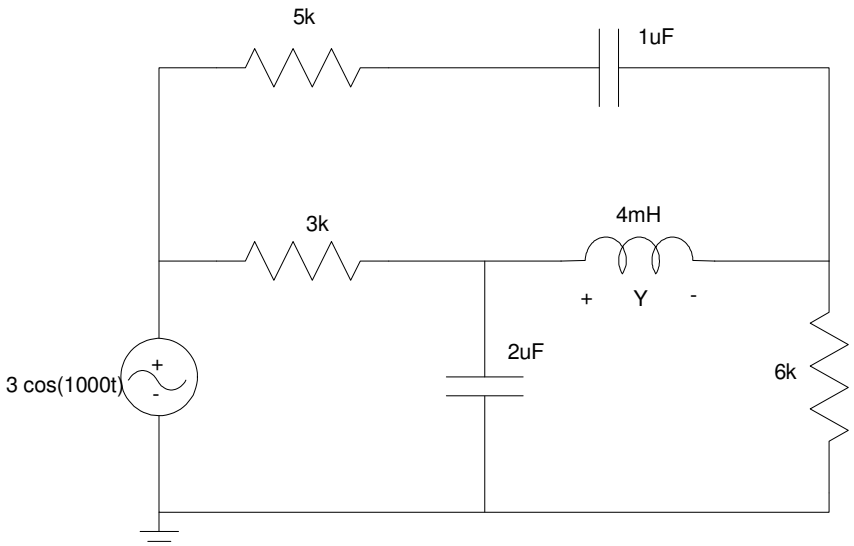
KCL with Phasors

	VI relationship	Phasor Notation
Voltage	$v(t) = a\cos(\omega t) + b\sin(\omega t)$	$V = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L\frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C\frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

Current loops also works with phasors

Example 1:

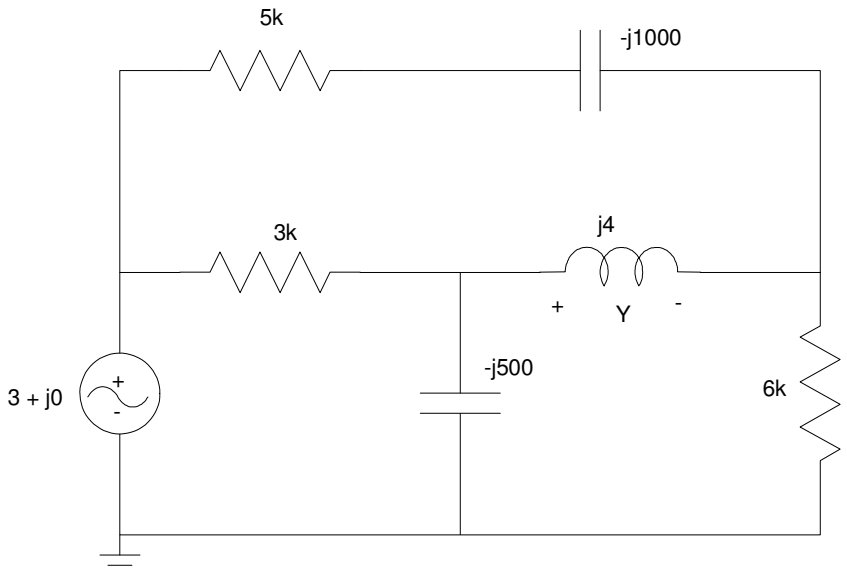


Example 1: Solve the following RLC circuits using current loops and voltage nodes.

First, convert to phaser notation. In this case,

$$\omega = 1000$$

due to the input's frequency.

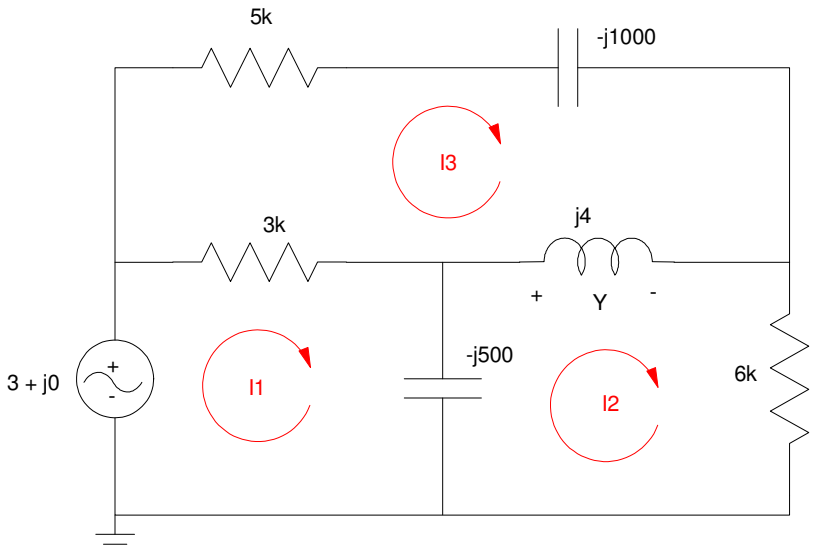


Step 1: Convert to phasor notation.

At this point, you could use current loops or voltage nodes.

