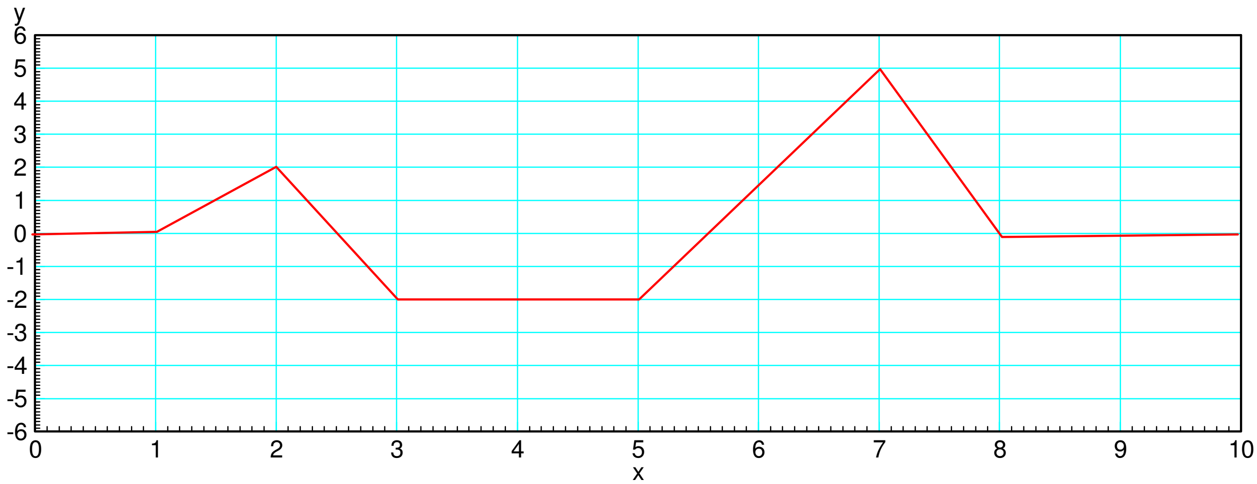
ECE 111 - Homework #8

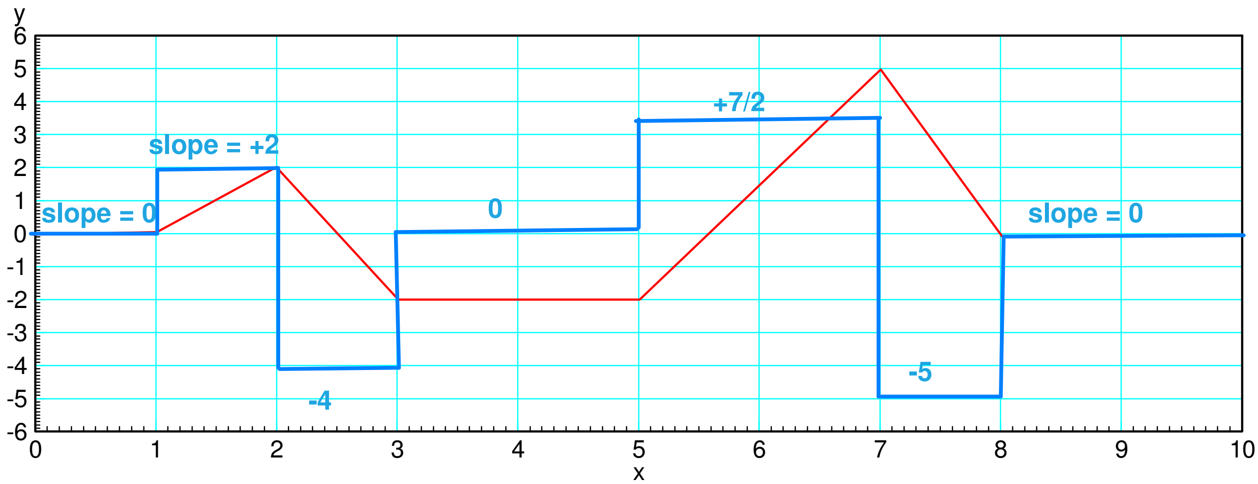
Week #8: ECE 351 Electromagnetics - Due Tuesday, March 7th

1) Assume the current flowing through a one Henry inductor is shown below. Sketch the voltage.

