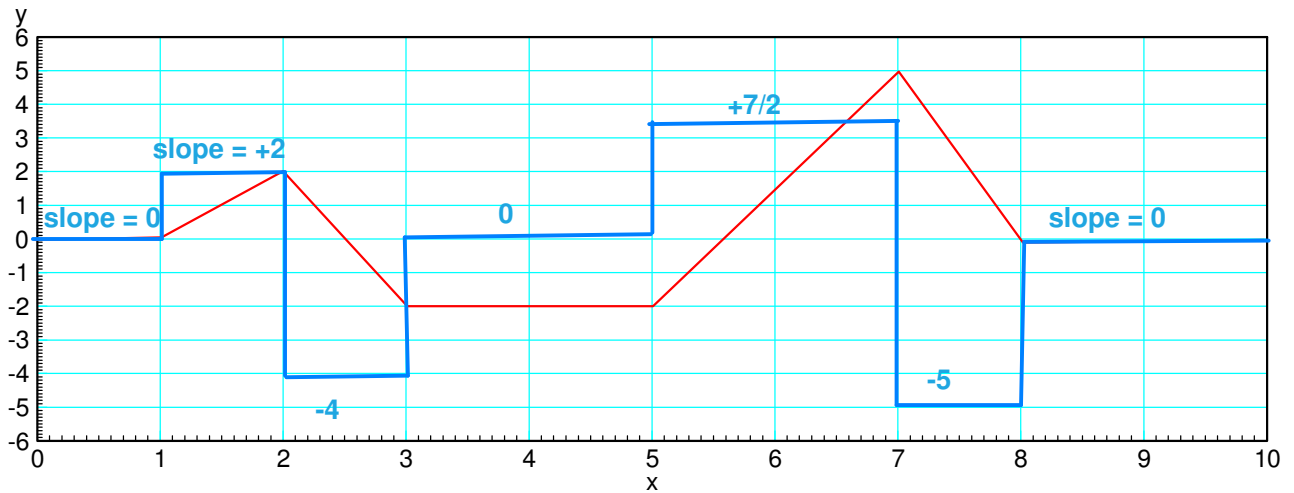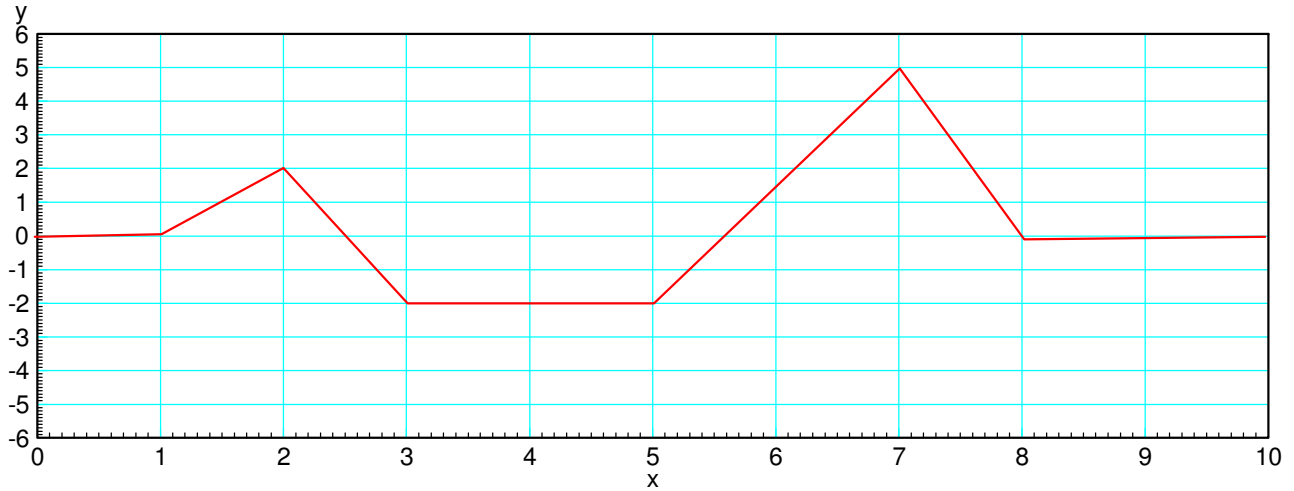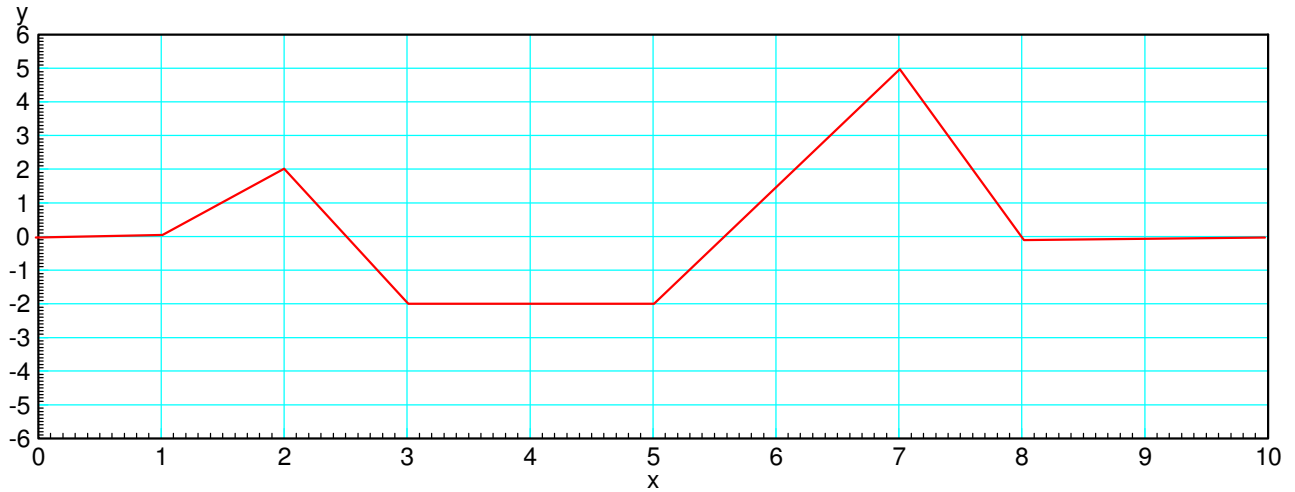# ECE 111 - Homework #4:

