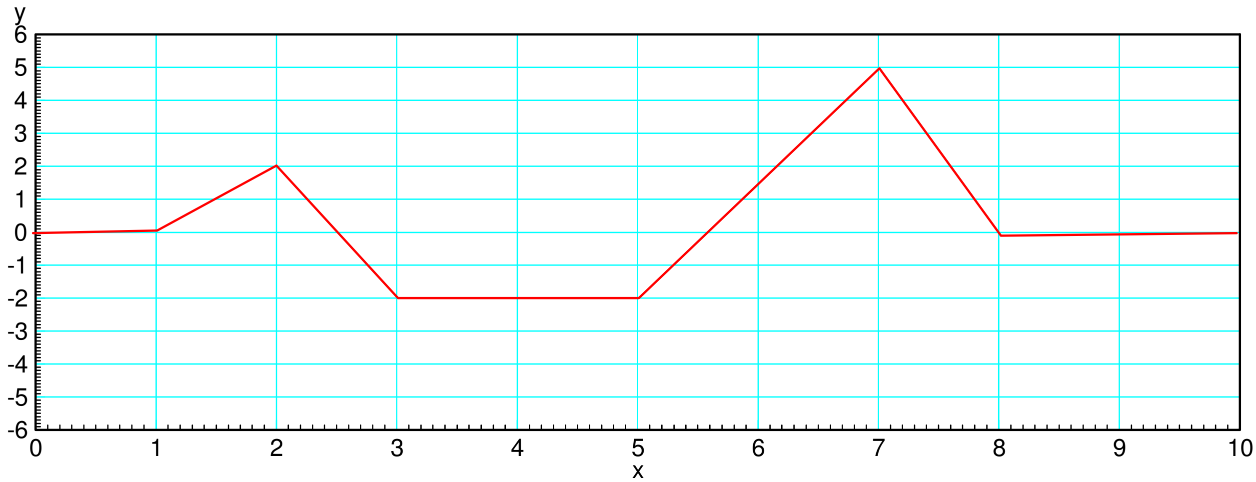
ECE 111 - Homework #7

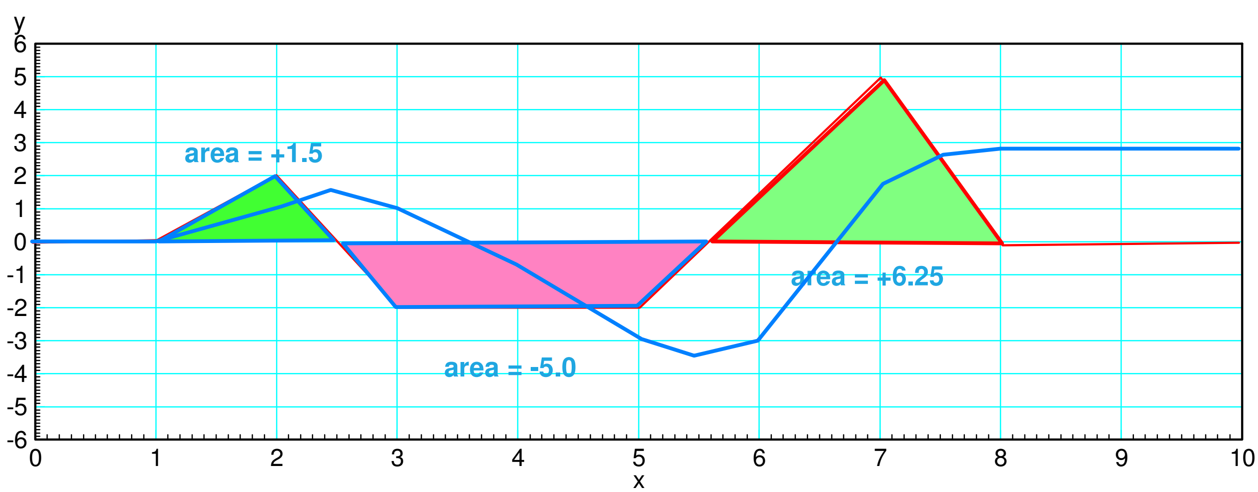
Week #7: ECE 311 Circuits II - Due Tuesday, February 28th

