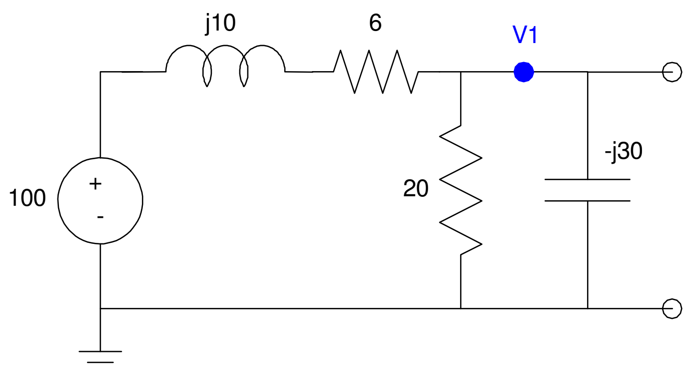
Thevenin Equivalents with Phasors

	VI relationship	Phasor Notation
Voltage	$v(t) = a\cos(\omega t) + b\sin(\omega t)$	$V = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L\frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C\frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

Thevenin equivalents also work with phasors - only you get complex numbers for the Thevenin voltage and Thevenin resistance.

Example 1: Determine

- The Thevenin equivalent for the following circuit,
- ZL for max power transfer, and
- The maximum power to a load



Solution: Combine the 20 Ohms and -j30 Ohms in parallel:

$$20 || -j30 = (13.846 - j9.231)\Omega$$

The Thevenin voltage by voltage division is

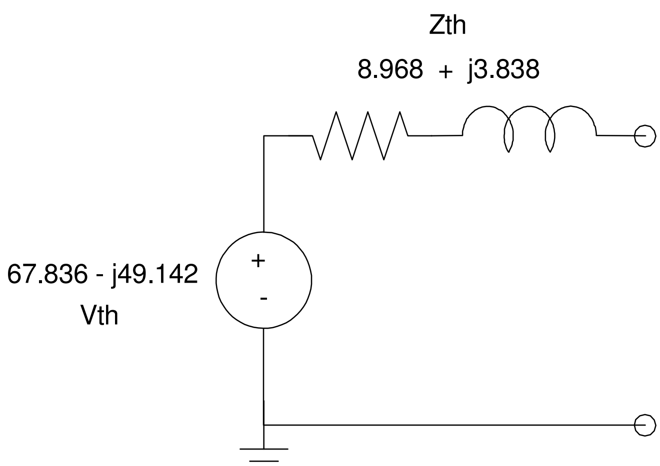
$$V_{th} = \left(\frac{(13.846 - j9.231)}{(13.846 - j9.231) + (6 + j10)}\right) 100 = 67.836 - j49.142$$

The Thevenin resistance is (turn off the voltage source and measure the resistance looking in:

$$Z_{th} = (-j30)||(20)||(6 + j10)$$

$$Z_{th} = 8.968 + j3.838$$

So the Thevenin equivalent is



Thevenin Equivalent: Same as before only now with complex numbers

AC Power

At DC, power is

$$P = VI = \frac{V^2}{R} = I^2 R$$

For AC, it's slightly different. Recall that AC voltages can be written three differnt ways

Vp = peak voltage

Vpp = peak-to-peak voltage

Vrms = rms voltage (DC equivalent voltage)

Power for AC deals with complex numbers for V, I, and Z. The general equation for AC power is

$$P = V_{rms} \cdot I_{rms}^* = \frac{1}{2} V_p \cdot I_p$$

P has two parts: real and complex

- The real part of P is the work done (or heat produced),
- The complex part of P is the energy that bounces back and forth (usually between inductors and capacitors).

Substituting

$$V = I \cdot Z$$

you get

$$P = Z \cdot I_{rms} \cdot I^*_{rms} = \frac{1}{2} \cdot Z \cdot I_p \cdot I^*_p$$

$$P = |I_{rms}|^2 \cdot Z = \frac{1}{2}|I_p|^2 \cdot Z$$

AC Power		
$P = V_{rms} \cdot I^*_{rms}$	$P = \lvert I_{rms}\rvert^2 \cdot Z$	$P = \frac{\lvert V_{rms}\rvert^2}{Z^*}$

The power to the load is the real part of P. Just like the DC case, this is a maximum when

$$real(Z_L) = real(Z_{th})$$

The complex part of the impedance reduces the current to the load. The maximum current is when the complex parts cancel, i.e.

