# Electronics I

Diodes and Solving Nonlinear Equations

## Objective

- Write N (nonlinear) equations to solve for N unknowns for a nonlinear circuit
- Solve for the current and voltage in a circuit using graphical methods (load-line analysis)
- Solve for the current and voltage in a circuit using numerical methods
- Simulate the voltage and current for an AC to DC converter

## Matlab Functions

- plot()
- fminsearch()
- m-files

## Diode VI Characteristics

A diode is a nonlinear circuit element which acts like a valve:

- The resistance is low when you try to force current to flow from the anode to cathode
- The resistance is high when you try to force current to flow from the cathode to the anode.

The symbol for a diode acts as a reminder of this:  it looks like an arrow which points in the direction current can flow
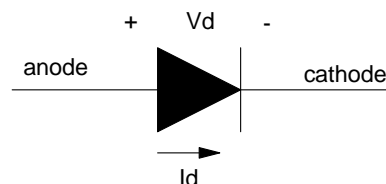


Symbol for a diode:  current only flows from anode to cathode

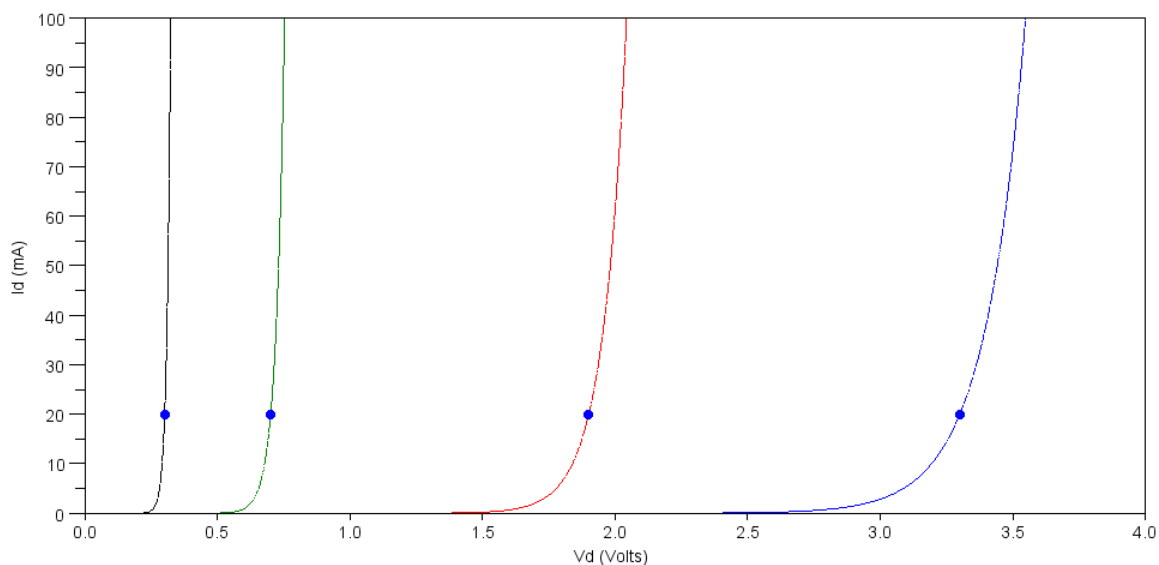The VI characteristics for a diode are

$$I_d = I_{dss}\left(\exp\left(\frac{V_d}{nV_T}\right) - 1\right)$$

$$V_d = nV_T\ln\left(\frac{I_d}{I_{dss}} + 1\right)$$

where

- Idss is a constant for the diode (typically 10-8A), and
- nVT is a constant depending upon the material (0.052 for silicon)

The VI characteristics for different types of diodes are shown below:

VI Characteristics for a Germanium (black), Silicon (green), Red LED (red), and Blue LED (blue)

These nonlinear curves make it difficult to describe the VI characteristics for a diode with a single number. Instead of specifying a curve, datasheets typically specify the voltage drop at 20mA as:

- Germanium:        0.3V @ 20mA
- Silicon:          0.7V @ 20mA
- Red LED:          1.9V @ 20mA
- Blue LED:         3.3V @ 20mA

As a rough approximation, you can assume this voltage drop is constant for $Id > 0$. As you can see from the above figure, the voltage does change with current slightly, but if a few tenths-of-a-volt doesn't bother you, this assumption makes diode circuit analysis a lot easier.

