ECE 111 - Homework #11

Week #11 - ECE 343 Signals- Due Tuesday, April 4th

Problem 1-5) Let x(t) be a function which is periodic in 2π

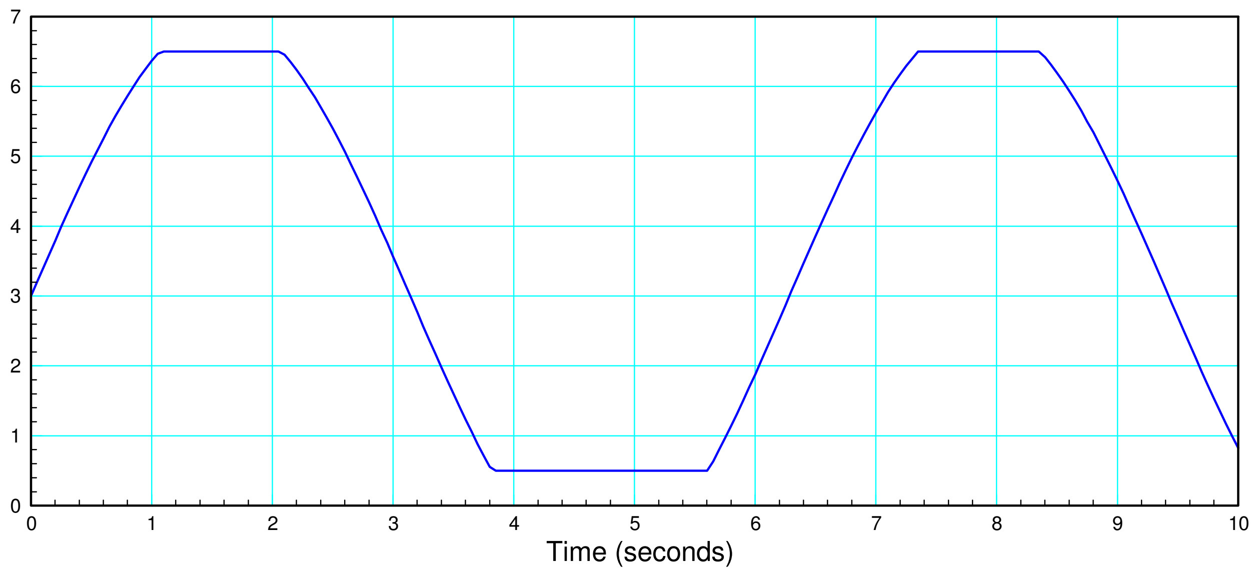
$$x(t) = x(t + 2\pi)$$

Over the interval $(0, 2\pi)$ x(t) is

$$x(t) = 4\sin(t) + 3$$

clipped at +6.5V and +0.5V. In Matlab:

```
t = [0:0.001:2*pi]';
x = 4*sin(t) + 3;
x = min(x, 6.5);
x = max(x, 0.5);
plot(t,x)
```



x(t) Note that x(t) repeats repeats every 2π seconds

Curve Fitting with a power series:

1) Using least squares, approximate x(t) over the interval (0, 2π) as

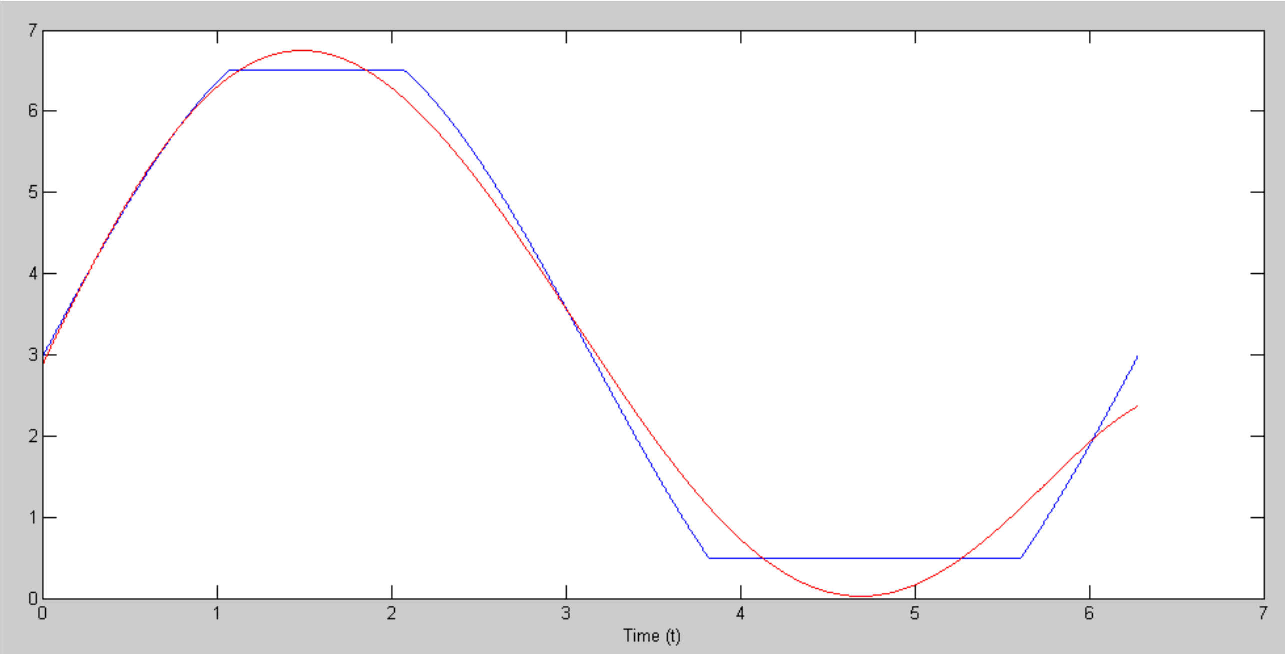
$$x(t) \approx a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