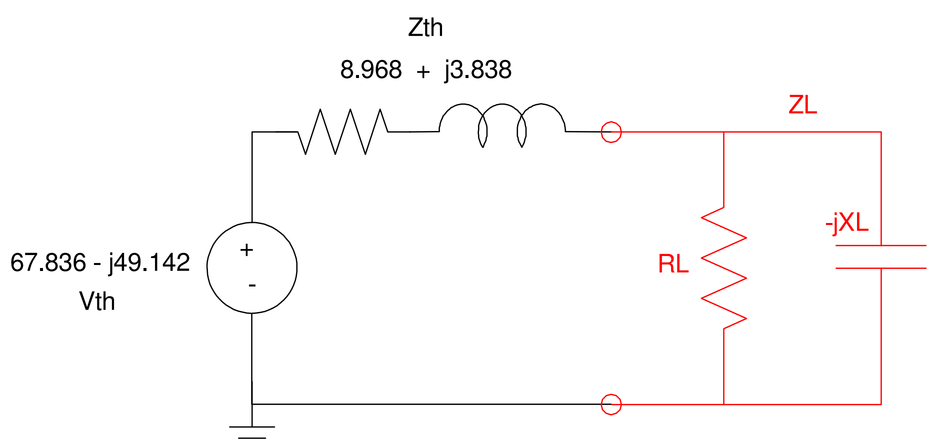
$$imag(Z_L) = -imag(Z_L)$$

The net result is

The maximum power to the load is when $Z_L = Z_{th}^*$

Example: Determine

- The load, ZL, which maximimizes the power to the load, and
- The power to the load (real and complex power)



Solution: The load should be the complex conjugate of Zth

$$Z_L = (8.968 + j3.838)^*$$

$$Z_L = 8.968 - j3.838$$

To find RL and jXL, add the inverses (since they are in parallel):

$$\frac{1}{Z_L} = \frac{1}{R_L} + \frac{1}{-jX_L}$$

$$R_L = 10.611\Omega$$

$$-jX_L = -j24.793\Omega$$

The power to the load is then

$$V_L = \left(\frac{(8.968-j3.838)}{(8.969-j3.838)+(8.968+j3.838)}\right) \cdot (67.836-j49.142)$$

$$V_L = 23.402-j39.087$$

Assuming units are rms:

$$P = \frac{|V_{rms}|^2}{Z^*} = \frac{|23.402-j39.087|}{(8.969-j3.838)^*}$$

$$= \frac{(45.557)^2}{8.969+j3.838}$$

$$= 4.293-j1.837 \quad \text{Watts}$$

Example 2: "Capacitors add Voltage"

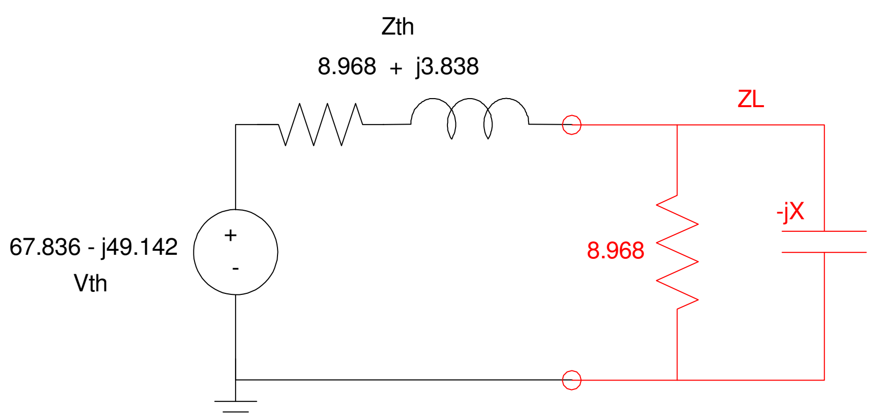
A saying you'll hear in the power industry is "capacitors add voltage". Utilities often add capacitors at the end of a transmission line to bring the voltage up. This works due to the load that utilities see are often inductive (all the motors attached to the line add +jX reactance). This is similar to the previous example.