1) Assume the current flowing through a one Farad capacitor is shown below. Sketch the voltage. Assume V(0) = 0. The voltage is the integral of the current (capacitors are integrators)

$$V = \frac{1}{C}\int I \cdot dt$$

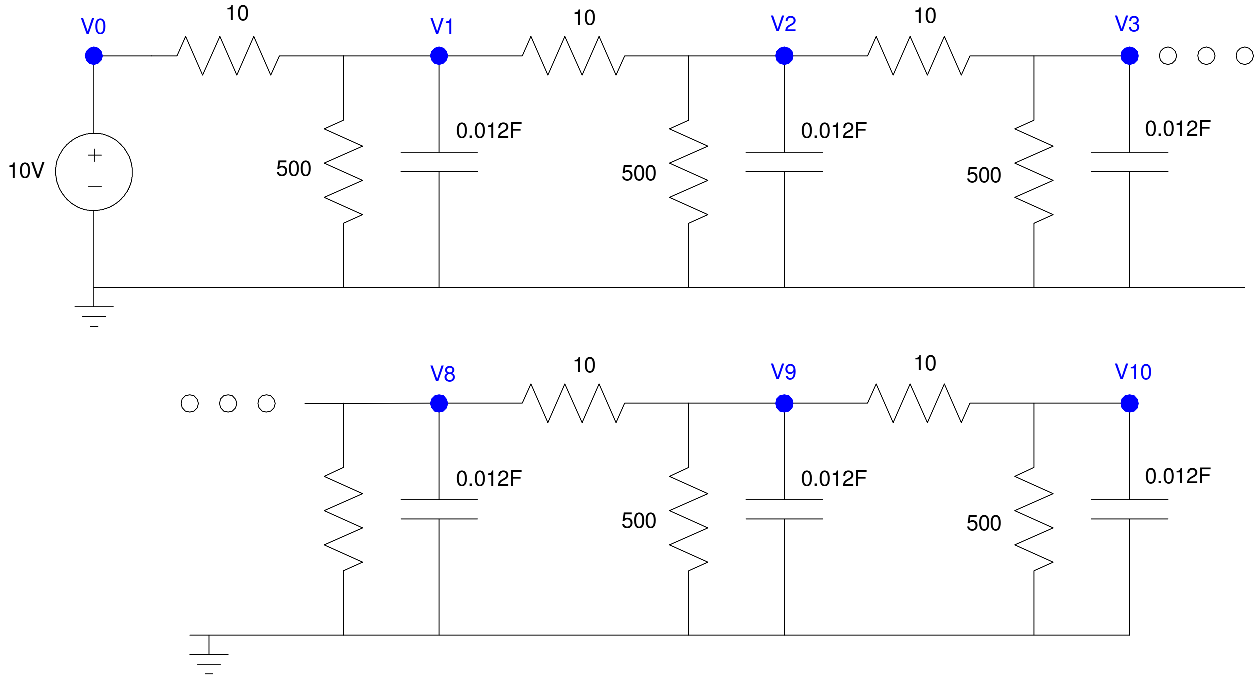


Since C = 1F, the votlage is simply the integral of the current (same as homework #4)



Problem 2-5: Assume a 10-stage RC filter (V0 .. V10)

10snip



Problem 2) Write the dynamics for this system as a set of ten coupled differential equations:

$$I_1 = C\frac{dV_1}{dt} = \sum(\text{current to node}V_1)$$

$$I_1 = 0.012\frac{dV_1}{dt} = \left(\frac{V_0-V_1}{10}\right) + \left(\frac{V_2-V_1}{10}\right) + \left(\frac{0-V_1}{500}\right)$$

$$0.012\frac{dV_1}{dt} = \left(\frac{1}{10}\right)V_0 - \left(\frac{1}{10}+\frac{1}{500}+\frac{1}{10}\right)V_1 + \left(\frac{1}{10}\right)V_2$$

$$\frac{dV_1}{dt} = 8.333V_0 - 16.833V_1 + 8.333V_2$$

The same pattern holds for nodes 2..9

$$\frac{dV_2}{dt} = 8.333V_1 - 16.833V_2 + 8.333V_3$$

$$\frac{dV_3}{dt} = 8.333V_2 - 16.833V_3 + 8.333V_4$$

$$\vdots$$

Node #10 is a little different since there is only one 10-Ohm resistor connected to it

$$I_{10} = 0.012\frac{dV_{10}}{dt} = \left(\frac{V_9-V_{10}}{10}\right) + \left(\frac{0-V_{10}}{500}\right)$$

$$\frac{dV_{10}}{dt} = 8.333V_9 - 8.50V_{10}$$

Forced Response for a 10-Node RC Filter (heat.m):