Plot x(t) along with it's approximation.

```
>> t = [0:0.001:2*pi]';
x = 4*sin(t) + 3;
x = min(x, 6.5);
x = max(x, 0.5);
plot(t,x)
>> B = [t.^0, t, t.^2, t.^3, t.^4, t.^5];
>> A = inv(B'*B)*B'*x

a0    2.9062
a1    4.4393
a2   -0.3072
a3   -0.9732
a4    0.2551
a5   -0.0176

>> plot(t,x,'b',t,B*A,'r')
>> xlabel('Time (t)')
>>
```



Note:

- This is a decent approximation to x(t), but
- This approximation doesn't help us find y(t)

Curve Fitting using a Fourier Series

2) Using least squares, approximate x(t) over the interval (0, 2π) as

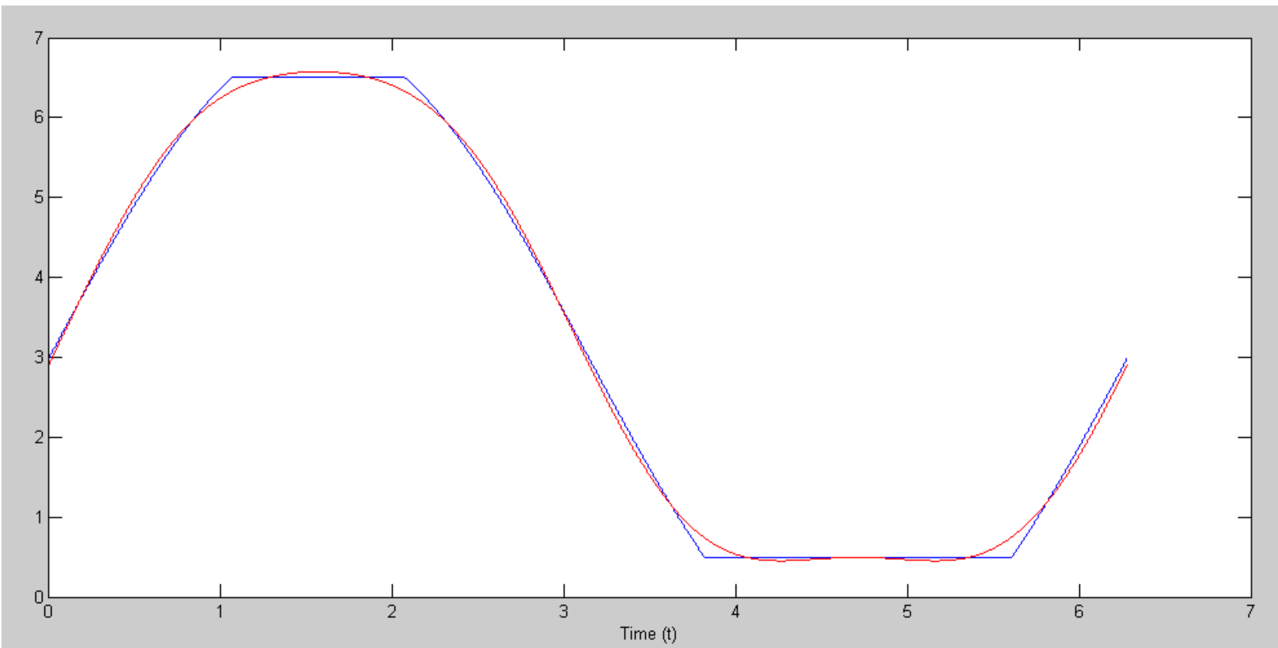
$$x(t) = a_0 + a_1\cos(t) + b_1\sin(t) + a_2\cos(2t) + b_2\sin(2t) + a_3\cos(3t) + b_3\sin(3t)$$