Week #4: Math 165 Calculus - Due Tuesday, February 7th

1) Sketch the derivative of the following funciton

If this is the balance of your checking account, how much money are you adding (positive) or withdrawing (negative) for the balance to be as shown?

2) Sketch the integral of the following funciton

If this is how much money you are depositing (positive) or withdrawing (negative) from your checking account, what is the balance at each instance?

area = +1.5

area = -5.0

area = +6.25
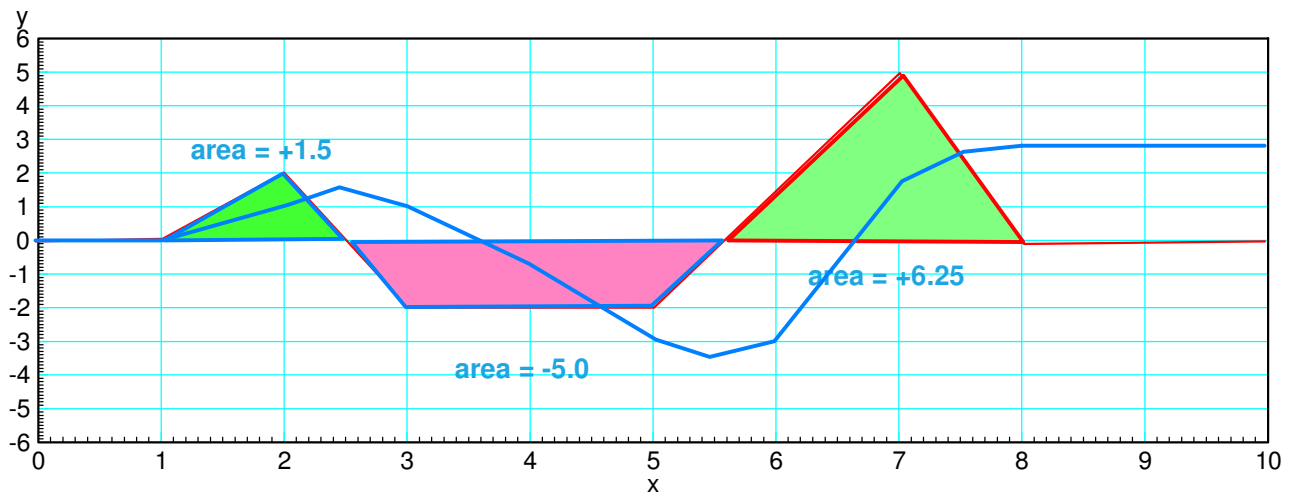
Numerical Differentiation:

3) Use numerical methods to determine the derivative of y:

