Superposition

Concept:

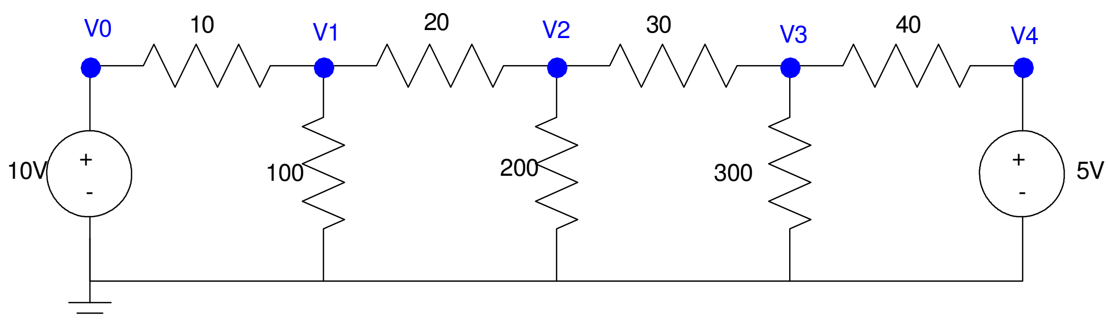
A circuit composed of resistors, inductors, capacitors, voltage sources, current sources, and dependent sources is a linear system. Linear systems have the property

$$f(a+b) = f(a) + f(b)$$

This, in a nut-shell, is superposition. If you have multiple inputs to a circuit, you can analyze the circuit with one element at a time. The answer will be the sum of the two.

Example 1:

Find the voltages V0 .. V4



Soution: Use superposition. First, assume the voltages are { 10V, 0V }. Using voltage division, the voltages are:

	V0	V1	V2	V3	V4
V0 = 10V V4 = 0V	10.00V	8.04 V	5.71 V	3.09 V	0.00 V

Next, assume the voltagtes are { 0V, 10V }. Using voltage division, the voltages are:

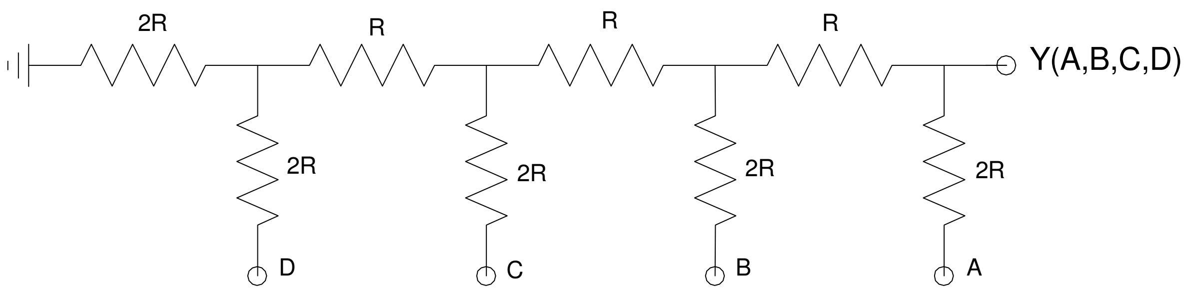
	V0	V1	V2	V3	V4
V0 = 0V V4 = 5V	0.00 V	0.386 V	1.24 V	2.69 V	5.00 V

By superposition, if both voltages are turned on, the voltages will be the sum of these two cases:

	V0	V1	V2	V3	V4
V0 = 10V V4 = 5V	10.00 V	8.42 V	6.95 V	5.78 V	5.00 V

Example 2: R-2R Ladder.

Determine Y as a funciton of A, B, C, and D



By superposition, we know that

$$Y = aA + bB + cC + dD$$

where {a, b, c, d} are constants. To determine the constants, turn off all the inputs but one and set the other to 1V.

Case 1: A = 1, B = C = D = 0.

