Phasor Impedances

Objective:

- Represent the impedance of a resistor, inductor, and capacitor as a complex number (termed its phasor impedance)
- Determine the impedance of RLC networks.

Resistors

From before, a current (or voltage)

$$i(t) = a\cos(\omega t) + b\sin(\omega t)$$

can be written in phasor form as

$$I = a - jb$$

The voltage produced by a current flowing through a resistor is

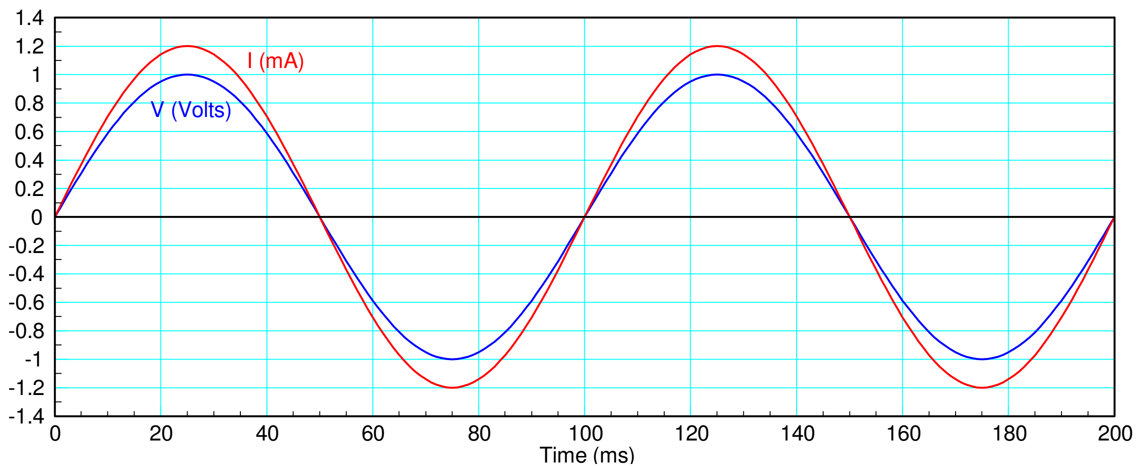
$$v(t) = i(t) \cdot R$$

or in phasor form

$$V = IR$$

The complex impedance of a resistor is R

For example: The current flowing through an 833 Ohm resistor is shown below: current is in phase with voltage for resistors.



Voltage and current are in phase for resistors

Capacitors

The current through a capacitor is

$$i(t) = C\,\frac{dv(t)}{dt}$$

If v(t) is a sinusoid:

$$v(t) = a\cos(\omega t) + b\sin(\omega t)$$

$$V = a - jb$$

then the current will be

$$i(t) = C \cdot \frac{d}{dt}(a\cos(\omega t) + b\sin(\omega t))$$

$$= C \cdot (-a\omega\sin(\omega t)) + b\omega\cos(\omega t))$$

$$I = C \cdot (ja\omega + b\omega) = j\omega C \cdot (a - jb)$$

The impedance is then the ratio:

$$Z = \frac{V}{I}$$

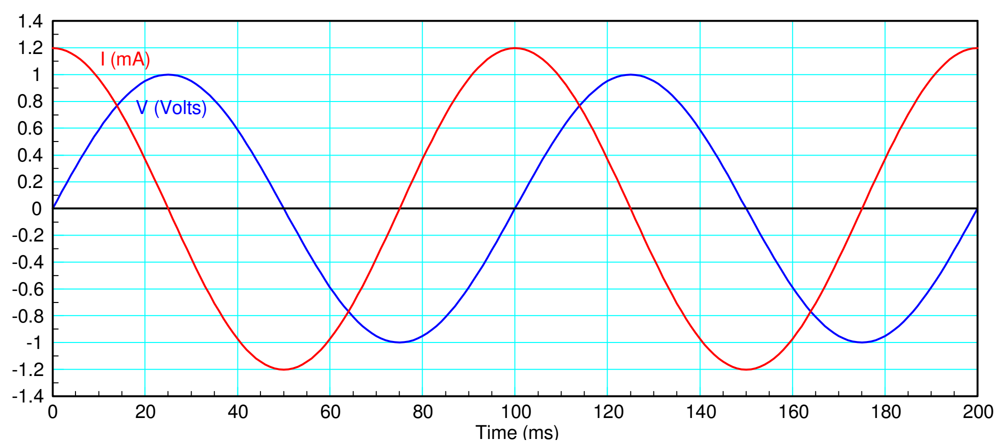
$$Z = \left(\frac{a-jb}{j\omega C\cdot(q-jb)}\right)$$