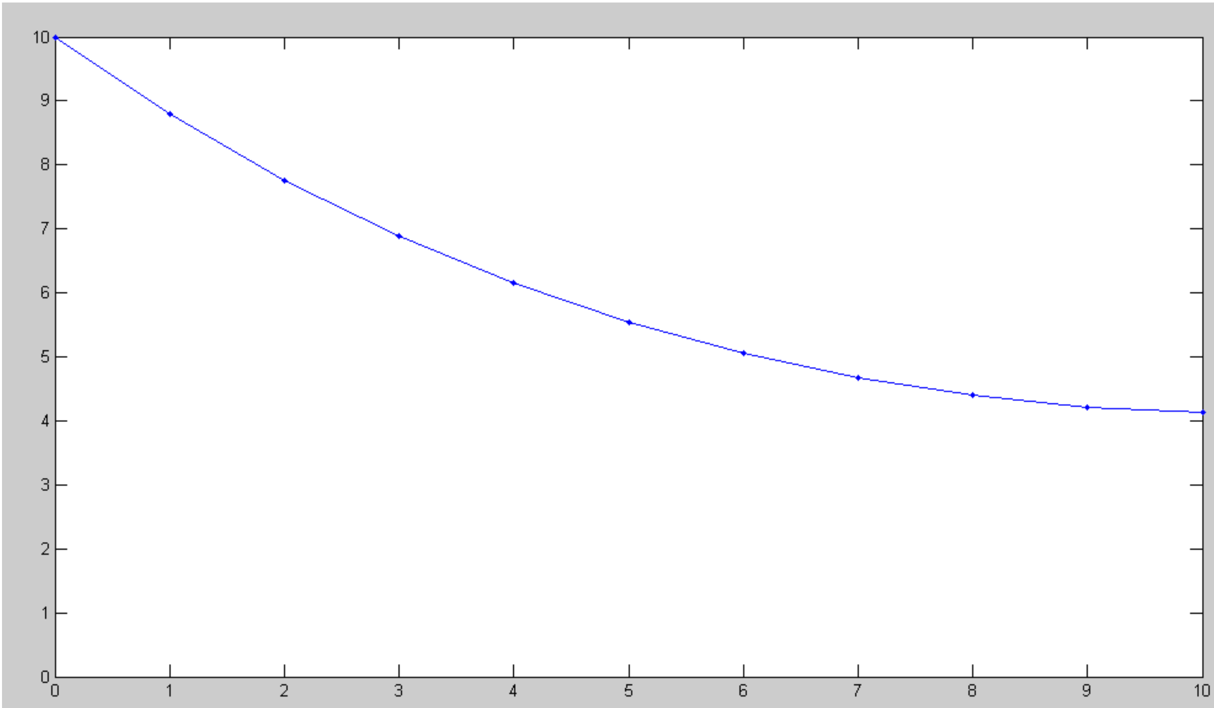
Problem 3) Using Matlab, solve these ten differential equations for $0 < t < 5$ s assuming

- The initial voltages are zero, and
- V0 = 10V.

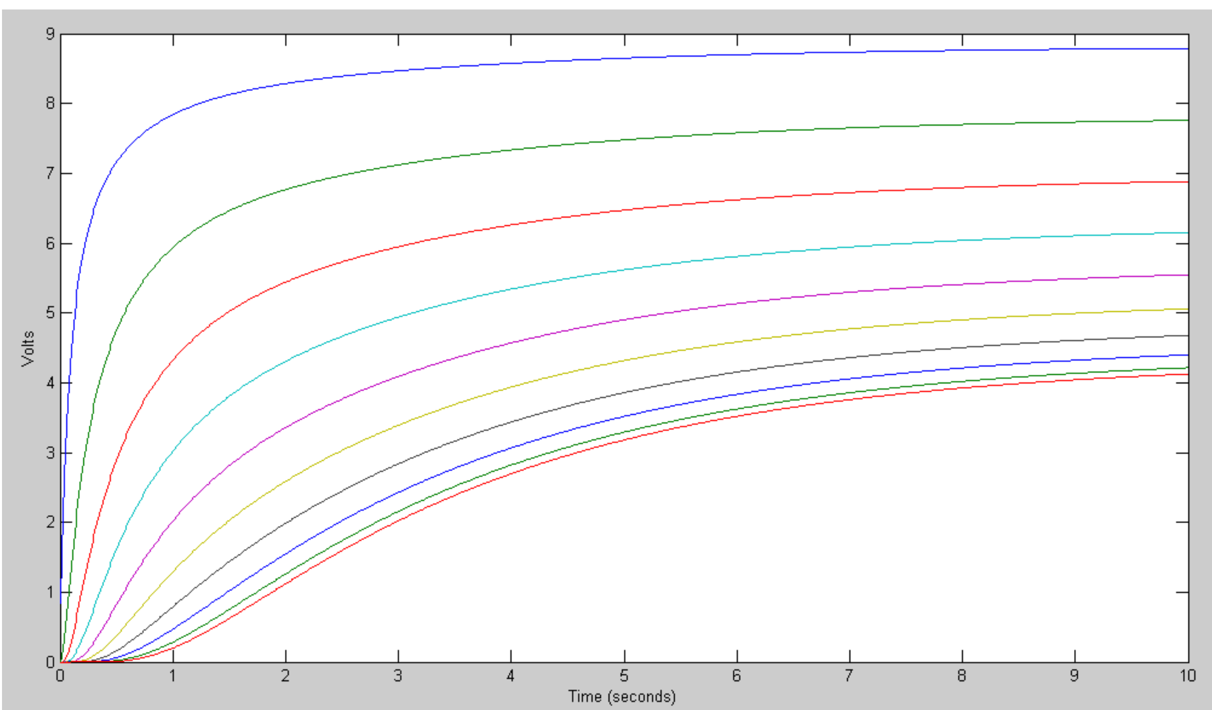
Matlab Code:

```
% ECE 111 Homework #7
V = zeros(10,1);
dV = zeros(10,1);
V0 = 10;
dt = 0.01;
t = 0;
y = [];
while(t < 10)
 dV(1) = 8.333*V0 - 16.833*V(1) + 8.333*V(2);
 dV(2) = 8.333*V(1) - 16.833*V(2) + 8.333*V(3);
 dV(3) = 8.333*V(2) - 16.833*V(3) + 8.333*V(4);
 dV(4) = 8.333*V(3) - 16.833*V(4) + 8.333*V(5);
 dV(5) = 8.333*V(4) - 16.833*V(5) + 8.333*V(6);
 dV(6) = 8.333*V(5) - 16.833*V(6) + 8.333*V(7);
 dV(7) = 8.333*V(6) - 16.833*V(7) + 8.333*V(8);
 dV(8) = 8.333*V(7) - 16.833*V(8) + 8.333*V(9);
 dV(9) = 8.333*V(8) - 16.833*V(9) + 8.333*V(10);
 dV(10) = 8.333*V(9) - 8.5*V(10);
 V = V + dV*dt;
 t = t + dt;
 plot([0:10], [V0;V], '.-');
 ylim([0,10]);
 pause(0.01);
  y = [y ; V'];
end
pause(3)

t = [1:length(y)]' * dt;
plot(t,y);
xlabel('Time (seconds)');
ylabel('Volts');
```