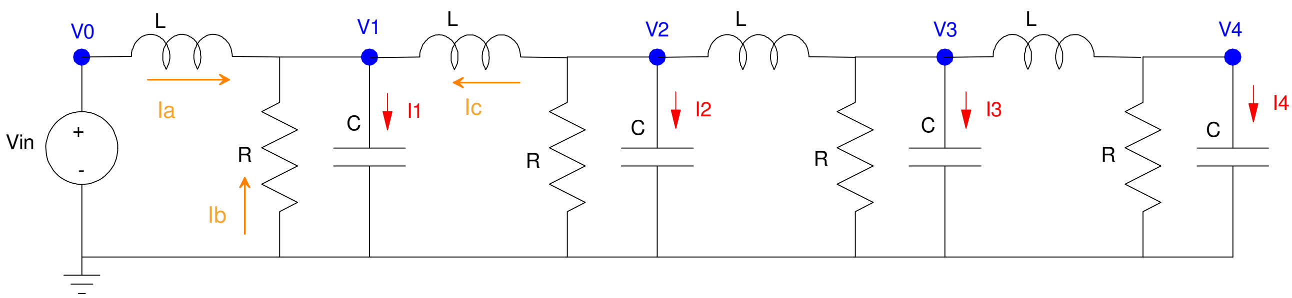
$$V = L\frac{dI}{dt}$$



The derivative (times one) gives the votlage



Problem 2-3) 4-Node RLC Circuit



R = 200Ω, C = 0.25F, L = 0.25H. Repeat for 30 nodes for problems 4-6

2) Write the dynamic equations for the following 4-stage RLC circuit. (i.e. write the node equations)

From conservation of current

$$I_1 = I_a + I_b + I_c$$

For capacitors (prime denoted differentiation - easier to write)

$$I = C\frac{dV}{dt} = CV'$$

$$I_1 = CV_1' = I_a + I_b + I_c$$

Differentiating again

$$CV_1'' = I_a' + I_b' + I_c'$$

For inductors

$$V = L\frac{dI}{dt} = LI'$$

$$V_0 - V_1 = LI_a'$$

$$I_a' = \left(\frac{V_0 - V_1}{L}\right)$$

$$I_c' = \left(\frac{V_2 - V_1}{L}\right)$$

For the resistor

$$I_b = \left(\frac{0 - V_1}{R}\right)$$

$$I_b' = -\left(\frac{1}{R}\right) V_1'$$

Substituting

$$CV_1'' = \left(\frac{V_0 - V_1}{L}\right) - \left(\frac{1}{R}\right) V_1' + \left(\frac{V_2 - V_1}{L}\right)$$

Grouping terms

$$V_1'' = \left(\frac{1}{LC}\right) V_0 - \left(\frac{2}{LC}\right) V_1 + \left(\frac{1}{LC}\right) V_2 - \left(\frac{1}{RC}\right) V_1'$$

ditto for the other nodes (except the last node where there is only a single 1/LC term)

$$V_2'' = \left(\frac{1}{LC}\right) V_1 - \left(\frac{2}{LC}\right) V_2 + \left(\frac{1}{LC}\right) V_3 - \left(\frac{1}{RC}\right) V_2'$$

$$V_3'' = \left(\frac{1}{LC}\right) V_2 - \left(\frac{2}{LC}\right) V_3 + \left(\frac{1}{LC}\right) V_4 - \left(\frac{1}{RC}\right) V_3'$$

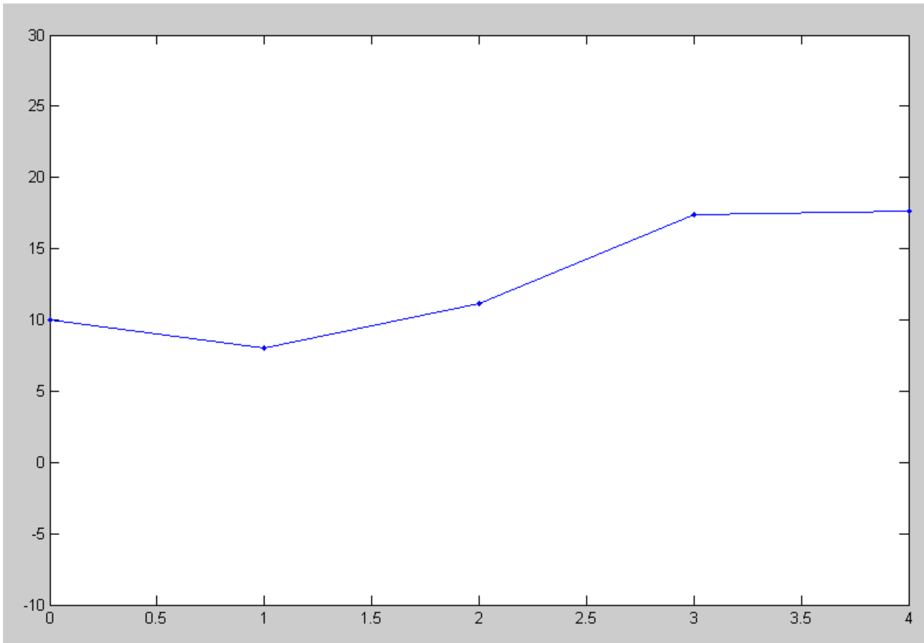
$$V_4'' = \left(\frac{1}{LC}\right) V_3 - \left(\frac{1}{LC}\right) V_4 - \left(\frac{1}{RC}\right) V_4'$$

