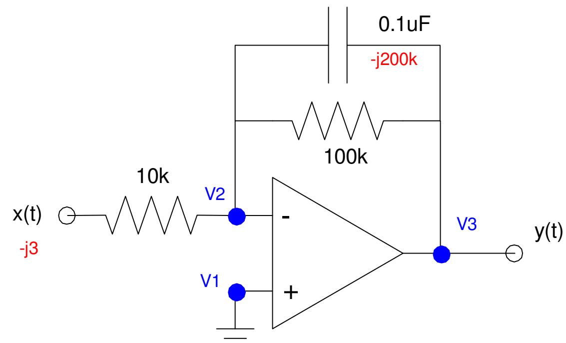
# Op-Amp Circuits with Phasors

## Single-Pole Low-Pass Filter

Find the voltage, y(t), for

$$x(t) = 3\sin(50t)$$




Solution: Convert to phasor notation

$$3\sin(50t) \rightarrow 0 - j3$$

$$0.1\mu F \rightarrow \frac{1}{j\omega C} = -j200k$$

Write the voltage node equations. At node V3, V+ = V-

$$V_2 = V_1 \qquad (1)$$

$$V_1 = 0 \qquad (2)$$

Sum the current to zero at node V2

$$\left(\frac{V_2-(-j3)}{10k}\right) + \left(\frac{V_2-V_3}{100k}\right) + \left(\frac{V_2-V_3}{-j200k}\right) = 0 \qquad (3)$$

Solve. Plug in V1 = V2 = 0 into (3)

$$\left(\frac{0-(-j3)}{10k}\right) + \left(\frac{0-V_3}{100k}\right) + \left(\frac{0-V_3}{-j200k}\right) = 0$$

$$\left(\frac{-1}{100k} + \frac{-1}{-j200k}\right) V_3 = \left(\frac{-j3}{10k}\right)$$