## Load-Line Analysis:

Suppose you have a circuit with a single diode, such as the following:
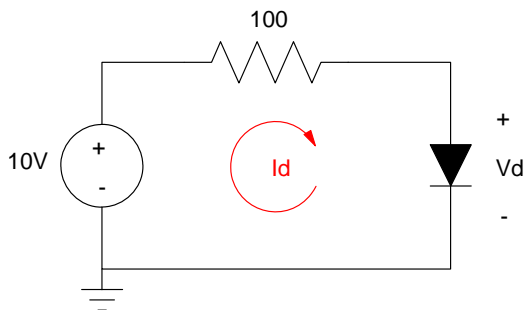
Here, you have two unknowns (Vd, Id), meaning you need two equations. One equation is the diode equation:

(1)    $V_d = nV_T \ln\left(\frac{I_d}{I_{dss}} + 1\right)$

A second equation comes from the 10V source and 100 Ohm resistor:

(2)    $V_d = 10 - 100 I_d$

One way to solve is to plot these two functions on the same graph: the solution is where the two functions overlap. For example, assume

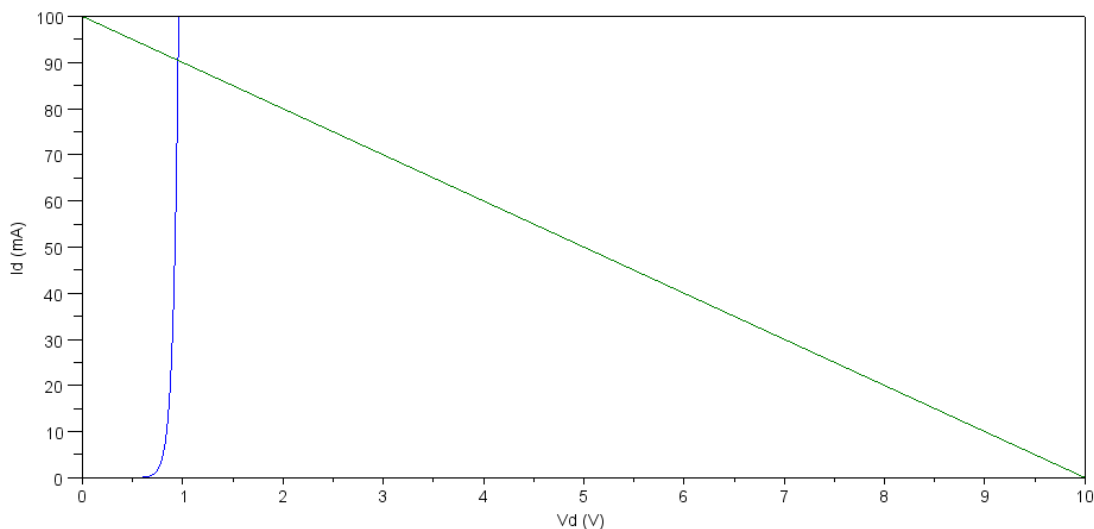nVT = 0.052                            (typical for silicon)

Idss = 1e-9

In MATLAB:

```
-->Idss = 1e-9;
-->nVT = 0.052;

-->Id = [0:0.001:100]' / 1000;

-->V1 = nVT * log(Id / Idss + 1);
-->V2 = 10 - 100*Id;

-->plot(V1,Id*1000,V2,Id*1000);
-->xlabel('Vd (V)');
-->ylabel('Id (mA)');
```



Graphical solution to 2 equations and 2 unknowns. The green line is called *the load line*

The net results is the intersection: (0.95V, 90.5mA)

## KVN and KCL for Diode Circuits

A second way to solve is to use KVN and KCL:  they always apply for circuit analysis - even if there are diodes present.  The only difference is you wind up with N nonlinear equations to solve.