Plugging in numbers (R = 200, C = 0.25F, L = 0.25H)

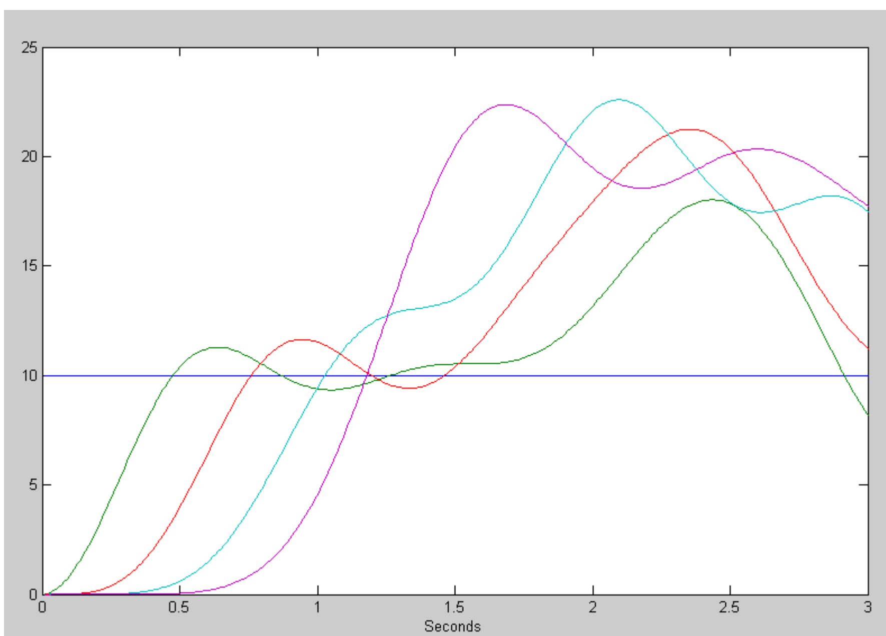
$$V_1'' = 16V_0 - 32V_1 + 16V_2 - 0.02V_1'$$
$$V_2'' = 16V_1 - 32V_2 + 16V_3 - 0.02V_2'$$
$$V_3'' = 16V_2 - 32V_3 + 16V_4 - 0.02V_3'$$
$$V_4'' = 16V_3 - 16V_4 - 0.02V_4'$$

3) Assume Vin = 10V and the initial conditions are zero ($V_1 = V_2 = V_3 = V_4 = 0$). Solve for the voltages at t = 3 seconds. *Hint: Solve numerically using Matlab*

Result at 3 seconds



Voltage at t = 3 seconds



Votlages vs. Time

Matlab Code:

```
V0 = 10;
V1 = 0;
V2 = 0;
V3 = 0;
V4 = 0;

dV1 = 0;
dV2 = 0;
dV3 = 0;
dV4 = 0;

V = [];

t = 0;
dt = 0.01;

while(t < 3)

% Calculate acceleration
   ddV1 = 16*V0 - 32*V1 + 16*V2 - 0.02*dV1;
   ddV2 = 16*V1 - 32*V2 + 16*V3 - 0.02*dV2;
   ddV3 = 16*V2 - 32*V3 + 16*V4 - 0.02*dV3;
   ddV4 = 16*V3 - 16*V4         - 0.02*dV4;

% Integrate once to get velocity
   dV1 = dV1 + ddV1*dt;
   dV2 = dV2 + ddV2*dt;
   dV3 = dV3 + ddV3*dt;
   dV4 = dV4 + ddV4*dt;

% integrate again to get position
   V1 = V1 + dV1*dt;
   V2 = V2 + dV2*dt;
   V3 = V3 + dV3*dt;
   V4 = V4 + dV4*dt;

   t = t + dt;

   plot([0,1,2,3,4],[V0,V1,V2,V3,V4],'.-');
   ylim([-10,30]);
   pause(0.01);

   V = [V ; V0, V1, V2, V3, V4];

end

pause(5);
clg
t = [1:length(V)]' * dt;
plot(t,V);
xlabel('Seconds');
xlim([0,3]);
```