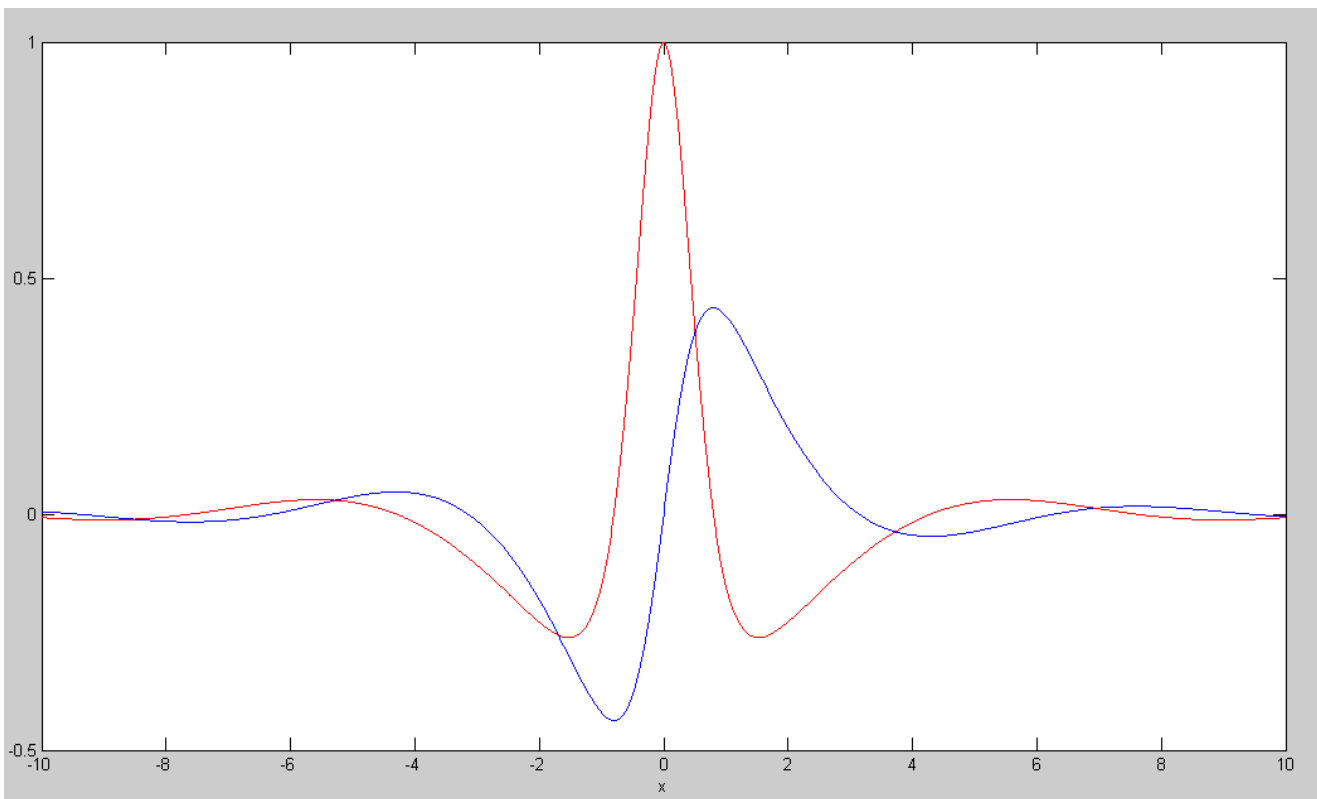$$y = \left(\frac{\sin(x)}{x^2+1} \right)$$

$$z = \frac{d}{dx}(y)$$

for -10 < x < 10. (a plot is sufficient).

Function derivative:

```
function [ dy ] = derivative( x, y )

dx = x(2) - x(1);
dy = 0*y;
n = length(y);
for i=2:n-1
  dy(i) = ( y(i+1) - y(i-1) ) / ( x(i+1) - x(i-1) );
end

dy(1) = (y(2) - y(1))/(x(2) - x(1));
dy(n) = (y(n) - y(n-1)) / (x(n) - x(n-1));

end
```