$$V_3 = 12 + j24 = 26.8\angle 63^0$$

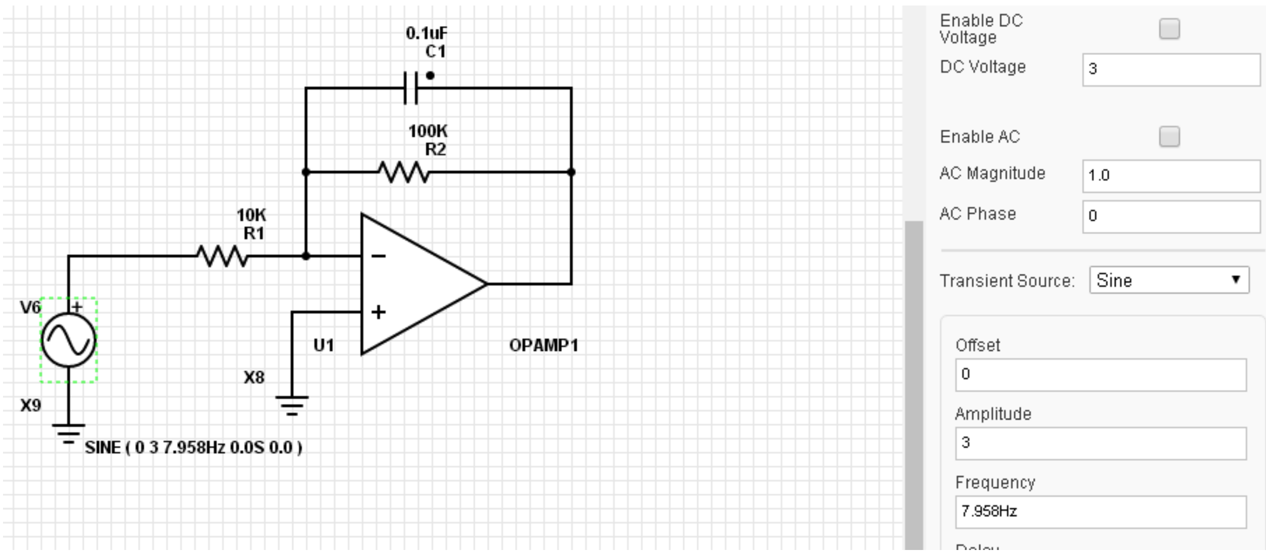
meaning

$$v_3(t) = 12\cos(50t) - 24\sin(50t)$$

$$v_3(t) = 26.8\cos(50t + 63^0)$$

Checking in PartSim: The input is

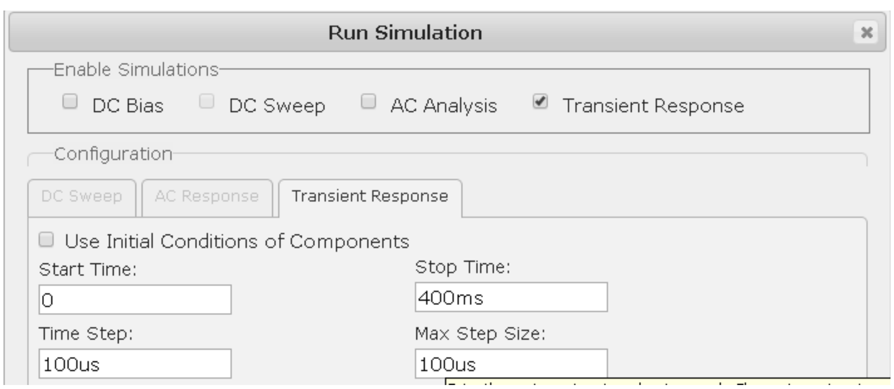
$$Hz = \frac{\omega}{2\pi} = \frac{50 \text{ rad/sec}}{2\pi} = 7.958Hz$$



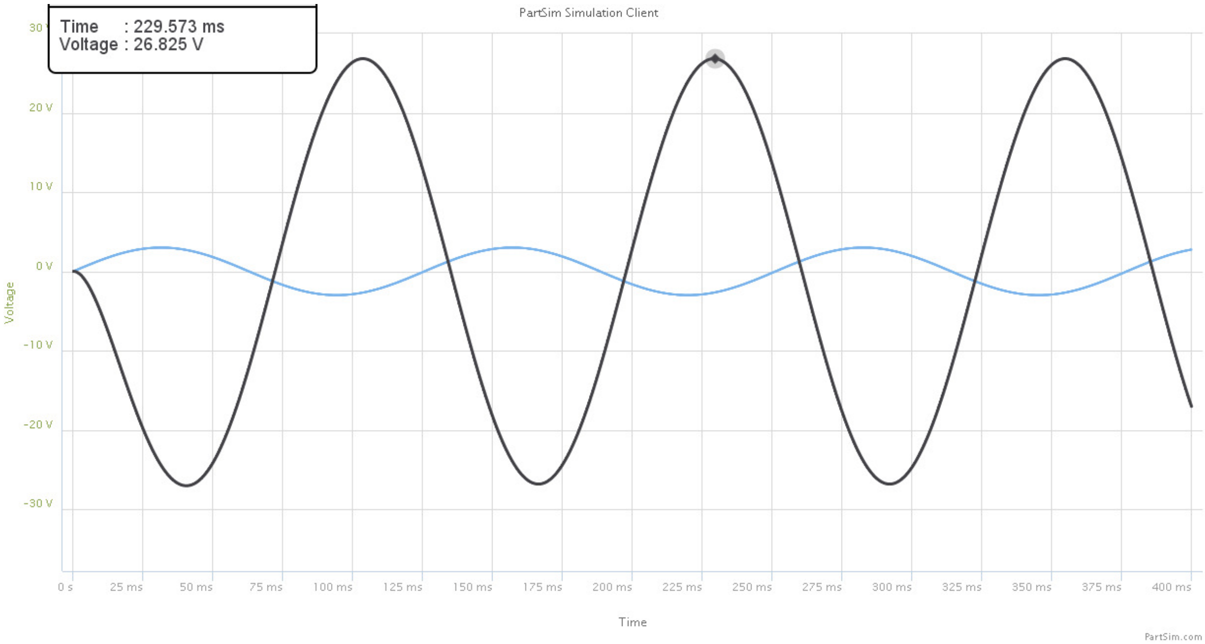

To see the output, select the probes to be the input and the output.

Run a transient simulation for 3 cycles

$$t_{\max} = 3T = \frac{3}{f} = \frac{3}{7.958Hz} = 377ms$$




This results in




Note:

- The period is 126ms (7.958Hz = 50 rad/sec)
- The peak is 26.825V ( vs. 26.8V calculated )
- The peak for the output ( cos(0) ) is 22ms ahead of the zero crossing for the input ( sin(0) ). This works out to

