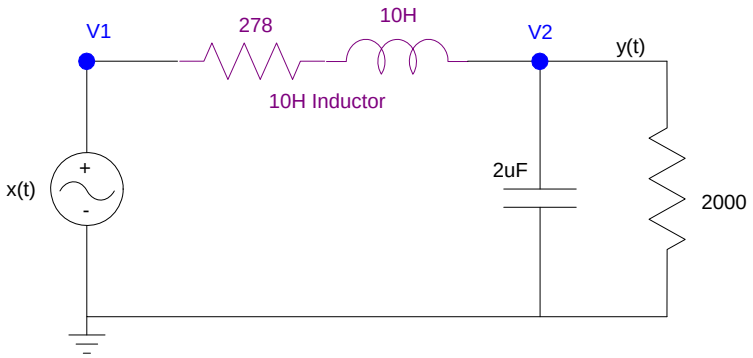


Superposition allows you to treat a circuit with multiple inputs as if there were multiple circuits, each with a single input.

Example: RLC Filter

For example, determing y(t) for the following circuit assuming

$$x(t) = 10 + 9\sin(100t) + 8\cos(200t)$$



Solution: Use superposition and treat this as three separate problems.

Part 1 (DC). Assume

$$x_1(t) = 10$$

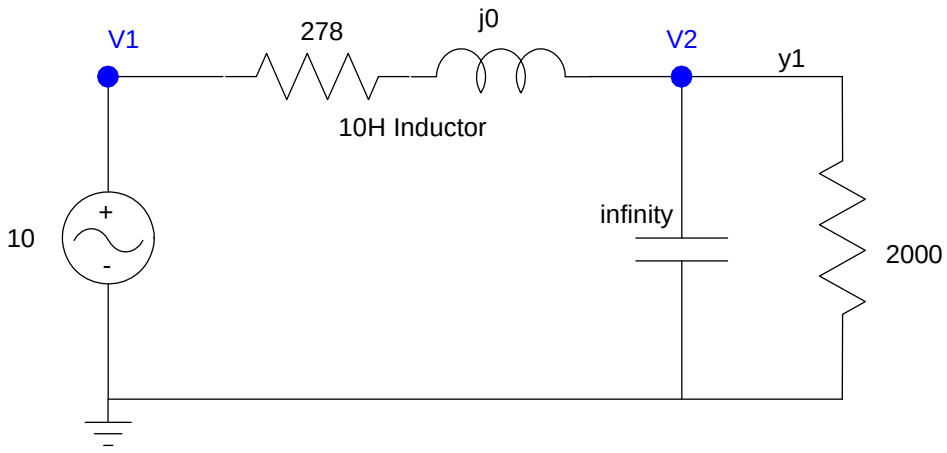
Analyze the circuit for this input.