Doing some algebra

$$\boxed{Z = \left(\frac{1}{j\omega C}\right)}$$

The complex impedance of a capacitor is $\frac{1}{j\omega C}$

For example, the current flowing through a capacitor with an impedance of -j833 Ohms is shown below: note that there is a 90 degree phase shift (1/j) and the current leads voltage for capacitors



Current leads voltage for capacitors

Inductors:

The VI relationship of an inductor is

$$v(t) = L\frac{di(t)}{dt}$$

Assume current is

$$i(t) = a\cos(\omega t) + b\sin(\omega t)$$

$$I = a - jb$$

The voltage is then

$$v(t) = L \cdot \frac{d}{dt}(a\cos(\omega t) + b\sin(\omega t))$$

$$v(t) = L \cdot (-a\omega\sin(\omega t) + b\omega\cos(\omega t))$$

$$V = L\omega \cdot (ja + b) = j\omega L \cdot (a - jb)$$

The impedance of an inductor is then

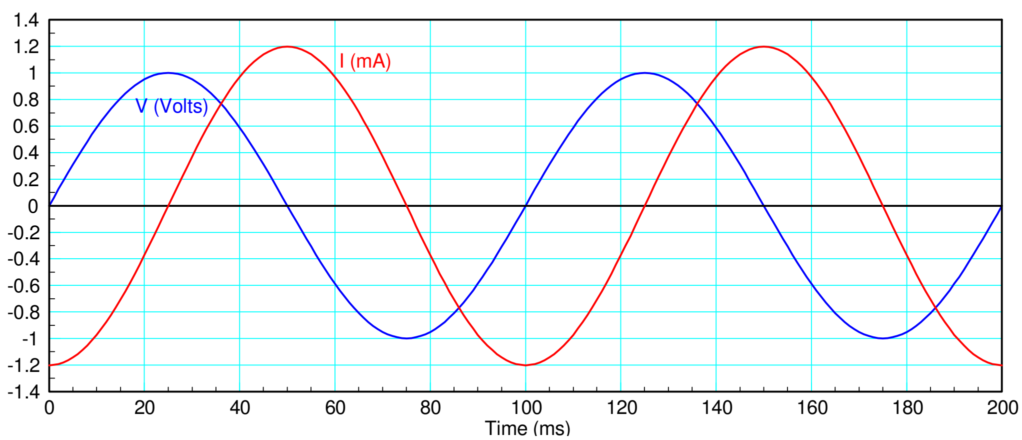
$$Z = \frac{V}{I}$$

$$Z = \left(\frac{j\omega L \cdot (a - jb)}{a - jb}\right)$$

$$\boxed{Z = j\omega L}$$

The complex impedance of an inductor is jwL

For example, the voltage and currents through an inductor with an impedance of j833 Ohms is shown below. Note that there is a 90 degree phase shift (j) and that voltage leads current for inductors.



Voltage leads current for inductors

ELI the ICE man: One way to remember the current - voltage relationship for capacitors and inductors is the phrase

ELI the ICE man

ELI: Voltage (E) leads current for inductors (L)

