ECE 111 - Homework #3

Week #3: Linear Algebra. Due Tuesday, January 31st

N equations & N unknowns

1) Solve for {x, y}

$$7x + 4y = 5$$

$$9x + 6y = -2$$

Place in matrix form

$$\begin{bmatrix} 7 & 4 \\ 9 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

Solve using Matlab

```
>> A = [7,4 ; 9,6]

     7     4
     9     6

>> B = [5;-2]

     5
    -2

>> C = inv(A)*B

x    6.3333
y   -9.8333
```

2) Solve for {x, y, z}

$$4x - 9y - 8z = -3$$
$$-6x - 4y + 7z = 10$$
$$5x - 9y + 4z = -9$$

Place in matrix form

$$\begin{bmatrix} 4 & -9 & -8 \\ -6 & -4 & 7 \\ 5 & -9 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \\ -9 \end{bmatrix}$$

Solve using Matlab

```
>> A = [4,-9,-8 ; -6,-4,7 ; 5,-9,4]

     4    -9    -8
    -6    -4     7
     5    -9     4

>> B = [-3;10;-9]

    -3
    10
    -9

>> C = inv(A)*B
```

x -1.9187
y -0.2171
z -0.3401

3) Solve for {a, b, c, d}

$$-a-6b+5c+4d=10$$
$$-2a+6c-6d=-3$$
$$6a-b-4c-7d=2$$
$$6a+3b+4c=-5$$

Place in matrix form

$$\begin{bmatrix} -1 & -6 & 5 & 4 \\ -2 & 0 & 6 & -6 \\ 6 & -1 & -4 & -7 \\ 6 & 3 & 4 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 10 \\ -3 \\ 2 \\ -5 \end{bmatrix}$$

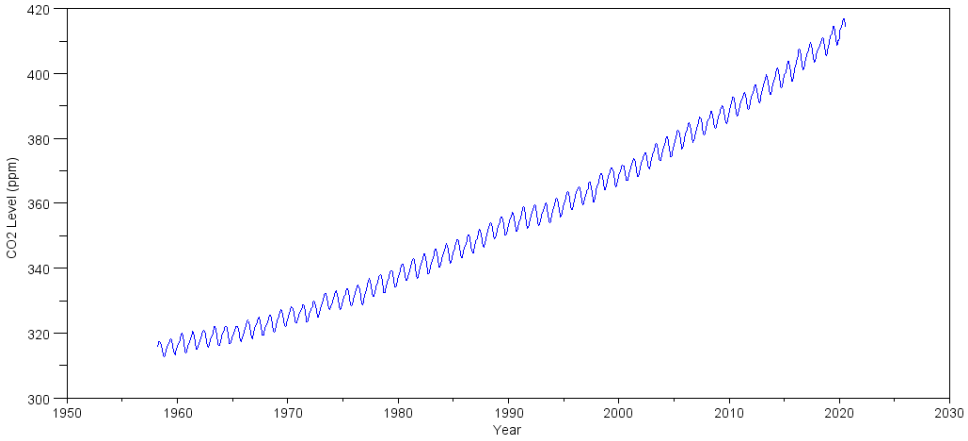
Solve using Matlab

```
>> A = [-1,-6,5,4 ; -2,0,6,-6 ; 6,-1,-4,-7 ; 6,3,4,0]

    -1    -6     5     4
    -2     0     6    -6
     6    -1    -4    -7
     6     3     4     0

>> B = [10;-3;2;-5]

    10
    -3
     2
    -5

>> C = inv(A)*B
```

a 0.1703
b -1.7213
c -0.2145
d 0.2287

Global CO2 Levels

The CO2 levels measured at Mauna Loa observatory for the past 52 years are:



https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_mm_mlo.txt
http://www.bisonacademy.com/ECE111/Code/CO2%20Levels.txt