For example, write the KVN and KCL equations for the following circuit:
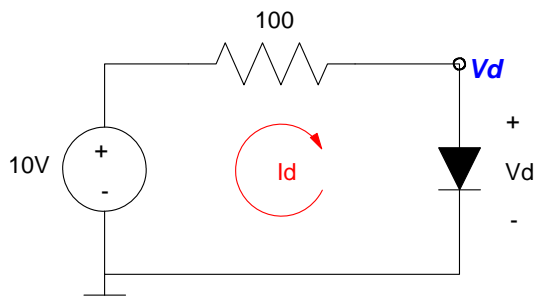


KVN:  Sum the currents from node Vd to zero.  Note that the current through the diode is defined by the diode equations.

$$\left(\frac{V_d - 10}{100}\right) + I_{dss}\left(\exp\left(\frac{V_d}{nV_T}\right) - 1\right) = 0$$

KCL:  Sum the voltages around the loop to zero.  Note that the voltage across the diode is defined by the diode equations:

$$-10 + 100 I_d + nV_T \ln\left(\frac{I_d}{I_{dss}} + 1\right) = 0$$

In both cases, you get one equation and one unknown.

**Example 2: Voltage Nodes: Find {V1, V2, V3} so that the currents balance.**
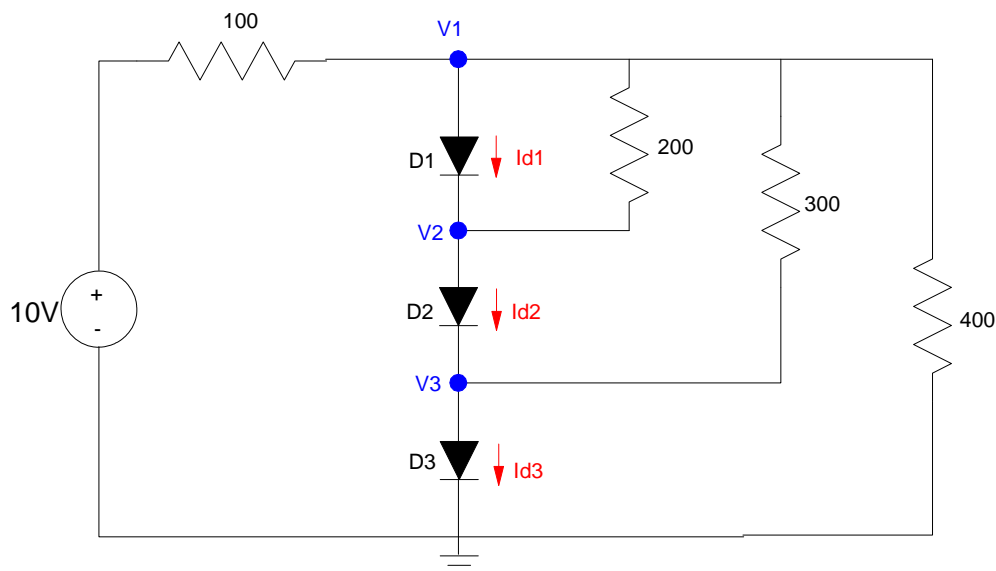


There are three voltage nodes, so you need to write three equations. First, write the equations for the currents through the diodes