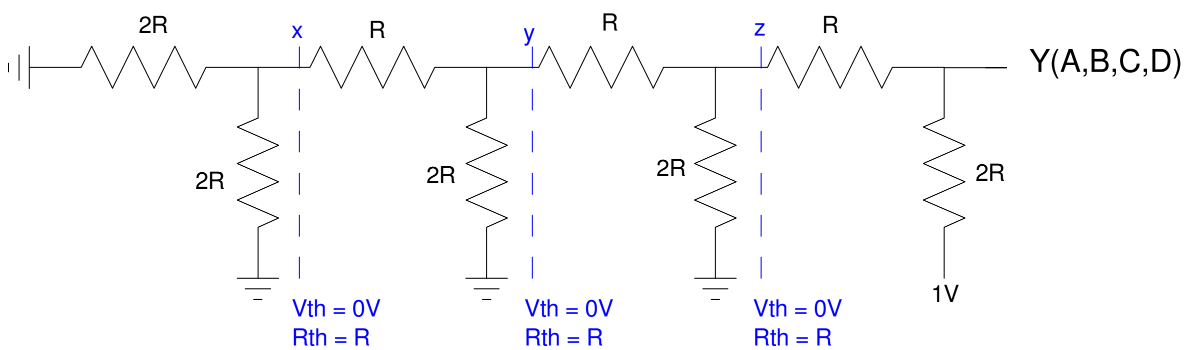
Take the Thevenin equivalent of the circuit looking left at x. All you see is 2R || 2R = R to ground.

Repeat at y. All you see is 2R || 2R = R to ground.

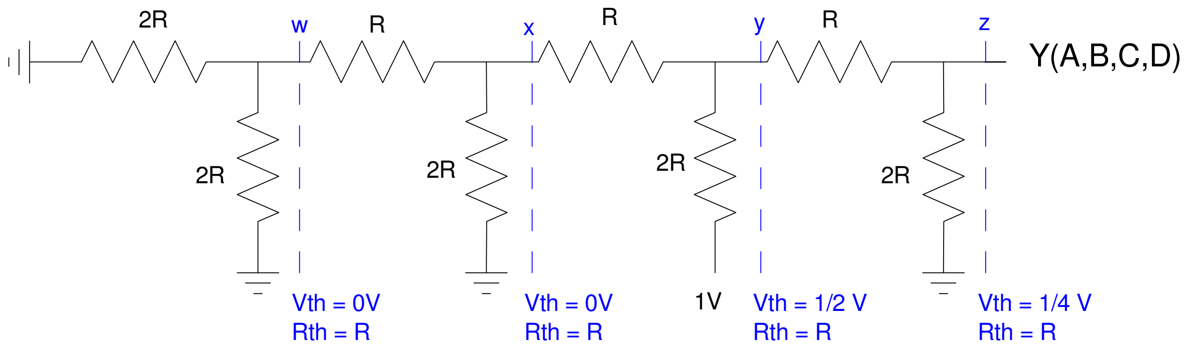
Repeat at z. All you see is 2R || 2R = R to ground.

By voltage division, Y = 1/2 volt.

a = 1/2



Case 2: A = 0, B = 1, C = D = 0.