Computing y using numerical methods

```
>> dx = 0.01;
>> x = [-10:dx:10]';
>> y = sin(x) ./ (x.^2 + 1);
>> z = derivative(x, y);
>> plot(x,y,'b',x,z,'r');
>> xlabel('x');
```


y(x) (blue) and its derivative (red)
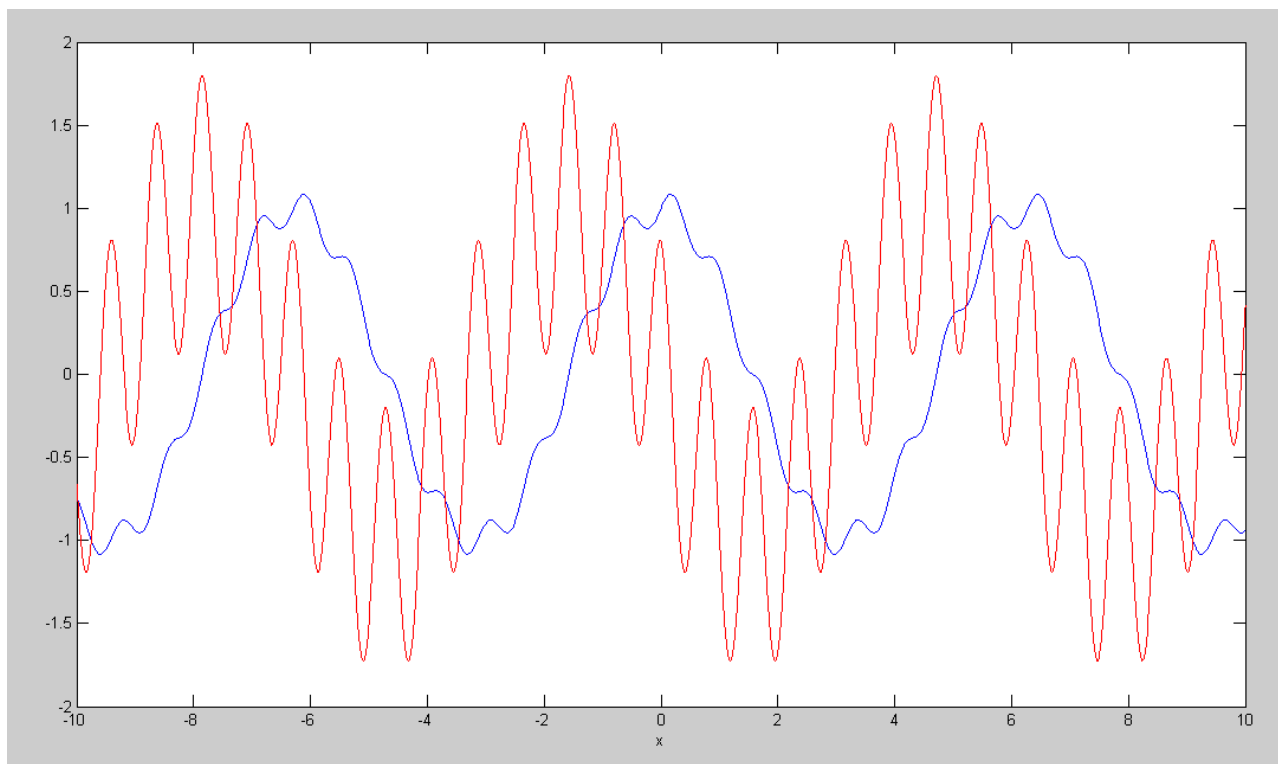
4) Use numerical methods to determine the derivative of y:

$$y = \cos(x) + 0.1\sin(8x)$$

$$z = \frac{d}{dx}(y)$$

for -10 < x < 10. (a plot is sufficient).

```
>> dx = 0.01;
>> x = [-10:dx:10]';
>> y = cos(x) + 0.1*sin(8*x);
>> dy = derivative(x, y);
>> plot(x,y,'b',x,dy,'r');
>> xlabel('x');
```

Note: differentiation amplifies high-frequency terms (usually bad)

y(x) (blue) and its derivative (red)

Numerical Integration

5) Use numerical methods to determine the intergral of y(x)