$$I_{d1} = I_{dss}\left(\exp\left(\frac{V_1-V_2}{nV_T}\right) - 1\right)$$

$$I_{d2} = I_{dss}\left(\exp\left(\frac{V_2-V_3}{nV_T}\right) - 1\right)$$

$$I_{d3} = I_{dss}\left(\exp\left(\frac{V_3-0}{nV_T}\right) - 1\right)$$

Now write the voltage node equations. Use ID1, ID2, and ID3 for the currents through the diodes (helps to prevent sign errors in your equations).
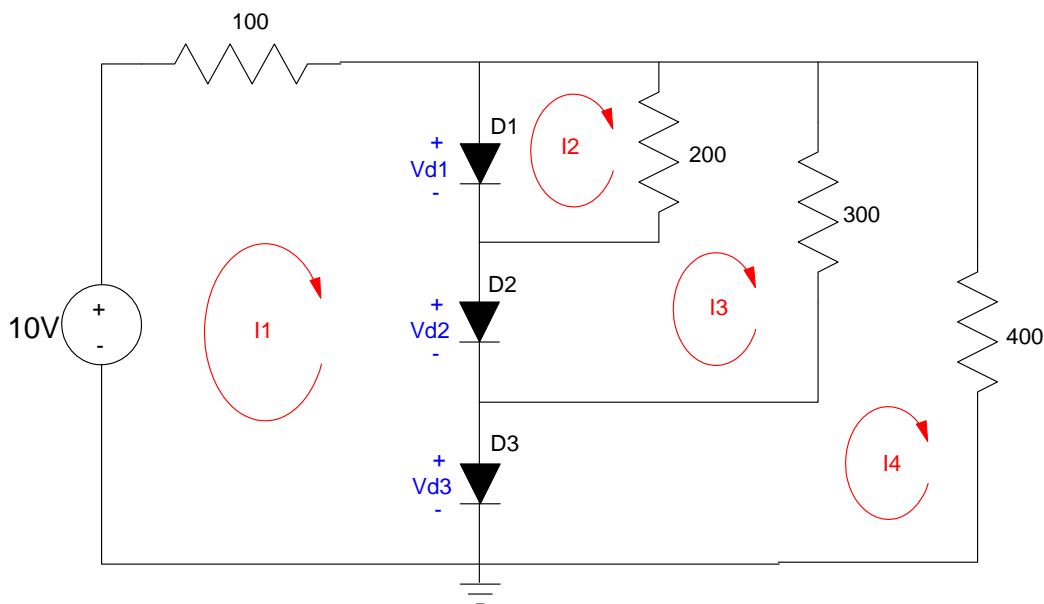
Node:

V1:    $\left(\frac{V_1-10}{100}\right) + I_{D1} + \left(\frac{V_1-V_2}{200}\right) + \left(\frac{V_1-V_3}{300}\right) + \left(\frac{V_1-0}{400}\right) = 0$

V2:    $\left(\frac{V_2-V_1}{200}\right) - I_{D1} + I_{D2} = 0$

V3:    $\left(\frac{V_3-V_1}{300}\right) - I_{D2} + I_{D3} = 0$

This gives 3 equations for 3 unknowns. They're not easy to solve, but in concept they should be solvable.

**Example 3: Current Loops**: Find {I1, I2, I3, I4} so that the voltages balance.



There are four current loops - so you need to write four equations to solve for four unknowns.

First, write the voltages across the diodes based upon the loop currents {I1, I2, I3, I4}:

$$V_{d1} = nV_T \ln\left(\frac{I_1 - I_2}{I_{dss}} + 1\right)$$

$$V_{d2} = nV_T \ln\left(\frac{I_1 - I_3}{I_{dss}} + 1\right)$$

$$V_{d3} = nV_T \ln\left(\frac{I_1 - I_4}{I_{dss}} + 1\right)$$

Next, sum the voltages to zero going around three closed paths:

Loop:

I1:     $-10 + 100I_1 + V_{d1} + V_{d2} + V_{d3} = 0$
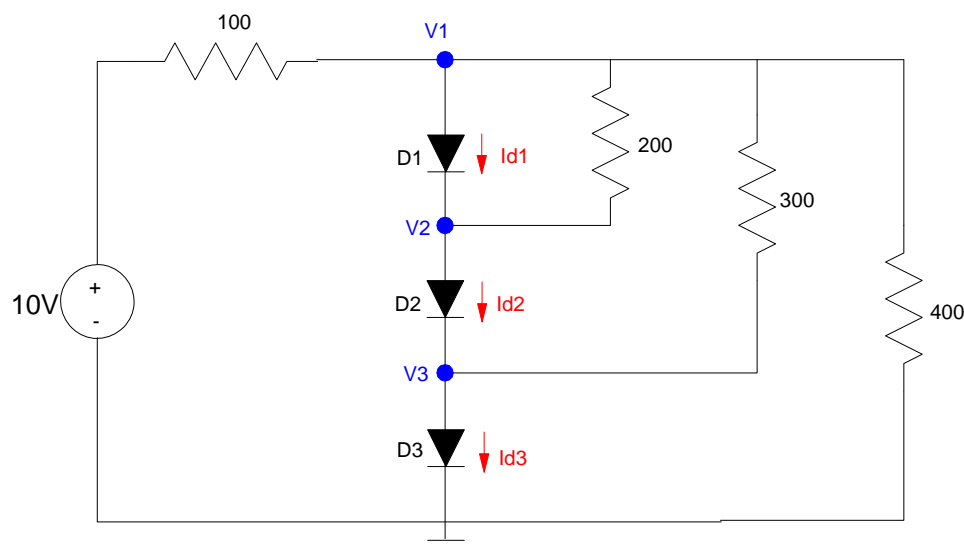
I2:     $-V_{d1} + 200(I_2 - I_3) = 0$

I3:     $-V_{d2} + 200(I_3 - I_2) + 300(I_3 - I_4) = 0$

I4:     $-V_{d3} + 300(I_4 - I_3) + 400I_4 = 0$

Again, you now have N equations for N unknowns.