Current Loops:

Using current loops, the first step is to define the loop currents (shown in blue):



Step 2 for Current Loops: Define the current loops (shown in blue)

Summing the voltage around each loop to zerod gives three equations for three unkowns:

I1: $-3+3000(I_1-I_3)-j500(I_1-I_2)=0$

I2: $-j500(I_2-I_1)+j4(I_2-I_3)+6000I_2=0$

I3: $-j1000(I_3)+5000(I_3)+j4(I_3-I_2)+3000(I_3-I_1)=0$

Grouping terms:

$$(3000-j500)I_1+(j500)I_2+(-3000)I_3=3$$

$$(j500)I_1+(6000-j496)I_2+(-j4)I_3=0$$

$$(-3000)I_1+(-j4)I_2+(8000-j996)I_3=0$$

Solving (using SciLab or MATLAB):

```
A = [3000-j*500,j*500,-3000;j*500,6000-j*496,-j*4;-3000,-j*4,8000-j*996]

    3000. - 500.i    500.i          - 3000.
    500.i            6000. - 496.i  - 4.i
  - 3000.          - 4.i              8000. - 996.i

I = inv(A) * [3;0;0]

I1:    0.0014008 + 0.0004598i
I2:    0.0000475 - 0.0001125i
I3:    0.0004962 + 0.0002342i
```

meaning (converting to mA)

$$i_1(t)=1.4008\cos(1000t)-0.4598\sin(1000t)\ \text{mA}$$

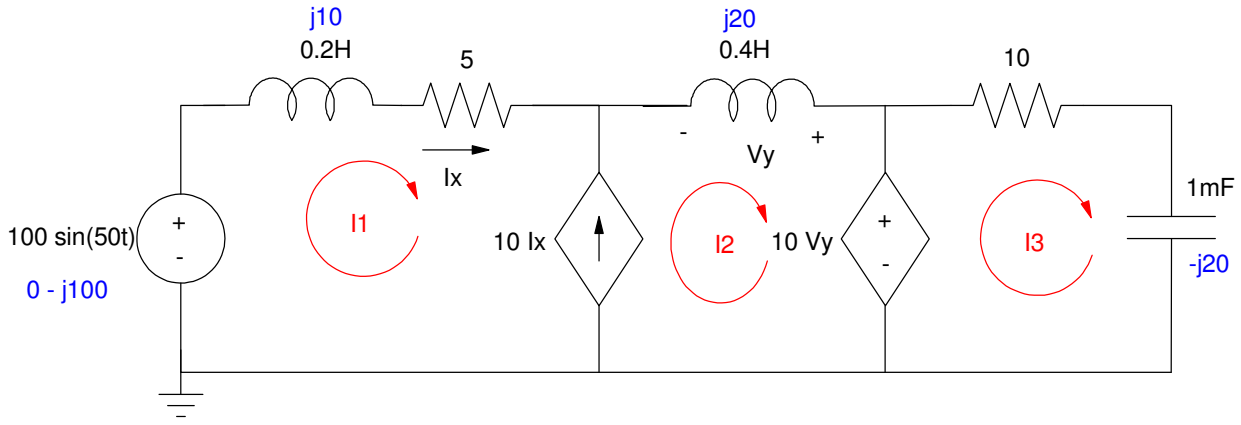
$$i_2(t)=0.0475\cos(1000t)+0.1125\sin(1000t)\ \text{mA}$$

$$i_3(t)=0.4962\cos(1000t)-0.2342\sin(1000t)\ \text{mA}$$

Current Loops with Dependent Sources

Not surprisingly, it also works with dependent sources

Example: Determine the currents, I1, I2, and I3



Step 1: Convert to the phasor domain: (shown in blue)

$$\omega = 50$$

$$100\sin(50t) \rightarrow 0 - j100$$

$$0.2H \rightarrow j\omega L = j10$$

$$0.4H \rightarrow j\omega L = j20$$

$$1mF \rightarrow \frac{1}{j\omega C} = -j20$$

Step 2: Write N equations for N uknowns

$$I_x = I_1$$

$$V_y = j20 \cdot I_2$$

$$10I_x = I_1 - I_2$$

$$-10V_y + 10I_3 - j20I_3 = 0$$

Super Loop

$$-(0 - j100) + (5 + j10)I_1 + j20 \cdot I_2 + (10 - j20)I_3 = 0$$

Group terms

$$I_x - I_1 = 0$$

$$V_y - j20 \cdot I_2 = 0$$

$$10I_x - I_1 + I_2 = 0$$

$$-10V_y + 10I_3 - j20I_3 = 0$$

$$(5 + j10)I_1 + j20 \cdot I_2 + (10 - j20)I_3 = -j100$$

Place in matrix form

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & -j20 & 0 & 0 & 1 \\ -1 & 1 & 0 & 10 & 0 \\ 0 & 0 & 10-j20 & 0 & -10 \\ (5+j10) & (j20) & (10-j20) & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_x \\ V_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -j100 \end{bmatrix}$$

Solve in Matlab

```
A = [-1,0,0,1,0 ; 0,-j*20,0,0,1];
A = [A; -1,1,0,10,0 ; 0,0,10-j*20,0,-10];
A = [A ; 5+j*10,j*20,10-j*20,0,0]

  - 1.            0       0             1.      0
    0           - 20.i    0             0       1.
  - 1.            1.      0             10.     0
    0             0       10. - 20.i    0     - 10.
    5. + 10.i     20.i    10. - 20.i    0       0

B = [0;0;0;0;-j*100]

    0
    0
    0
    0
  - 100.i
```

```
I = inv(A)*B

I1    0.0507611  -0.0001288i
I2   -0.4568498  +0.0011595i
I3    3.6501607  -1.8366755i
Ix    0.0507611  -0.0001288i
Vy   -0.0231903  -9.136997i
```

meaning

$$i_1(t) = 0.0507\cos(50t) + 0.0001288\sin(50t)$$

$$i_2(t) = -0.4568\cos(50t) - 0.0011\sin(50t)$$

$$i_3(t) = -0.0232\cos(50t) + 1.8367\sin(50t)$$