$$y = \left(\frac{\sin(x)}{x^2+1} \right)$$
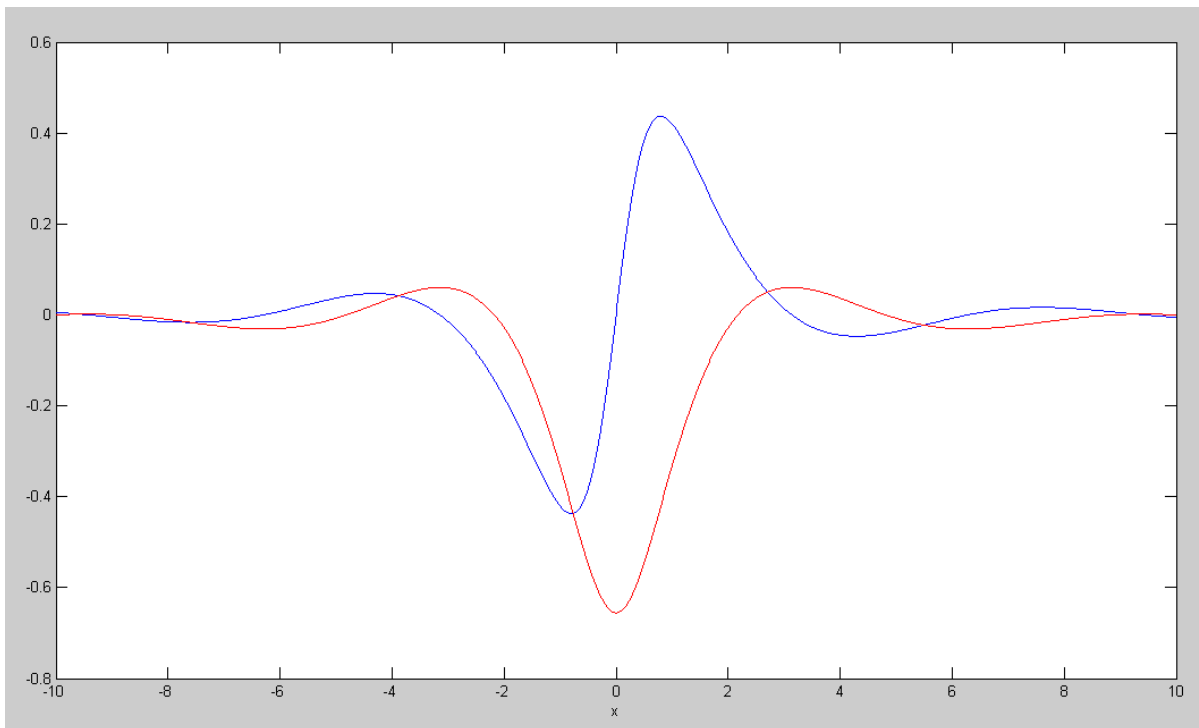
$$z = \int y \cdot dx$$

for -10 < x < 10. (a plot is sufficient). Assume z(-10) = 0.

Use the following function (uses trapezoid rule for integration)

```
function [y ] = Integrate ( x, dy )

npt = length(x);
y = 0*dy;
for i=2:npt
 y(i) = y(i-1) + 0.5*(dy(i) + dy(i-1)) * (x(i) - x(i-1));
end
end
```

Solving in Matlab

```
>> dx = 0.01;
>> x = [-10:dx:10]';
>> y = sin(x) ./ (x.^2 + 1);
>> z = Integrate(x, y);
>> plot(x,y,'b',x,z,'r');
>> xlabel('x');
```


y(x) (blue) and its integral (red)

Note: Numerical integration works for any function you can input into Matlab. Even ones you don't know
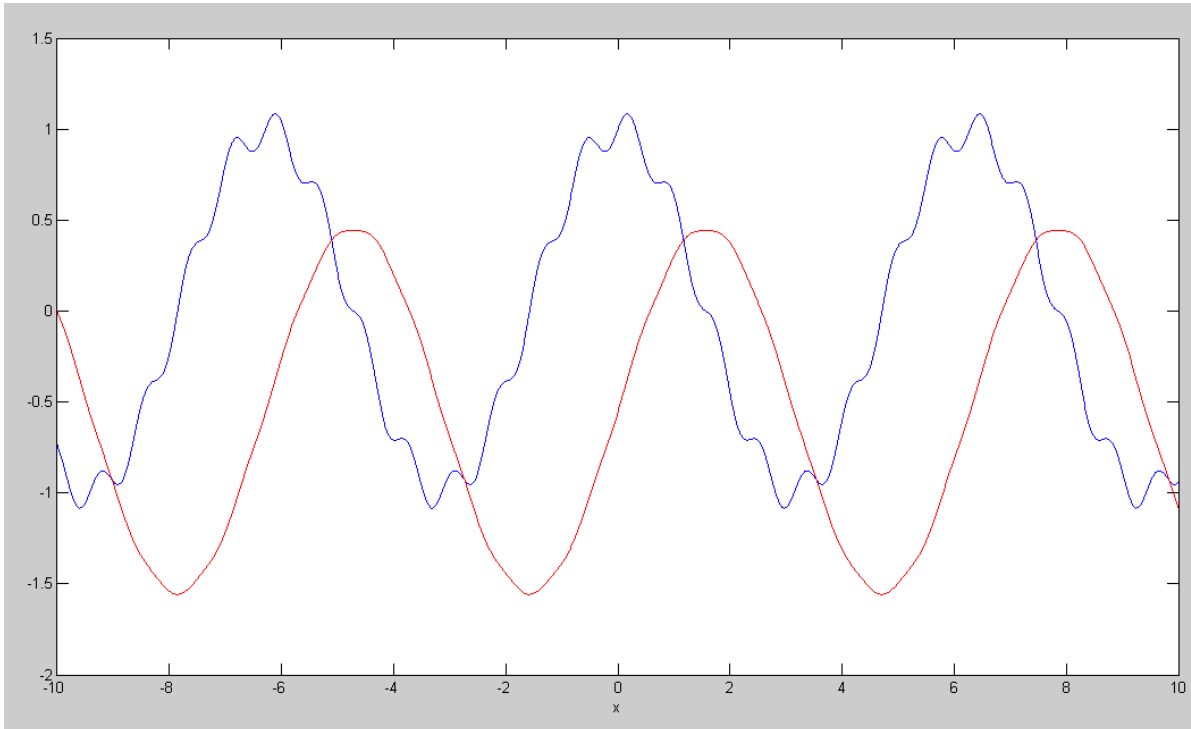how to do by hand. (very big plus)