Resulting Graph (for 0..10 seconds)

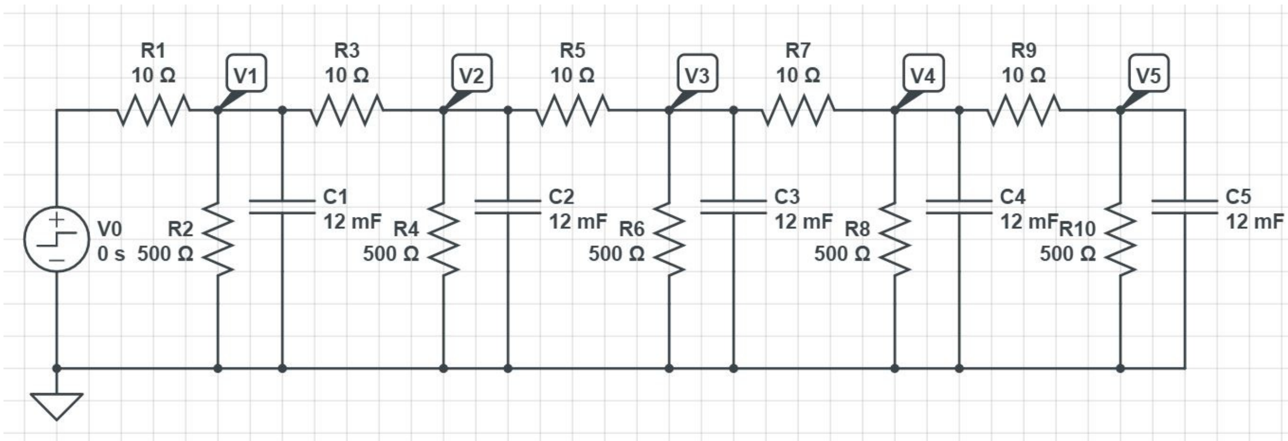


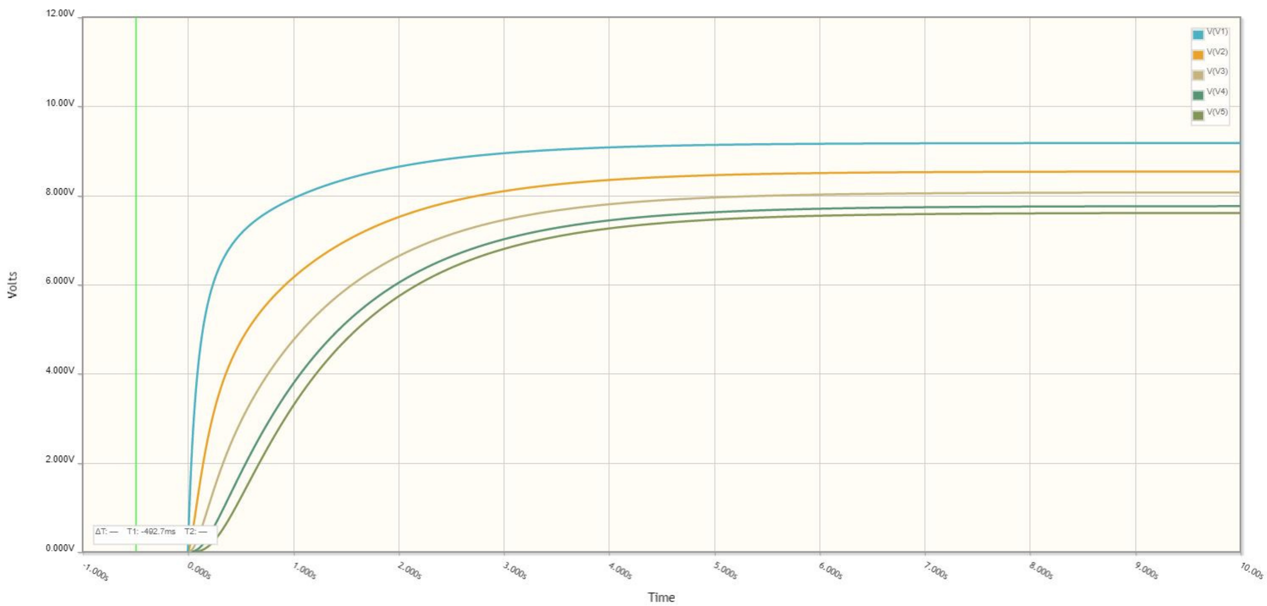
Voltage at Eac Node at t = 10 second



Votlage of each node vs. Time

Problem 4) Using CircuitLab, find the response of this circuit to a 10V step input. *note: It's OK if you only build this circuit to 3 nodes...*





This is almost the same result as Matlab (a little different since there are only four capacitors rather than ten)

Natural Response

Problem 5) Assume V0 = 0V. Determine the initial conditions of V1..V10 so that

- The maximum voltage is 10V and
- 5a) The voltages go to zero as slow as possible
- 5b) The voltages go to zero as fast as possible.

Simulate the response for these initial conditions in Matlab.

This is an eigenvector problem

- A is a 10x10 matrix
- A has ten eigenvalues (how the system behaves)
- A has ten eigenvectors (what behaves that way)

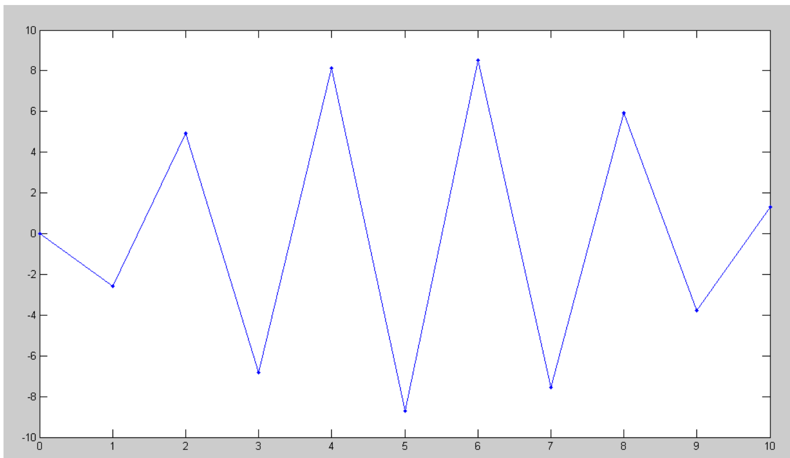
The slow eigenvector decays as per its eigenvector (red)

The fast eigenvector decays as per its eigenvector (blue)