Problem 4-6) 30-Node RLC Circuit (hint: modify the program Wave.m)

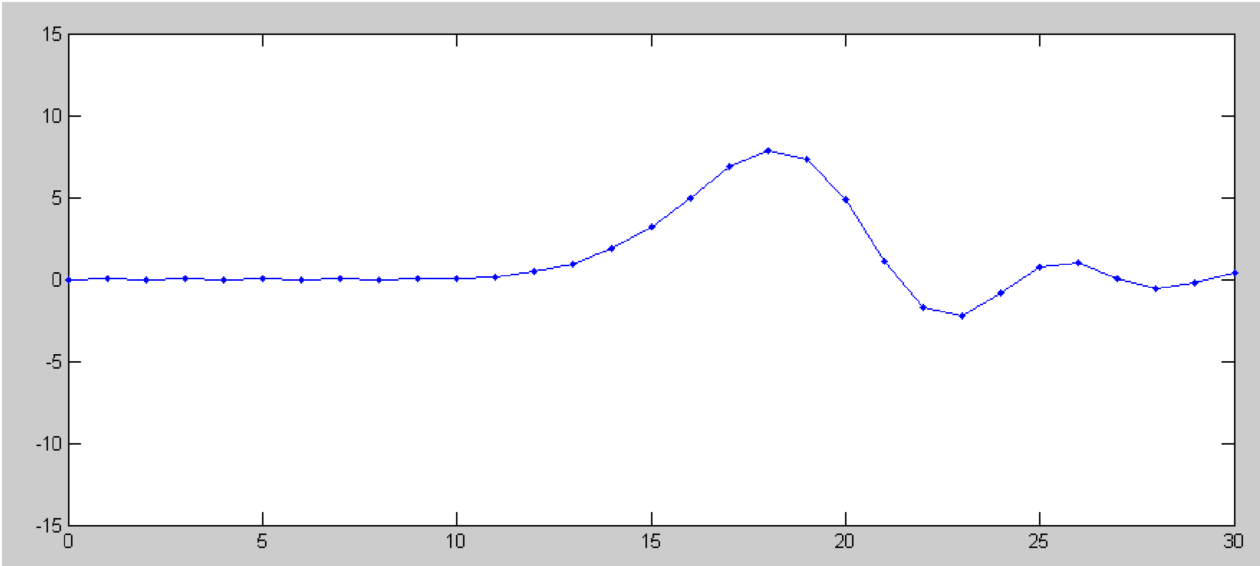
4) Expand the RLC circuit from problem #2 to 30 nodes. Plot the voltage at t = 12 seconds (just after the reflection) for $1 / R_{30}C = 0.01$

Note: This is where for-loops are really useful. Rather than copying each equation 30 times, just use a for-loop.

- Node #1 needs to be separate since V[0] is not valid in Matlab (indicies must be 1 or more)
- Node #30 needs to be separate since it's equation is slightly different
- Nodes 2..29 can be in a for-loop