6) Use numerical methods to determine the integral of y(x)

$$y = \int (\cos(x) + 0.1 \sin(8x)) \cdot dx$$

$$z = \int y \cdot dx$$

for -10 < x < 10. (a plot is sufficient). Assume z(-10) = 0.

```
>> x  =  [-10:dx:10]';
>> y  =  cos(x)  +  0.1*sin(8*x);
>> z  =  Integrate(x, y);
>> plot(x,y,'b',x,z,'r');
>> xlabel('x');
```


y(x) (blue) and its integral (red)

Note: Integration attenuates high-frequency terms (usually good)
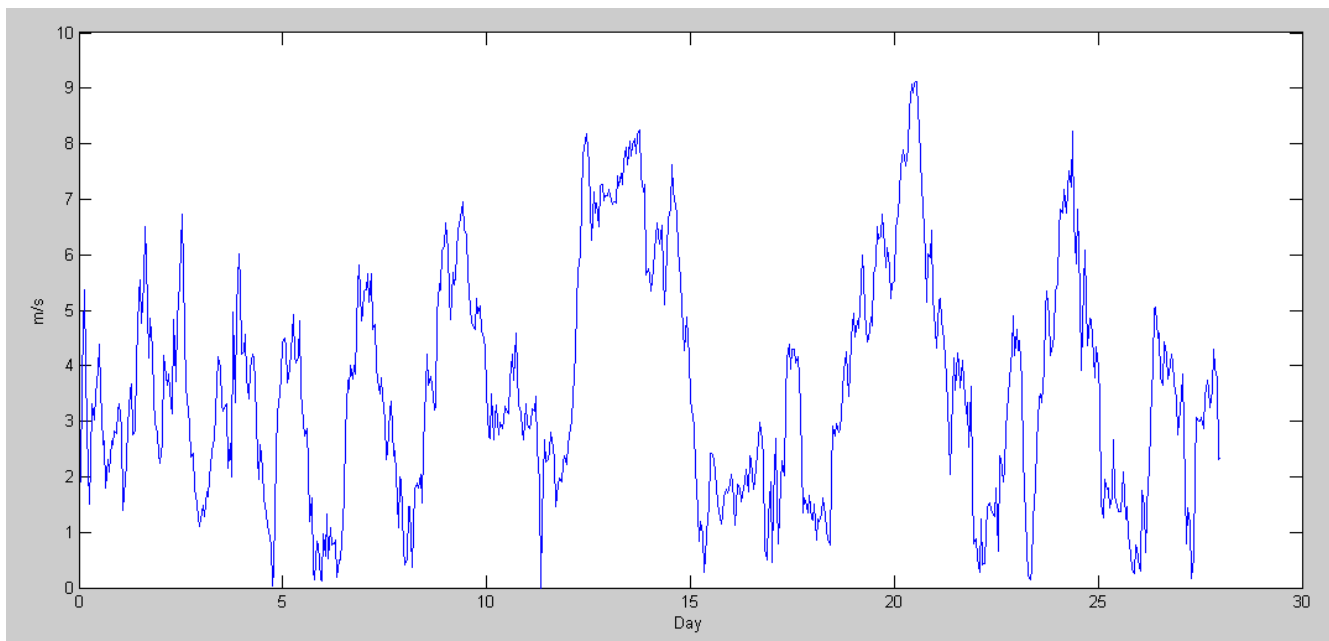
Wind Energy