By adding capacitors to the load,

- The +jX reactance is reduced, which
- Increases the current to the load, which
- Increases the voltage at the load.

This only works up to a point: once you have cancelled all of the inductance (+jX), adding more capacitors will actually redice the voltage.

For example, determine the voltage of the following circuit vs. -jX:



In Matlab:

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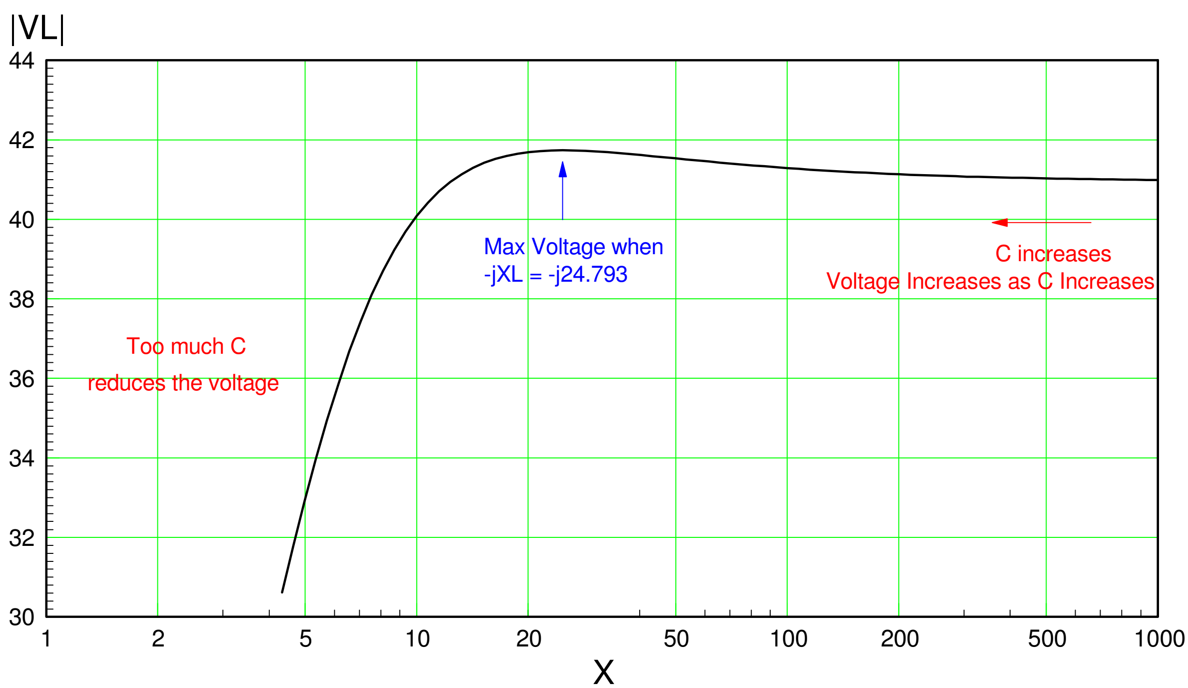
X = logspace(0,3,100)';
Vth = 67.836 - j*49.142
Zth = 8.968 + j*3.838;

VL = 0*X;