## Numerical Solutions

**Example 2:  Voltage Nodes:**  Solve the three diode circuit.  Using the previous voltage node equations,



i) Guess the voltages ( V1, V2, V3 )

ii) Compute the diode currents:

$$I_{d1} = I_{dss}\left(\exp\left(\frac{V_1 - V_2}{nV_T}\right) - 1\right)$$

$$I_{d2} = I_{dss}\left(\exp\left(\frac{V_2 - V_3}{nV_T}\right) - 1\right)$$

$$I_{d3} = I_{dss}\left(\exp\left(\frac{V_3 - 0}{nV_T}\right) - 1\right)$$

iii)  Find the excess current (error) from each node:

$$e_1 = \left(\frac{V_1 - 10}{100}\right) + I_{d1} + \left(\frac{V_1 - V_2}{200}\right) + \left(\frac{V_1 - V_3}{300}\right) + \left(\frac{V_1}{400}\right)$$

$$e_2 = \left(\frac{V_2 - V_1}{200}\right) - I_{d1} + I_{d2} = 0$$

$$e_3 = \left(\frac{V_3 - V_1}{300}\right) - I_{d2} + I_{d3} = 0$$

iv) Compute the sum square error

$$J = e_1^2 + e_2^2 + e_3^2$$

In Matlab, create a cost function:

```
function [ J ] = cost( z )

   V1 = z(1);
   V2 = z(2);
   V3 = z(3);

   Idss = 1e-9;
   nVt = 0.052;

   Id1 = Idss* exp( (V1 - V2)/nVt - 1 );
   Id2 = Idss* exp( (V2 - V3)/nVt - 1 );
   Id3 = Idss* exp( (V3 - 0)/nVt - 1 );

   e1 = (V1 - 10)/100 + Id1 + (V1-V2)/200 + (V1-V3)/300 + (V1/400);
   e2 = (V2-V1)/200 - Id1 + Id2;
   e3 = (V3-V1)/300 - Id2 + Id3;

   J = (e1)^2 + (e2)^2 + (e3)^2;

   end
```

Solving with *fminsearch*
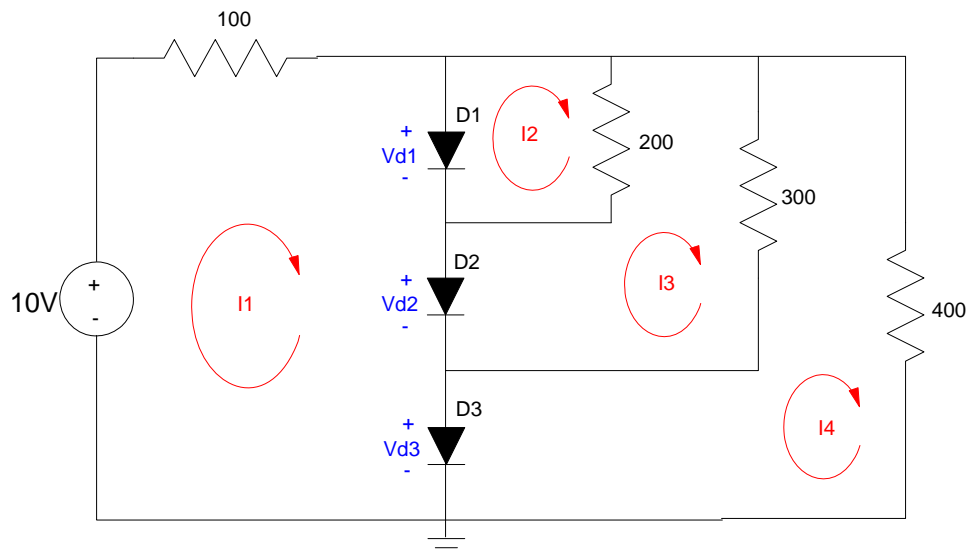
```
>> [z,e] = fminsearch('cost',[3,2,1])

z =     2.9421    1.9664    0.9860

e =     4.8665e-010
```