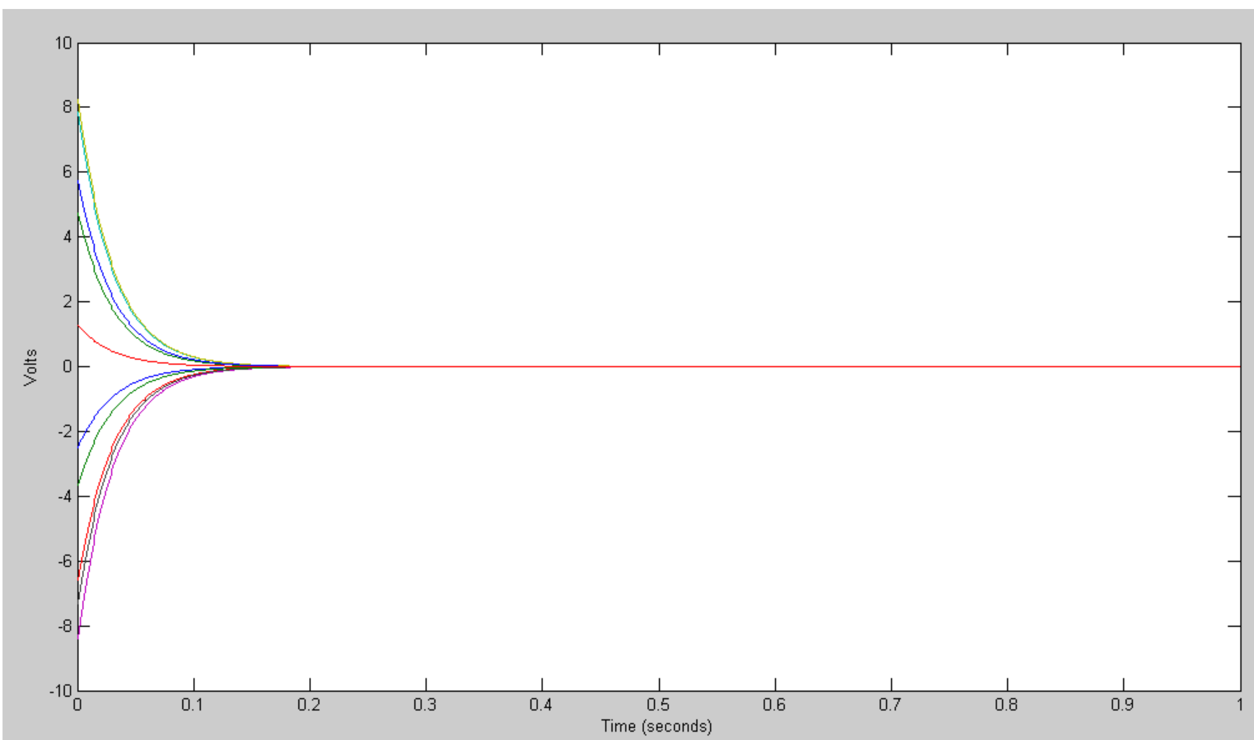
```
>> A = zeros(10,10);
>> for i=1:9
A(i,i) = -16.833;
A(i+1,i) = 8.333;
A(i,i+1) = 8.333;
end
>> A(10,10) = -8.5;
>> A

  -16.8330    8.3330         0         0         0         0         0         0         0         0
    8.3330  -16.8330    8.3330         0         0         0         0         0         0         0
         0    8.3330  -16.8330    8.3330         0         0         0         0         0         0
         0         0    8.3330  -16.8330    8.3330         0         0         0         0         0
         0         0         0    8.3330  -16.8330    8.3330         0         0         0         0
         0         0         0         0    8.3330  -16.8330    8.3330         0         0         0
         0         0         0         0         0    8.3330  -16.8330    8.3330         0         0
         0         0         0         0         0         0    8.3330  -16.8330    8.3330         0
         0         0         0         0         0         0         0    8.3330  -16.8330    8.3330
         0         0         0         0         0         0         0         0    8.3330   -8.5000

>> [M,V] = eig(A)

M =

     fast                                                                                    slow
   -0.1286   -0.2459    0.3412    0.4063    0.4352    0.4255    0.3780    0.2969   -0.1894    0.0650
    0.2459    0.4063   -0.4255   -0.2969   -0.0650    0.1894    0.3780    0.4352   -0.3412    0.1286
   -0.3412   -0.4255    0.1894   -0.1894   -0.4255   -0.3412   -0.0000    0.3412   -0.4255    0.1894
    0.4063    0.2969    0.1894    0.4352    0.1286   -0.3412   -0.3780    0.0650   -0.4255    0.2459
   -0.4352   -0.0650   -0.4255   -0.1286    0.4063    0.1894   -0.3780   -0.2459   -0.3412    0.2969
    0.4255   -0.1894    0.3412   -0.3412   -0.1894    0.4255   -0.0000   -0.4255   -0.1894    0.3412
   -0.3780    0.3780    0.0000    0.3780   -0.3780   -0.0000    0.3780   -0.3780    0.0000    0.3780
    0.2969   -0.4352   -0.3412    0.0650    0.2459   -0.4255    0.3780   -0.1286    0.1894    0.4063
   -0.1894    0.3412    0.4255   -0.4255    0.3412   -0.1894    0.0000    0.1894    0.3412    0.4255
    0.0650   -0.1286   -0.1894    0.2459   -0.2969    0.3412   -0.3780    0.4063    0.4255    0.4352


V =

  -32.7586  -30.6031  -27.2241  -22.9218  -18.0785  -13.1245   -8.5000   -4.6160   -1.8175   -0.3531
```