At w looking left, all you see is 2R || 2R = R

At x looking left, all you see is 2R || 2R = R

At y looking left, all you get

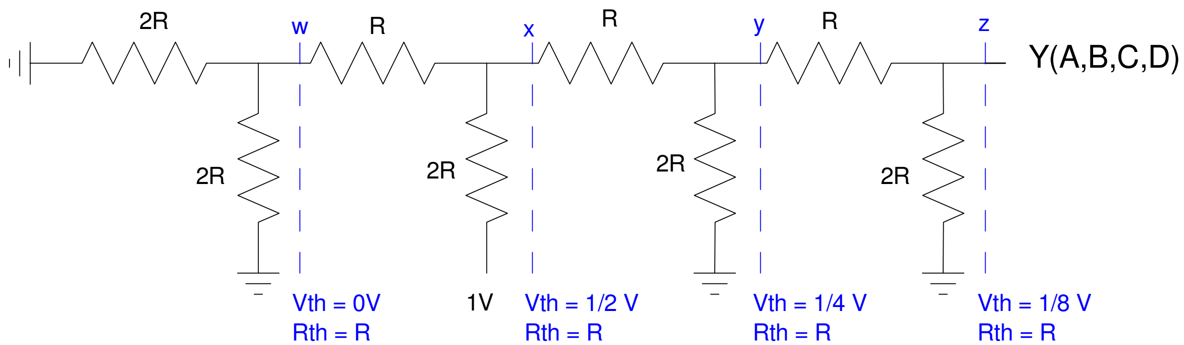
- Rth = 2R || 2R = R
- Vth = 1/2 V

At z looking left, you get

- Rth = 2R || 2R = R
- Vth = 1/2 * 1/2 = 1/4

b = 1/4

Case 3: A = B = D = 0. C = 1.



Taking the Thevenin equivalents looking left, you wind up with

Y = 1/8 V (c = 1/8)

The net result is

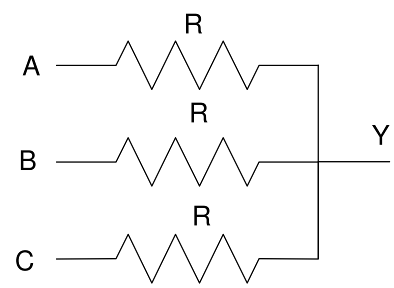
Y = 1/2 A + 1/4 B + 1/8 C + 1/16 D

Example 3: Weighted Average

Case 1: Design a circuit so that Y is the average of { A, B, C }

$$Y = \left(\frac{A+B+C}{3}\right)$$

Solution:



Writing the voltage node equation at Y:

$$\left(\frac{Y-A}{R}\right) + \left(\frac{Y-B}{R}\right) + \left(\frac{Y-C}{R}\right) = 0$$

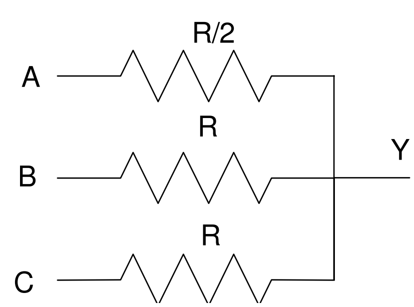
$$3Y = A + B + C$$

$$Y = \left(\frac{A+B+C}{3}\right)$$

Case 2: Design a circuit so that Y is the weighted average:

$$Y = \left(\frac{2A+B+C}{4}\right)$$

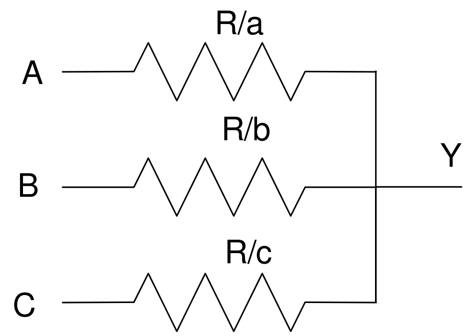
Solution: Add A twice. This is equivalent to reducing R by 2:



Case 3: Design a circuit to implement

$$Y = \left(\frac{aA+bB+cC}{a+b+c} \right)$$

Solution:



Y is the weighted average of A, B, C

Level Shifting

Assume A is an analog signal in the range of -10V .. +10V. Design a circuit to shift this voltage to the range of 0..5V.

Solution: Y is related to A as

$$Y = \frac{1}{4}A + 2.5$$

Assume you have a +5V power supply and a 0V power supply. This can be rewritten as

$$Y = \frac{1}{4}A + \frac{1}{2}(5V) + \frac{1}{4}(0V)$$

or

$$Y = \left(\frac{1 \cdot (A) + 2(\cdot 5V) + 1 \cdot (0V)}{4} \right)$$

A circuit which shifts a -10V to +10V signal (A) to a 0..5V signal (Y) is thus