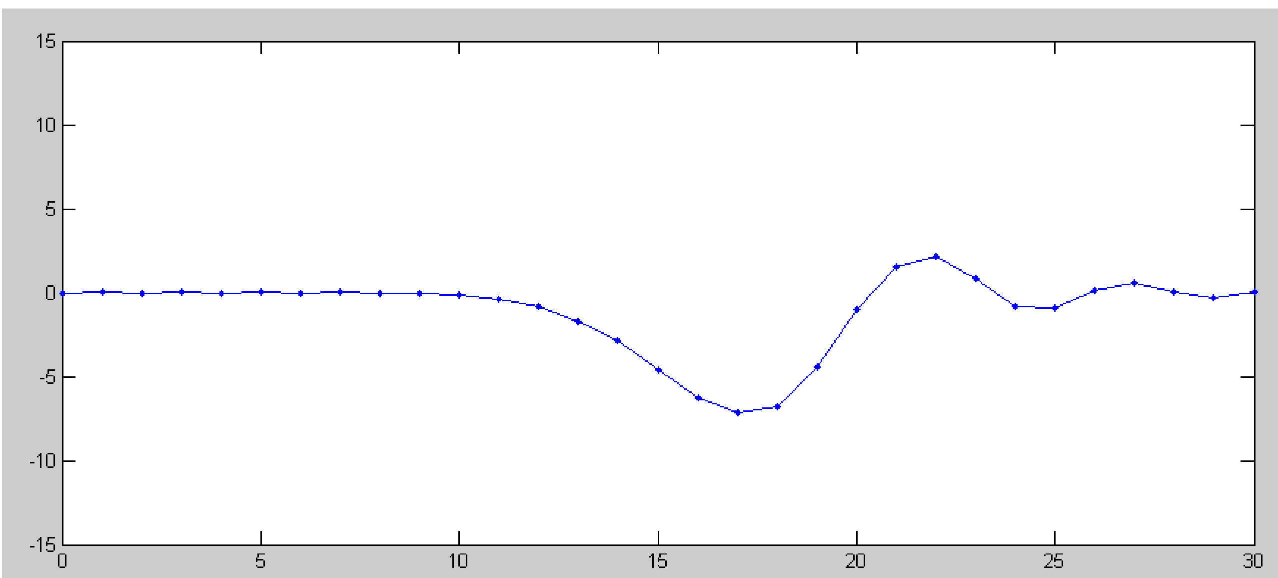
Code:

```
V0 = 10;
V = zeros(30,1);
dV = zeros(30,1);
ddV = zeros(30,1);

t = 0;
dt = 0.01;

while(t < 12)

   if (t < 2) V0 = 10 * ( ( sin(0.5*pi*t) )^2 );
      else V0 = 0;
   end

% Calculate acceleration
   ddV(1) = 16*V0 - 32*V(1) + 16*V(2) - 0.02*dV(1);

   for n=2:29
      ddV(n) = 16*V(n-1) - 32*V(n) + 16*V(n+1) - 0.02*dV(n);
   end

   ddV(30) = 16*V(29) - 16*V(30) - 0.01*dV(30);

% integrate to get velocity
   dV = dV + ddV*dt;

% integrate again to get position
   V  = V + dV*dt;

   t = t + dt;

   plot([0:30],[V0;V],'.-');
   ylim([-15,15]);
   pause(0.01);

end
```



Voltage at t = 12 for 1/RC(30) = 0.01. If too small, you get a positive reflection.

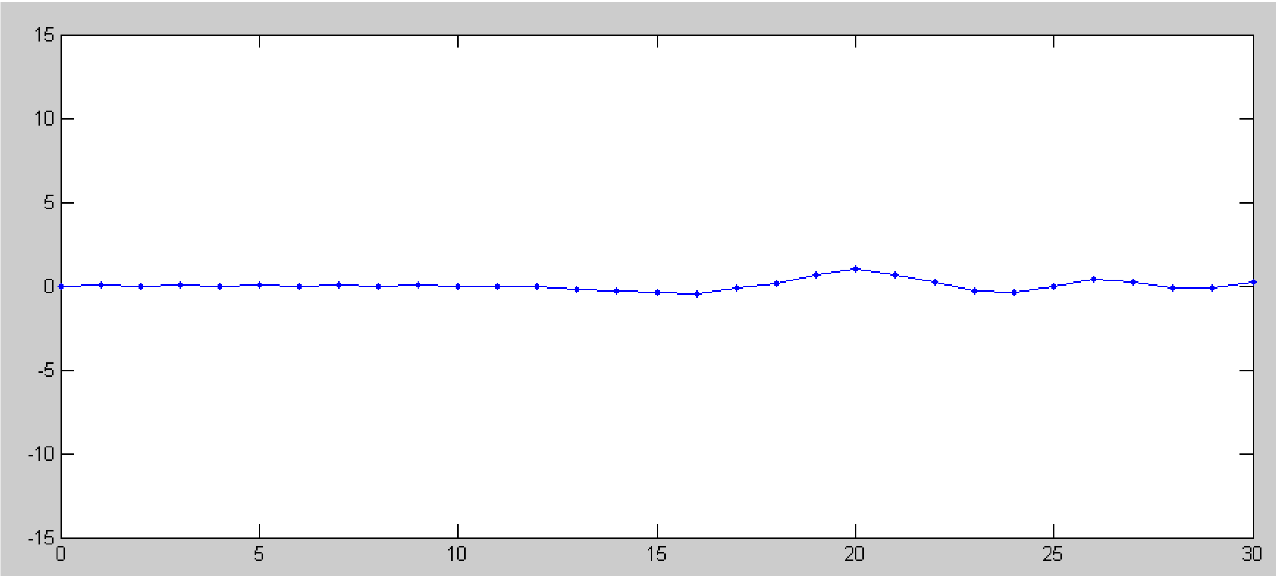
5) Plot the voltage at t = 8 seconds for $1 / R_{30}C = 100$



Voltage at t = 12 for 1/RC(30) = 100. If too large, you get a negative reflection.

6) Determine experimentally R_{30} so that the reflection is almost zero

By trial and error, 1/RC(30) = 4.00



Voltage at t = 12 for 1/RC(30) = 4.00. If just right, the reflection is almost zero.