Plot x(t) along with it's approximation.

```
>> B = [t.^0, cos(t), sin(t), cos(2*t), sin(2*t), cos(3*t), sin(3*t)];
>> A = inv(B'*B)*B'*x

a0    3.2278
a1    0.0000
b1    3.3767
a2   -0.3074
b2    0.0000
a3    0.0000
b3    0.3366

>> plot(t,x,'b',t,B*A,'r')
>> xlabel('Time (t)')
>>
```



Note:

- This is also a decent approximation for x(t)
- The result *is* useful since the results is a bunch of sine waves
- I like sine waves. I know how to find y(t) when x(t) is a sine wave.

3) Determine x(t) in terms of its Fourier Transform out to 3 rad/sec

Using least squares, the answer was:

```
a0    3.2278
a1    0.0000
b1    3.3767
a2   -0.3074
b2    0.0000
a3    0.0000
b3    0.3366
```

Another way to get the same result is to compute the Fourier coefficients. Note that you get the same answer (either method is valid)

Moral:

- Fourier Transform is nothing more than a least-squares curve fit
- Where the basis is made up of sine waves

```
>> a0 = mean(x)

a0 =    3.2278

>> a1 = 2*mean(x .* cos(t))

a1 =  7.7770e-004

>> b1 = 2*mean(x .* sin(t))

b1 =    3.3763

>> a2 = 2*mean(x .* cos(2*t))

a2 =   -0.3067

>> b2 = 2*mean(x .* sin(2*t))

b2 = -1.0166e-007

>> a3 = 2*mean(x .* cos(3*t))

a3 =  7.7784e-004

>> b3 = 2*mean(x .* sin(3*t))

b3 =    0.3366

>>
```

Superposition:

Assume X and Y are related by

$$Y = \left(\frac{2}{s^2+0.3s+1.5}\right)X$$

4) Using the results from problem 2 & 3, determine y(t) assuming

$$x(t) = a_0$$

$$x(t) = 3.2278$$

$$s = j0$$

$$Y = \left(\frac{2}{s^2+0.3s+1.5}\right)_{s=j0} \cdot (3.2778)$$

$$Y = 4.3037$$

$$y_0(t) = 4.3037$$

In Matlab:

```
>> s = 0;
>> X0 = a0;
>> Y0 = ( 2 / (s^2 + 0.3*s + 1.5) ) * X0

Y0 =    4.3037
```

5) Using the results from problem 2 & 3, determine y(t) assuming

$$x(t) = a_1 \cos(t) + b_1 \sin(t)$$

$$x(t) = 3.3763 \sin(t)$$

Solving using phasors

$$s = j1$$

$$X = 0 - j3.3763$$

$$Y = \left(\frac{2}{s^2+0.3s+1.5} \right)_{s=j1} \cdot (0 - j3.3763)$$

$$Y = -5.9558 - j9.9316$$

$$y_1(t) = -5.9558 \cos(t) + 9.9316 \sin(t)$$

In Matlab

```
>> s = j*1;
>> X1 = a1 - j*b1

X1 =   0.0008 - 3.3763i

>> s = j*1;
>> Y1 = ( 2 / (s^2 + 0.3*s + 1.5) ) * X1

Y1 =  -5.9558 - 9.9316i
```

6) Using the results from problem 2 & 3, determine y(t) assuming

$$x(t) = a_2 \cos(2t) + b_2 \sin(2t)$$

$$x(t) = -0.3067 \cos(2t)$$

Solving using phasors

$$s = j2$$

$$X = -0.3067$$

$$Y = \left(\frac{2}{s^2 + 0.3s + 1.5} \right)_{s=j2} \cdot (-0.3067)$$

$$Y = 0.2320 + j0.0557$$

meaning

$$y_2(t) = 0.2320 \cos(2t) - 0.0557 \sin(2t)$$

In matlab

```
> s = j*2;
>> X2 = a2 - j*b2

X2 =  -0.3067 + 0.0000i

>> Y2 = ( 2 / (s^2 + 0.3*s + 1.5) ) * X2

Y2 =   0.2320 + 0.0557i
```