The solution found by MATLAB is
- $V1 = 2.9421V$
- $V2 = 1.9664V$
- $V3 = 0.9860V$

**Example 3: Current Loops:** Solve the three diode circuit. Using the previous current loop equations,



i) Guess the currents ( I1, I2, I3, I4 )

ii) Compute the diode voltages:

$$V_{d1} = nV_T \ln\left(\frac{I_1 - I_2}{I_{dss}} + 1\right)$$

$$V_{d2} = nV_T \ln\left(\frac{I_1 - I_3}{I_{dss}} + 1\right)$$

$$V_{d3} = nV_T \ln\left(\frac{I_1 - I_4}{I_{dss}} + 1\right)$$

iii) Compute the excess voltage (error) around each loop:

$$e_1 = -10 + 100I_1 + V_{d1} + V_{d2} + V_{d3}$$

$$e_2 = -V_{d1} + 200(I_2 - I_3)$$

$$e_3 = -V_{d2} + 200(I_3 - I_2) + 300(I_3 - I_4)$$

$$e_4 = -V_{d3} + 300(I_4 - I_3) + 400I_4$$

iv) Compute the sum-squared error:

$$J = e_1^2 + e_2^2 + e_3^2 + e_4^2$$

In Matlab, create a cost function:

```
function [ J ] = cost( z )

    I1 = z(1);
    I2 = z(2);
    I3 = z(3);
    I4 = z(4);

    Idss = 1e-9;
    nVt = 0.052;

    Vd1 = nVt * log( (I1 - I2)/Idss + 1 );
    Vd2 = nVt * log( (I1 - I3)/Idss + 1 );
    Vd3 = nVt * log( (I1 - I4)/Idss + 1 );

    e1 = -10 + 100*I1 + Vd1 + Vd2 + Vd3;
    e2 = -Vd1 + 200*(I2 - I3);
    e3 = -Vd2 + 200*(I3 - I2) + 300*(I3 - I4);
    e4 = -Vd3 + 300*(I4 - I3) + 400*I4;

    J = e1^2 + e2^2 + e3^2 + e4^2;

    end
```

Solving with fminsearch(),

```
>> [z,e] = fminsearch('cost',[4,3,2,1])

z =     0.0721    0.0178    0.0132    0.0070

e =  2.2259e-005
```

The error is almost zero:  this is the answer
- $I_a = 72.1mA$
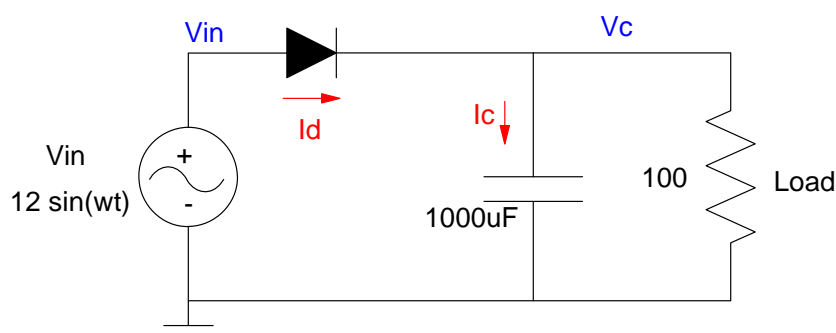- $I_b = 17.8mA$
- $I_c = 13.2mA$
- $I_d = 7.0mA$

## Sidelight:  AC to DC Converters

A commonly used diode circuit converts 60Hz AC to DC.

- The diode only allows positive current to pass (rectifier)
- The capacitor acts like a battery, powering the circuit while the AC source goes negative.