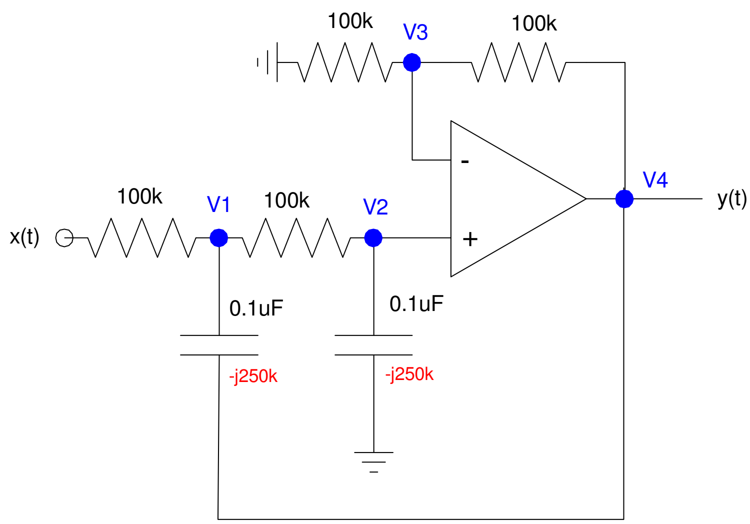
$$\phi = \left(\frac{\text{22ms time lead}}{\text{126ms period}}\right) \cdot 360^0 = 62.8^0 \text{ phase shift ( vs. 63 degrees computed )}$$

Also note that if you change the frequency of x(t), you have to resolve the entire problem.

## Example 2: Two-Pole Op-Amp Circuit

Find y(t) for

$$x(t) = 3\cos(40t)$$




Step 1: Convert to phasors

$$3\cos(40t) \rightarrow 3 + j0$$

$$0.1\mu F \rightarrow \frac{1}{j\omega C} = -j250k$$

Step 2: Write the voltage node equations. With 4 nodes, we need 4 equations. Start with the easy one: for negative feedback, V+ = V-

$$V_2 = V_3 \qquad (1)$$

Now write three more

$$\left(\frac{V_1-3}{100k}\right) + \left(\frac{V_1-V_4}{-j250k}\right) + \left(\frac{V_1-V_2}{100k}\right) = 0 \qquad (2)$$

$$\left(\frac{V_2-V_1}{100k}\right) + \left(\frac{V_2-0}{-j250k}\right) = 0 \qquad (3)$$

$$\left(\frac{V_3-0}{100k}\right) + \left(\frac{V_3-V_4}{100k}\right) = 0 \qquad (4)$$

Step 3: Solve. Group terms

$$V_2 - V_3 = 0$$

$$\left(\frac{1}{100k} + \frac{1}{-j250k} + \frac{1}{100k}\right)V_1 - \left(\frac{1}{100k}\right)V_2 - \left(\frac{1}{-j250k}\right)V_4 = \left(\frac{3}{100k}\right)$$

$$\left(\frac{-1}{100k}\right)V_1 + \left(\frac{1}{100k} + \frac{1}{-j250k}\right)V_2 = 0$$

$$\left(\frac{1}{100k} + \frac{1}{100k}\right)V_3 + \left(\frac{-1}{100k}\right)V_4 = 0$$

Place in matrix form

$$\begin{bmatrix} 0 & 1 & -1 & 0 \\ \left(\frac{1}{100k} + \frac{1}{-j250k} + \frac{1}{100k}\right) & \left(\frac{-1}{100k}\right) & 0 & \left(\frac{-1}{-j250k}\right) \\ \left(\frac{-1}{100k}\right) & \left(\frac{1}{100k} + \frac{1}{-j250k}\right) & 0 & 0 \\ 0 & 0 & \left(\frac{1}{100k} + \frac{1}{100k}\right) & \left(\frac{-1}{100k}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \left(\frac{3}{100k}\right) \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