7) Load the 4-weeks worth of wind-speed data from NDAWN. (close to your home town if you're from North Dakota). Plot this in MATLAB as wind speed vs hour.

https://ndawn.ndsu.nodak.edu/

```
>> Wind = [ <paste data
   }
>> length(Wind)
   672

>> hr = [1:672]';
>> plot(hr/24,Wind)
>> xlabel('Day');
>> ylabel('m/s');
```


Wind speed in Fargo from Dec 5, 2022 to Jan 2, 2023

8) Write a function in Matlab where you pass the wind speed at 180m (about 2.2x the wind speed at the ground) and it returns the power generated by a Vestas V172-7.2MW

Wind Speed (m/s)	0..3	4	5	6	7	8	9	10	11	12	13+
kW	0	53	496	1,096	1,882	2,857	3,968	5,102	6,100	6,803	7,200

https://nozebra.ipapercms.dk/Vestas/Communication/Productbrochure/enventus/enventus-platform-brochure/?page=1

8a) Determine a function in Matlab to approximate this curve.

```
function [kW] = PowerCurve( Wind )

x = [3,4,5,6,7,8,9,10,11,12,13]';
y = [0,53,496,1096,1882,2857,3968,5102,6100,6803,7200]';
B = [x.^3, x.^2, x, x.^0];
A = inv(B'*B)*B'*y;

kW = 0*Wind;
for i=1:length(Wind)
    if(Wind(i) < 3)
        kW(i) = 0;
    elseif(Wind(i) > 13)
        kW(i) = 7200;
    else
        kW(i) = [Wind(i)^3, Wind(i)^2, Wind(i), 1]*A;
    end
end

plot(x,y,'r',Wind,kW,'r.')
end
```
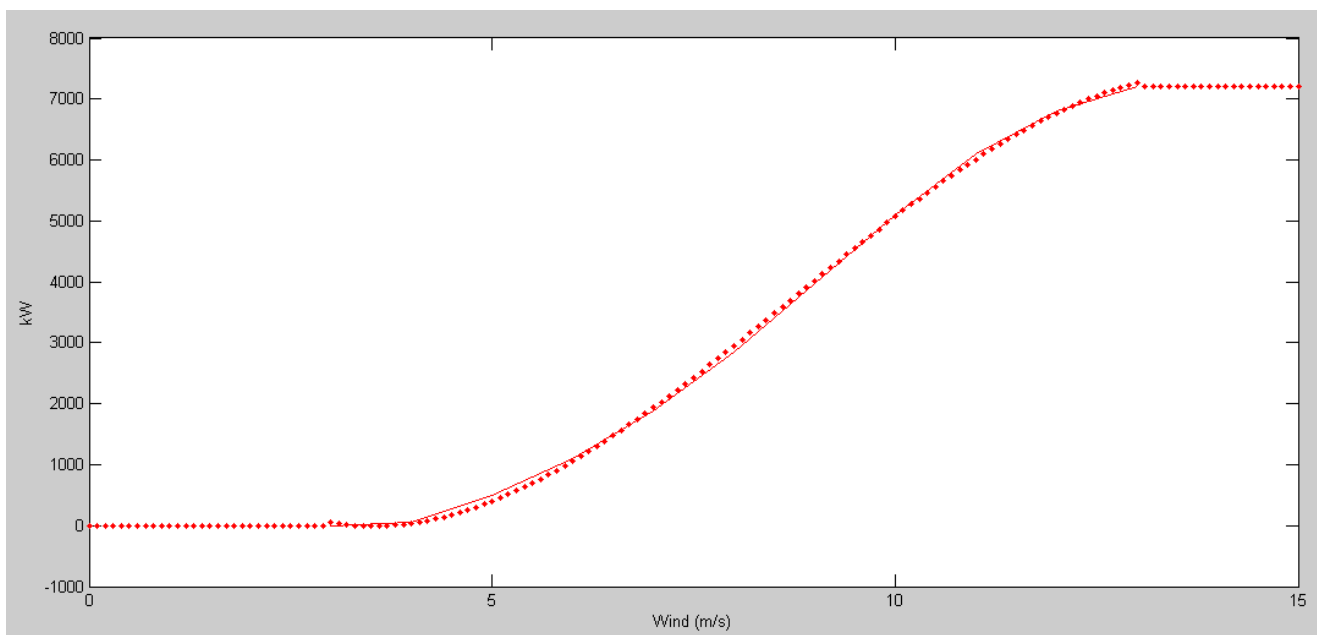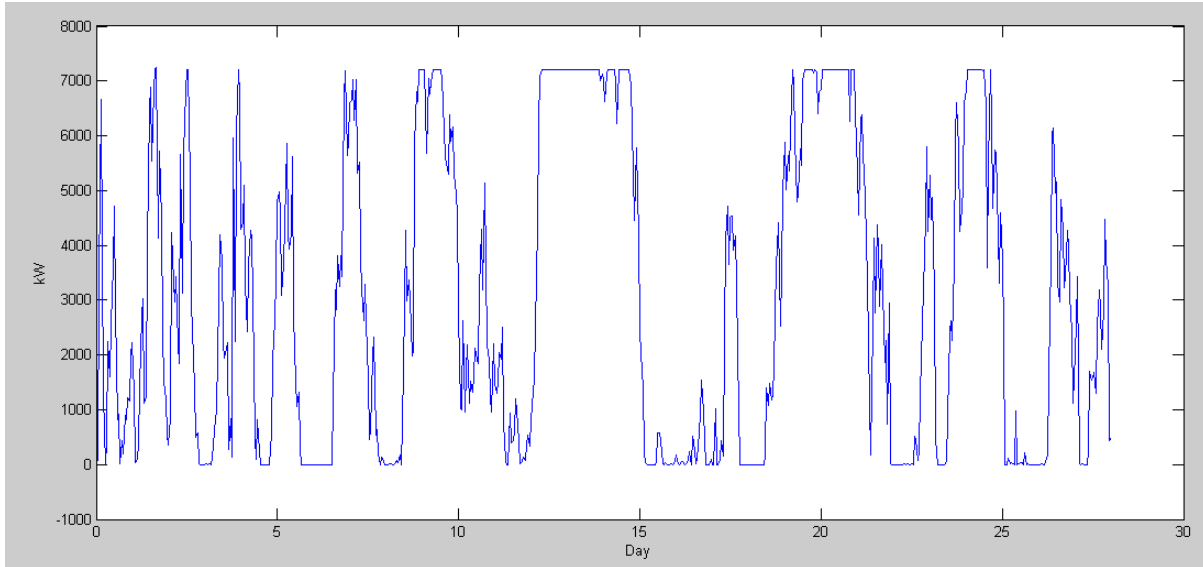
Checking this in Matlab

```
>> Wind = [0:0.1:15]';
>> kW = PowerCurve(Wind);
```