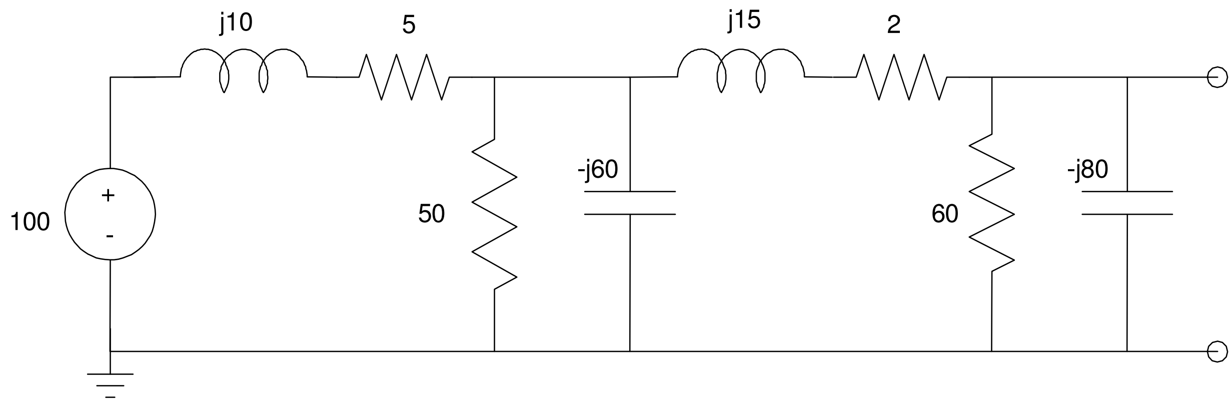
for i=1:length(X)
  ZL = 1 / ( 1/8.968 + 1/(-j*X(i)));
   VL(i) = abs( ( ZL / (ZL + Zth) ) * Vth );
   end

plot(X, VL);
```



Example 3: Source Transformations

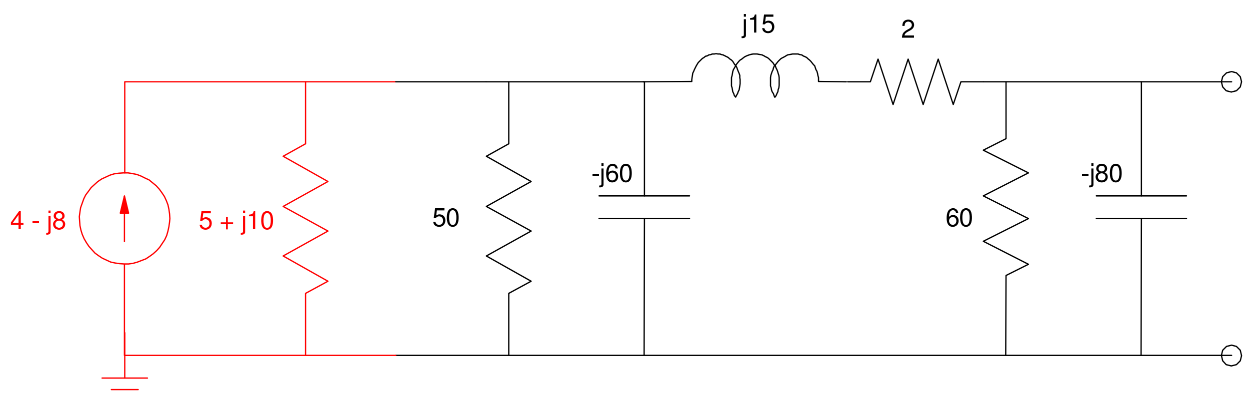
Source transformations also work with complex numbers. For example, determine the Thevenin equivalent for the following circuit:



Step 1: Convert to a Norton equivalent

$$Z_N = Z_{th} = 5 + j10$$

$$I_N = \frac{V_{th}}{Z_{th}} = 4 - j8$$



Combine impedances in parallel

$$(5 + j10)||(50)||(-j60) = 7.883 + j8.321$$

Convert to Thevenin

$$Z_{th} = Z_N = 7.883 + j8.321$$

$$V_{th} = I_N \cdot Z_N = (4 - j8) \cdot (7.883 + j8.321)$$

$$V_{th} = 98.102 - j29.781$$

Now find the Thevenin equivalent.

By voltage division:

$$V_{th} = \left(\frac{(38.40 - j28.80)}{(38.40 - j28.80) + (7.883 - j29.781) + (2 + j15)} \right) (98.102 - j29.781)$$

$$V_{th} = 68.701 - j74.405$$

Zth: Turn off the source and measure the impedance

$$Z_{th} = ((7.83 - j29.781) + (2 + j15)) || (60) || (-j80)$$

$$Z_{th} = 20.077 + j14.931$$