Problem:  Determine the voltage Vc and the current Id for the following circuit:

- C = 10000 uF
- R = 100 Ohms
- $\omega = 360$  rad/sec



Solution:  The current through the diode is:

$$I_d = I_{dss}\left(\exp\left(\frac{V_{in}-V_c}{nV_T}\right) - 1\right)$$

The current to the load is

$$I_{load} = \frac{V_c}{100}$$

The excess current charges (or discharges if negative) the capacitor

$$I_c = I_d - I_{load}$$

$$C\frac{dV_c}{dt} = I_c$$

In MATLAB, simulate this with a step size of

- dt = 0.00002 seconds

(found by trial-and-error.  Reduce the step size until the numerical errors are small, but not so small that the simulation runs too slowly.)  Just to speed it up, plot the data every 10th iteration.

```
Idss = 1e-9;
nVt = 0.052;

Vc = 0;

t = 0;
dt = 0.00002;
```

```
C = 1000e-6;
R = 100;

clf

while(t < 0.05)

   for i=1:10

      Vin = 12*sin(360*t);
      Id = Idss*(exp( (Vin - Vc)/nVt ) - 1);
      Ic = Id - Vc/R;

      dVc = Ic / C;

      t = t + dt;
      Vc = Vc + dVc * dt;
      end

   subplot(211);
   plot(t,Vin,'.',t,Vc,'.');
   hold on

   subplot(212);
   plot(t,Id,'.');

   pause(0.01);
   hold on
   end

   hold off

>> subplot(211);
>> xlabel('Time (s)');
>> ylabel('Vin & Vc');
>> subplot(212);
>> xlabel('Time (s)');
>> ylabel('Id (A)');
```
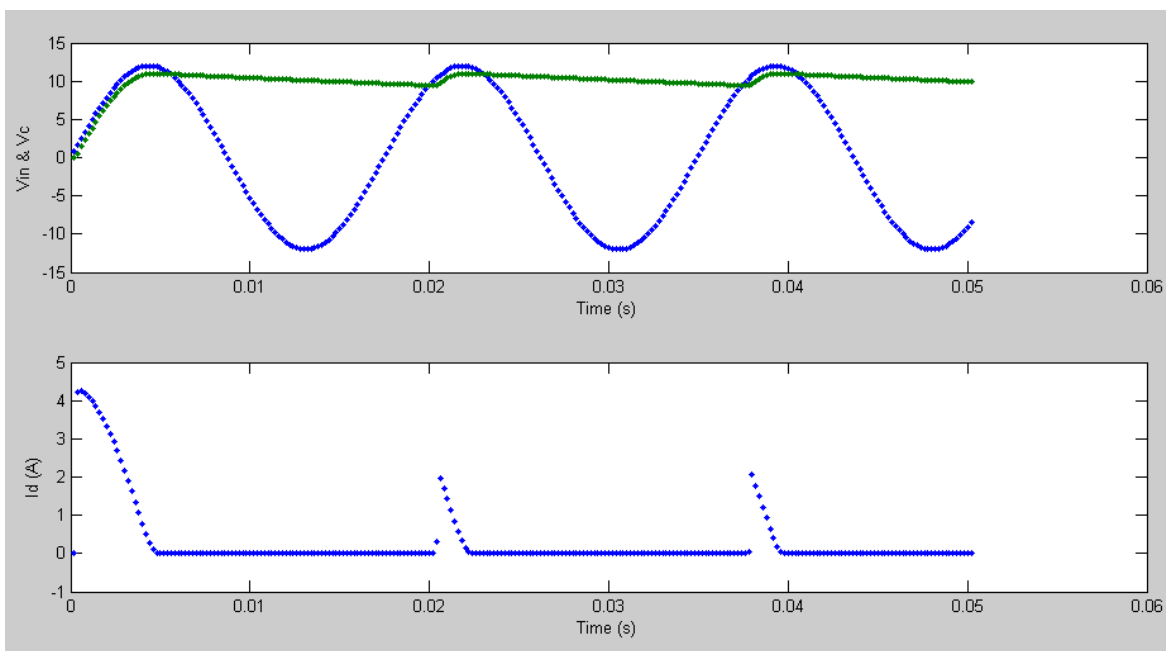
Simulated Voltage and Current for a 1/2 Wave Rectifier

Note:

- The voltage at the load (Vc: green line top) is 0.7V below the input (blue line). This is due to the voltage drop across the diode.

- When the input voltage drops below Vc, the diode turns off. At this point, the capacitor is powering the load.

- When the input voltage exceeds Vc, the diode turns on. During this brief time, the source recharges the capacitor.