Change the Matlab code for the fast eigenvector

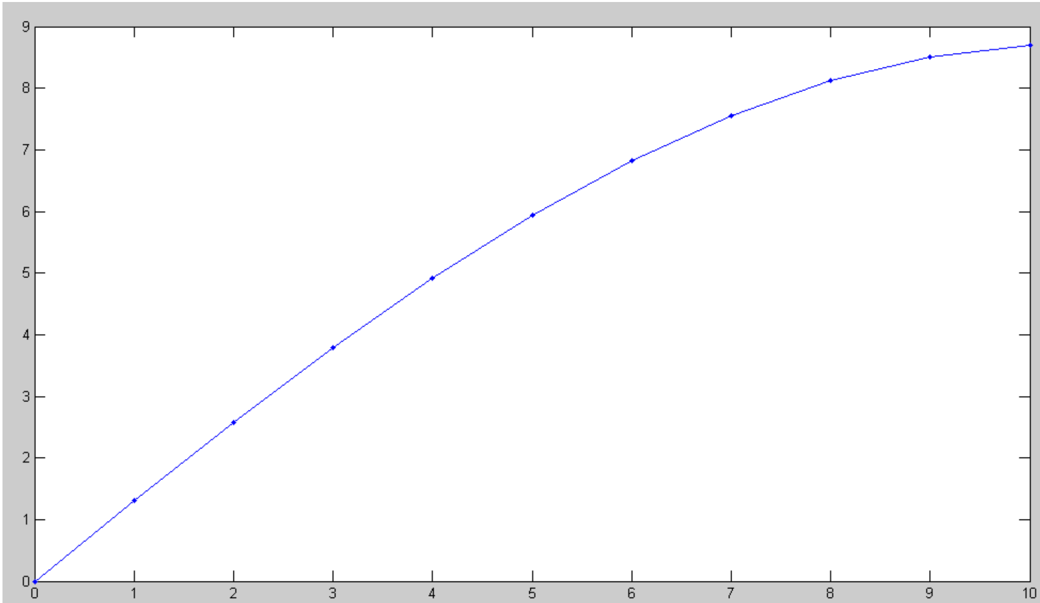
```
% ECE 111 Homework #7
V = M(:,1) * 20;
dV = zeros(10,1);
V0 = 0;
dt = 0.001;
t = 0;
y = [];
while(t < 1)
 dV(1) = 8.333*V0 - 16.833*V(1) + 8.333*V(2);
 etc
```