Power Curve (solid red) and Curve Fit (dotted red)

8b) Use this function to compute how much power a Vestas V172-7.2MW wind turbine would produce from the wind data your found in problem 3.

```
>> kW = PowerCurve(Wind * 2.2);
>> hr = [1:length(Wind)]';
>> plot(hr/24, kW)
>> xlabel('Day');
>> ylabel('kW');
```


Approximate Energy a Vestas V172-7.2MW Wind Turbine Would Produce

9) It takes 1.78 pounds of North Dakota lignite coal to produce 1kWh of electricity. How many pounds of coal does this wind turbine offset over 4 weeks?

```
>> mean(kW)
ans =   3061.2
```
The average output of this wind turbine would be 30061MW
   • 42.5% of the nameplate rating of 7200MW
   • 42.5% utilization factor

```
>> kWh = sum(kW) * 1
kWh =   2.0571e+006
```

Over these four weeks, the wind turbine would produce 2.057 million kWh

```
>> Pounds = kWh * 1.78

Pounds =   3.6616e+006
```

This offsets 3.66 million pounds of coal (and the CO2 & other waste it would produce)

```
>> Dollars = kWh * 0.11

Dollars =   2.2628e+005
```
At 11 cents per kWh, the wind turbine would generate $226,280 over this four week span

10. Assume
   - This wind turbine costs $9.36 million to build ($1300 / kW), and
   - You get $0.11 / kWh for the energy you produce.

How long will it take for this wind turbine to pay for itself?

```
>> Days = 9.36e6 / (Dollars/28)

Days =   1.1582e+003

>> Years = Days / 365

Years =    3.1732
```

Extrapolating, the wind turbine should pay for itself in 3.17 years. It is a very good invenstment.

https://nozebra.ipapercms.dk/Vestas/Communication/Productbrochure/enventus/enventus-platform-brochure/?page=1