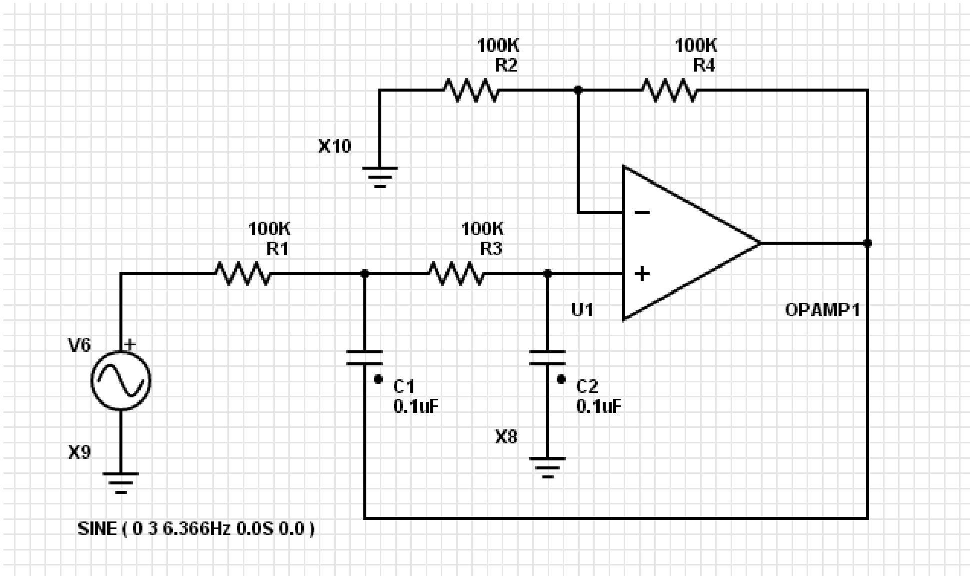
```
a1 = [0,1,-1,0];
a2 = [1/100000 + 1/(-j*250000) + 1/100000, -1/100000, 0, 1/(j*250000)]
a3 = [-1/100000, 1/100000 + 1/(-j*250000), 0, 0]
a4 = [0,0,2/100000, -1/100000]

A = [a1;a2;a3;a4]

    0                        1.                    - 1.         0
    0.00002 + 0.000004i    - 0.00001                 0        - 0.000004i
  - 0.00001                  0.00001 + 0.000004i     0          0
    0                        0                       0.00002  - 0.00001

B = [0;3/100000;0;0]

    0.
    0.00003
    0.
    0.

V = inv(A)*B

V1    3.4658041 - 0.2218115i
V2   2.9112754 - 1.3863216i
V3    2.9112754 - 1.3863216i
V4    5.8225508 - 2.7726433i
```

So

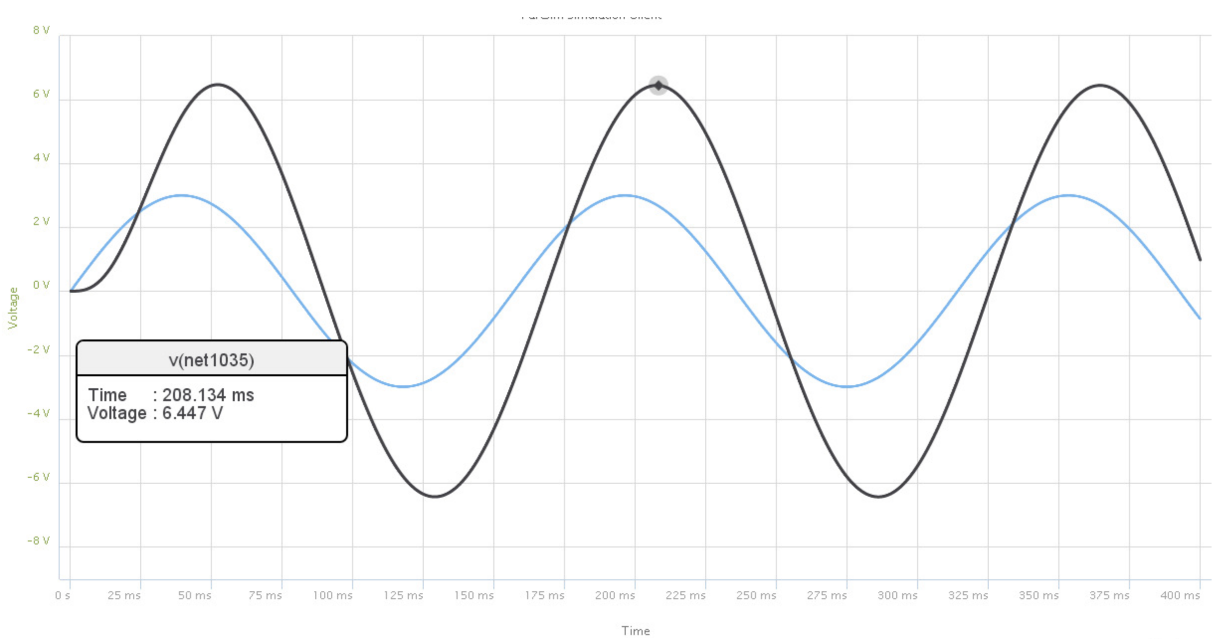
$$v_4(t) = 5.822\cos(40t) + 2.77\sin(40t) \qquad \text{rectangular form}$$

$$v_4(t) = 6.447\cos(40t - 25^0) \qquad \text{polar form}$$

Checking in PartSim: Note that 40 rad/sec = 6.366Hz




Running a transient simulation for 400ms (about 3 cycles)




The output peak is 6.447V ( vs. 6.447V computed)

The output is delayed by 9.9ms from the input. The phase shift is

$$\phi = \left(\frac{9.9\text{ms delay}}{157\text{ms period}}\right) \cdot 360^0 = 22.7^0 \text{ delay} \qquad \text{( vs. 25 degrees computed )}$$