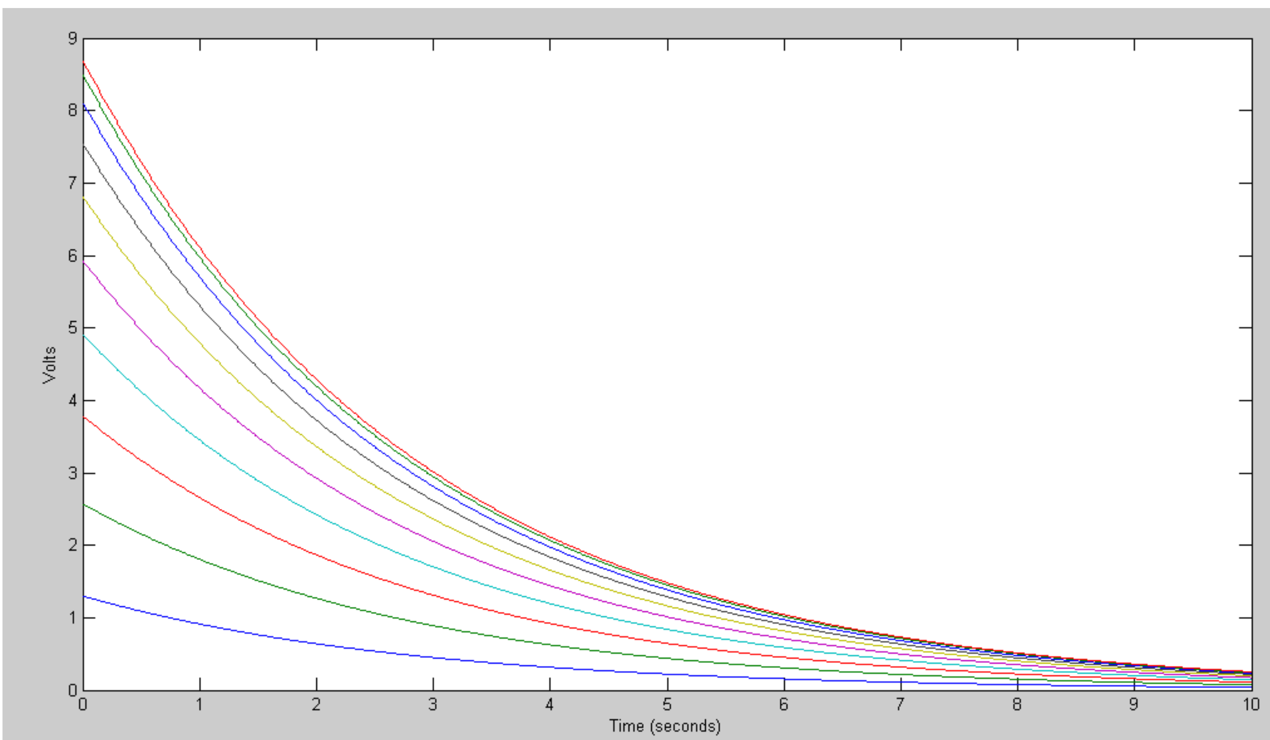
Fast Eigenvector



The fast eigenvector decays quickly (as exp(-32.75t)



Slow Eigenvector

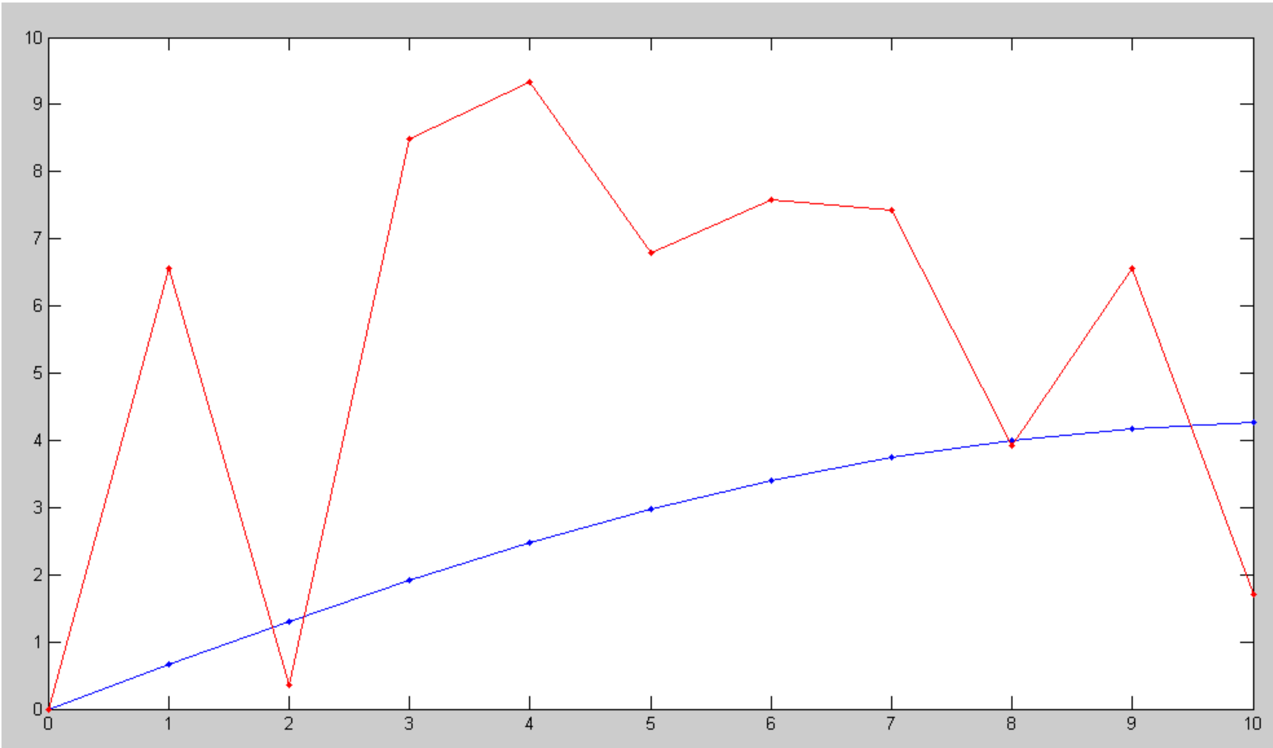


The slow eigenvector decays slowly

Problem 6) Assume Vin = 0V. Pick random voltages for V1 .. V10 in the range of (0V, 10V):

```
V = 10 * rand(10,1)
```

Plot the votlages at t = 2. Which eigenvector does it look like?



Initial Voltage (red) and votlage after 2 seconds (blue)

After two seconds, the voltage looks like the slow eigenvector