$$\omega = 0$$

$$L \rightarrow j\omega L = 0$$

$$C \rightarrow \frac{1}{j\omega C} = \infty$$

The circuit then becomes



Equivalent Circuit for x(t) = 10

From voltage division

$$y_1 = \left(\frac{2000}{2000+278}\right) \cdot 10$$

$$y_1 = 8.7796V$$

Part 2: Assume

$$x_2(t) = 9\sin(100t)$$

Using Phasor analysis

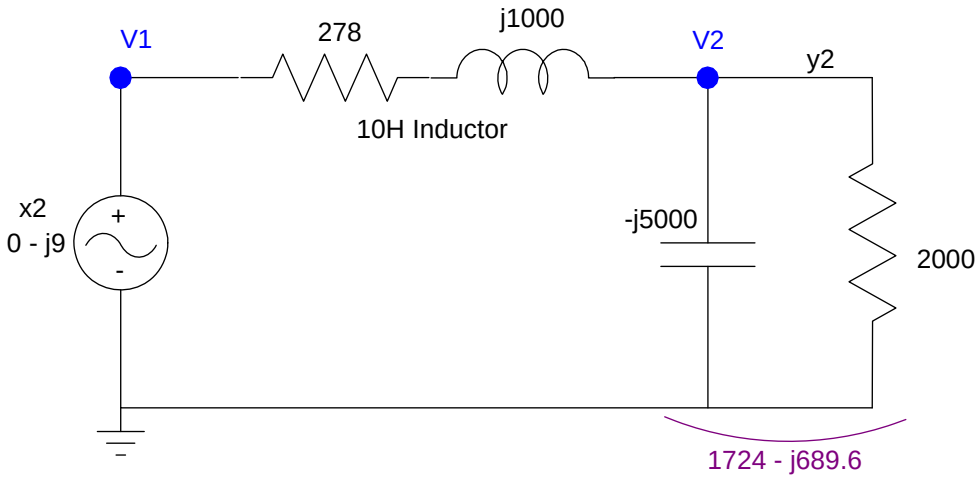
$$\omega = 100$$

$$X = 0 - j9$$

$$L \rightarrow j\omega L = j1000$$

$$C \rightarrow \frac{1}{j\omega C} = -j5000$$

The circuit at 100 rad/sec becomes:



Using phasor analysis, the capacitor and 2000 resistor in parallel are

$$\left(\frac{1}{2000} + \frac{1}{-j5000}\right)^{-1} = 1724 - j689.6\Omega$$

By voltage division

$$Y_2 = \left(\frac{(1724 - j689.6)}{(1724 - j689.6) + (278 + j1000)}\right) \cdot (0 - j9)$$

$$Y_2 = -4.201 - j7.099$$

Converting back to time

$$y_2(t) = -4.201\cos(100t) + 7.099\sin(100t)$$

Part 3: Assume

$$x_3(t) = 8\cos(200t)$$

Using Phasor analysis

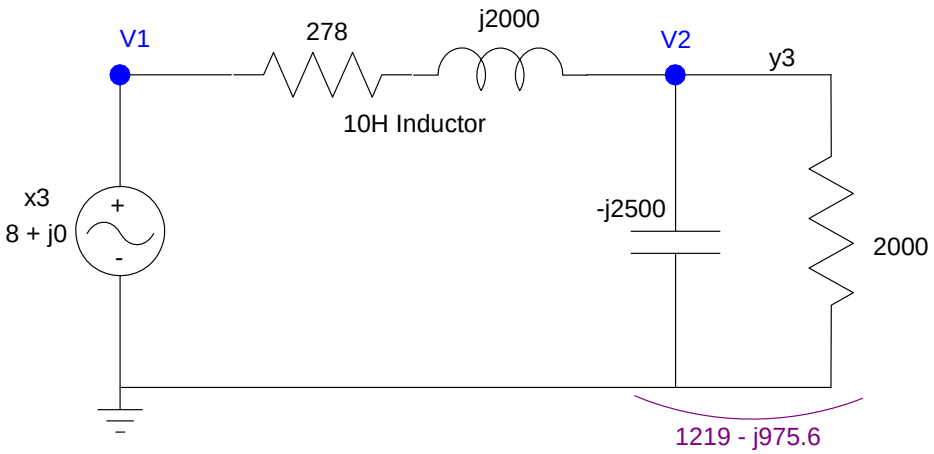
$$\omega = 200$$

$$X = 8 + j0$$

$$L \rightarrow j\omega L = j2000$$

$$C \rightarrow \frac{1}{j\omega C} = -j2500$$

The circuit at 200 rad/sec becomes:



The 2000 resistor and capacitor in parallel become

$$\left(\frac{1}{2000} + \frac{1}{-j2500}\right)^{-1} = 1219 - j975.6$$

By voltage division

$$Y = \left(\frac{1219 - j975.6}{(1219 - j975.6) + (278 - j2000)}\right) \cdot (8 + j0)$$

$$Y = 2.009 - j6.586$$

meaning

$$y_3(t) = 2.009\cos(200t) + 6.586\sin(200t)$$

The total answer is the sum of the three parts

$$y(t) = y_1 + y_2 + y_3$$

$$y(t) = 8.7796 - 4.201\cos(100t) + 7.099\sin(100t) + 2.009\cos(200t) + 6.586\sin(200t)$$