ICE: Current (I) leads voltage (E) for capacitors

Example: Determine the impedance of a

- 100 Ohm resistor, a
- 100mH inductor, and a
- 1uF capacitor

at { 0, 100 Hz, and 10kHz }

Solution: Note that

$$\omega = 2\pi f$$

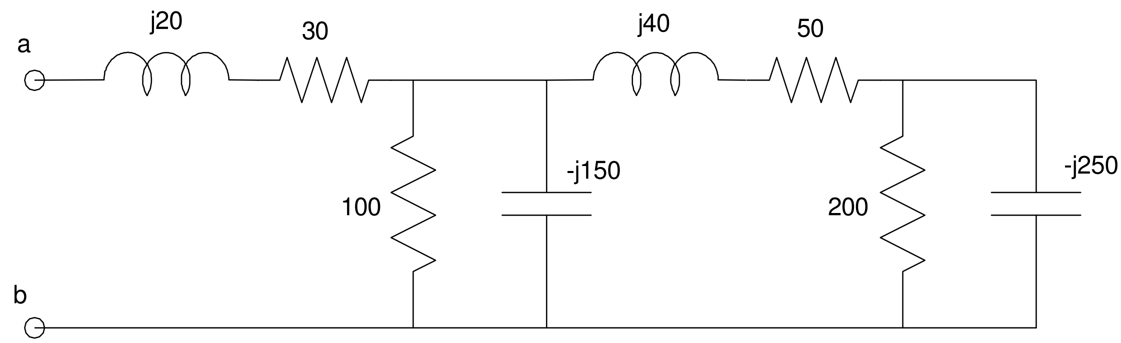
f (Hz)	0 Hz	100 Hz	10 kHz
w (rad/sec)	0	628.3	62,831
R = 100 Ohms	Z = 100	Z = 100	Z = 100
L = 100mH	Z = 0	Z = j62.83	Z = j6283.1
C = 1uF	Z = infinity	Z = -j1591	Z = -j15.91

- Resistors don't change with freqency:
- Inductors are short circuits at DC (0 Ohms) and have a +j impedance
- Capacitors are open circuits at DC (infinity Ohms) and have a -j impedance.

RLC Networks:

If you have a circuit with resistors, inductors, and capacitors, the impedance will be a complex number. The same rules that apply to resitors (parallel, series) apply when using complex impedances.

Example 1: Determine the complex impdeance Zab



Solution: Going right to left:

200 and -j250 are in parallel:

$$200 \parallel -j250 = \left(\frac{1}{200} + \frac{1}{-j250} \right)^{-1} = 121.95 - j97.56$$

This is in series with (50 + j40) Ohms

$$(121.95 - j97.56) + (50 + j40) = 171.95 - j57.56$$

which is in parallel with -j150 and 100 Ohms

$$(171.95 - j57.56) || (-j150) || (100) = 50.30 - j27.80$$

which is in series with 20+j30 Ohms

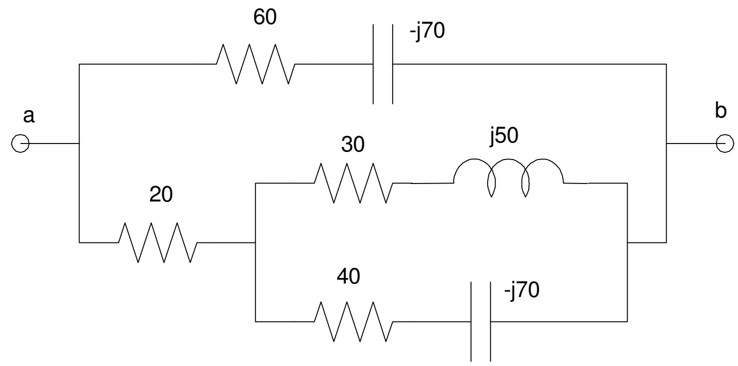
$$(50.30 - j27.80) + (20 + j30) = 70.30 + j2.20$$

answer:

$$Z_{ab} = 70.30 + j2.20 \quad \text{Ohms}$$

note: it really helps to have a calculator that does complex numbers. I recommend an HP35s or the Free42 app on a cell phone

Example 2: Determine the complex impedance Zab



Solution: Go inside out

$$(40 - j70) || (30 + j50) = 62.45 + j16.42$$

$$(62.45 + j16.42) + 20 = 82.45 + j16.42$$

$$(82.45 + j16.42) || (60 - j70) = 48.56 - j15.34$$

answer:

$$Z_{ab} = 48.56 - j15.34 \quad \text{Ohms}$$