7) Using the results from problem 2 & 3, determine y(t) assuming

$$x(t) = a_3 \cos(3t) + b_3 \sin(3t)$$

$$x(t) = 0.3366 \sin(3t)$$

Find y(t) using phasors

$$s = j3$$

$$X = 0 - j0.3366$$

$$Y = \left(\frac{2}{s^2 + 0.3s + 1.5} \right)_{s=j3} \cdot (0 - j0.3366)$$

$$Y = -0.0108 + j0.08855$$

meaning

$$y_3(t) = -0.0108 \cos(3t) - 0.08855 \sin(3t)$$

In Matlab

```
>> s = j*3;
>> X3 = a3 - j*b3

X3 =   0.0008 - 0.3366i

>> Y3 = ( 2 / (s^2 + 0.3*s + 1.5) ) * X3

Y3 =  -0.0108 + 0.0885i
```

8) Plot y(t) where y(t) is the sum of the results from problems 4..7

Add up all the inputs to get x(t)

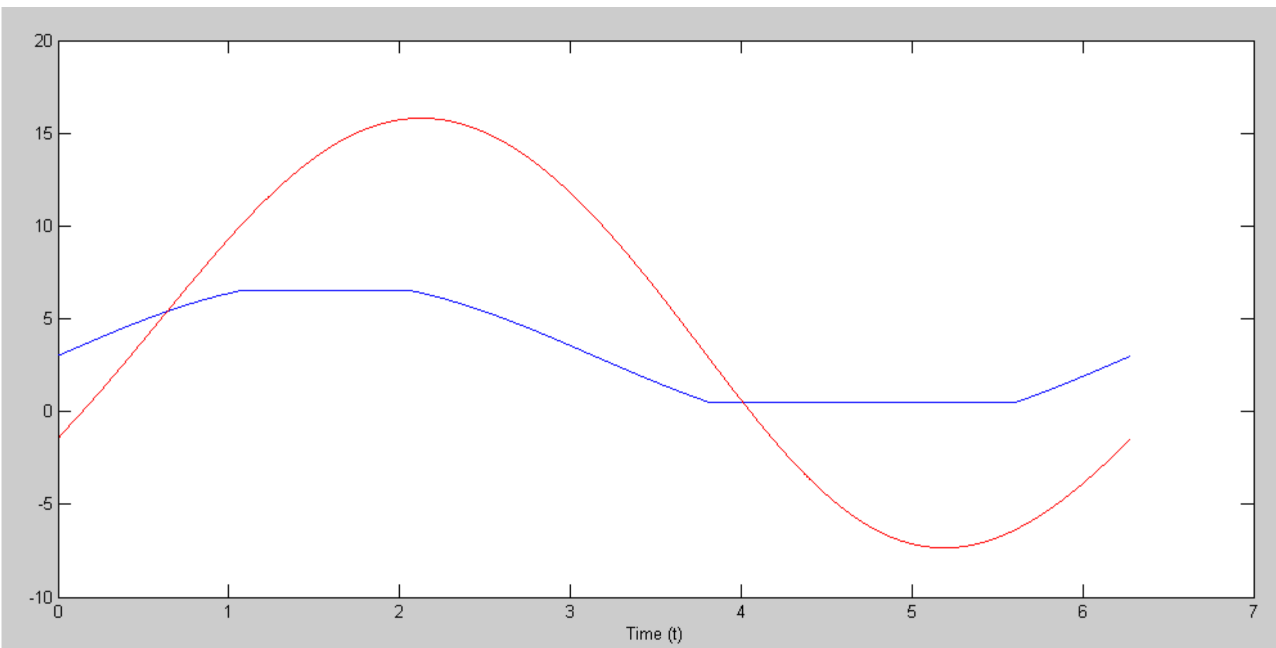
Add up all of the results to get y(t)

$$y(t) = y_0 + y_1 + y_2 + y_3$$

$$\begin{aligned} y(t) = &\quad 4.3037 \\ &-5.9558\cos(t) + 9.9316\sin(t) \\ &0.2320\cos(2t) - 0.0557\sin(2t) \\ &-0.0108\cos(3t) - 0.08855\sin(3t) \end{aligned}$$

In Matlab

```
>> y = 0*t + Y0;
>> y = y + real(Y1)*cos(t) - imag(Y1)*sin(t);
>> y = y + real(Y2)*cos(2*t) - imag(Y2)*sin(2*t);
>> y = y + real(Y3)*cos(3*t) - imag(Y3)*sin(3*t);
>> plot(t,x,'b',t,y,'r')
>> xlabel('Time (t)')
```



Note:

- In theory, you have to go out to infinity.
- In practice, the terms usually go to zero pretty fast. You can get a good approximation just using a couple of terms.

By converting x(t) into a bunch of sine waves,

- A problem which is very difficult to solve has been turned into
- Multiple problems that are easy to solve