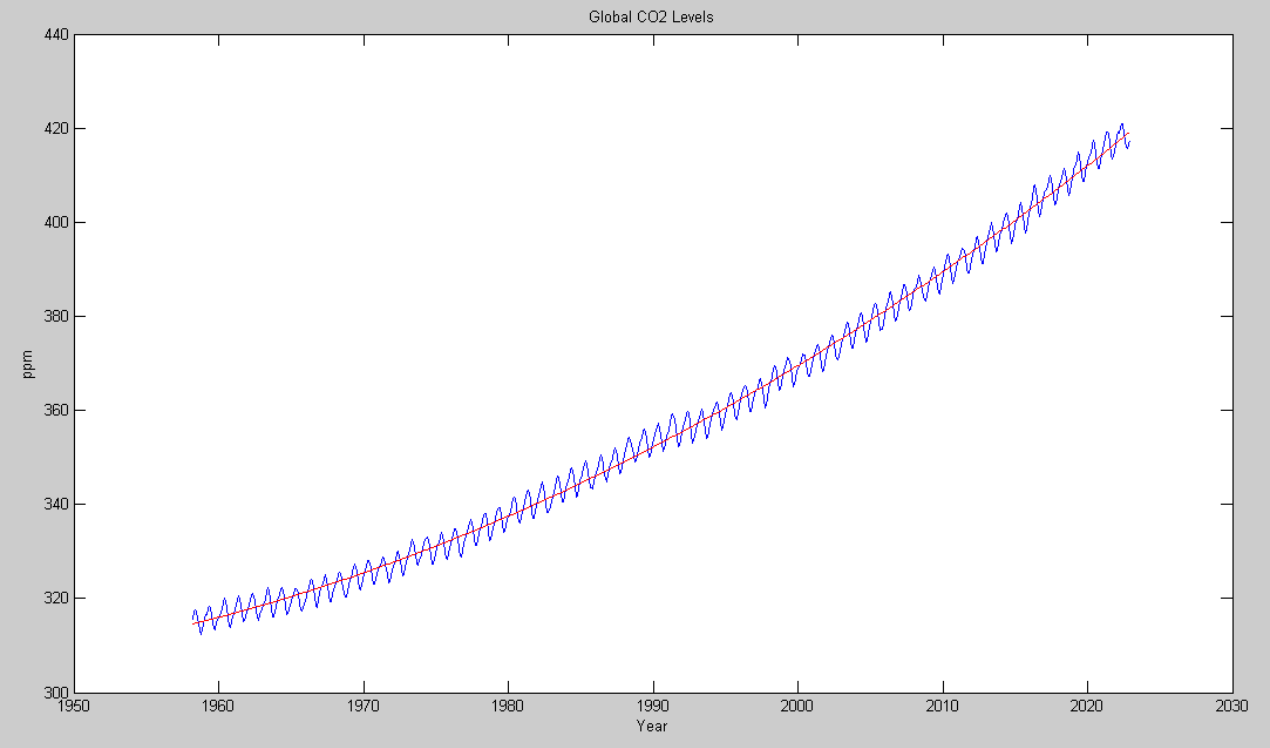
Problem 4) Determine a parabolic curve fit for this data in the form of

$$CO_2 \approx ay^2 + by + c$$

where 'y' is the year.

In Matlab

```
>> DATA = [  <paste data here>
   ];
>> year = DATA(:,3);
>> CO2 = DATA(:,4);
>> plot(year,CO2)

>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*CO2

A =

  1.0e+004 *

   0.000001312554484
  -0.005063966937742
   4.914667966306818



>> plot(year,CO2,'b',year, B*A, 'r');
>> xlabel('Year');
>> ylabel('ppm');
>> title('Global CO2 Levels');
```



From this data, when do you predict that we will hit

- 400ppm?
- 2000 ppm of CO2? (the same as what was observed during the Permian extinction)

Matlab doesn't have a f(x) = 400 function. It *does* have a f(x) = 0 function (roots). Change the prolem from

$$ay^2 + by + c = 400$$

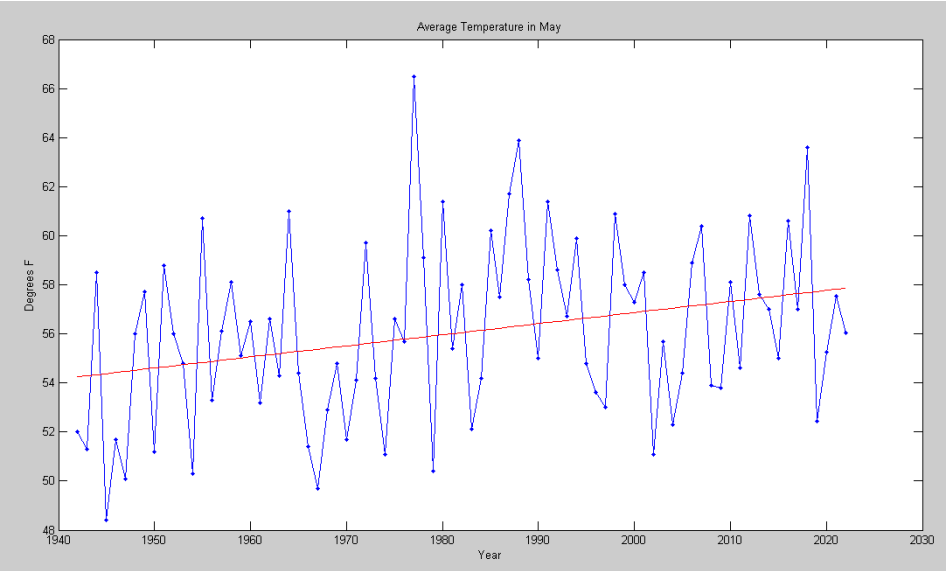
to

$$ay^2 + by + (c - 400) = 0$$

```
>> roots(A - [0;0;400])

   2014.81      Year when we hit 400ppm
   1843.28      stray solution

>> roots(A - [0;0;2000])

   2288.57      year we hit 2000ppm (266 years from now)
   1569.52      stray solution

>>
```

Fargo Temperatures

5) The average temperature in May for Fargo, ND is as follows:



5a) Determine a curve fit of the form of $T = ay + b$

Inport the data

```
>> DATA = [ <paste data>
   ];
>> year = DATA(:,1);
>> May = DATA(:,6);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*May

a   0.045224254742591
b -33.580522282557865

>> plot(year,May,'b.-',year, B*A, 'r');
>> xlabel('Year');
>> ylabel('Degrees F');
>> title('Average Temperature in May')
```

$T \approx 0.04522 \cdot year - 33.5805$ degrees F

5b) How much has Fargo warmed up over the past 80 years?

```
>> a = A(1);
>> 80*a
   3.6179
```

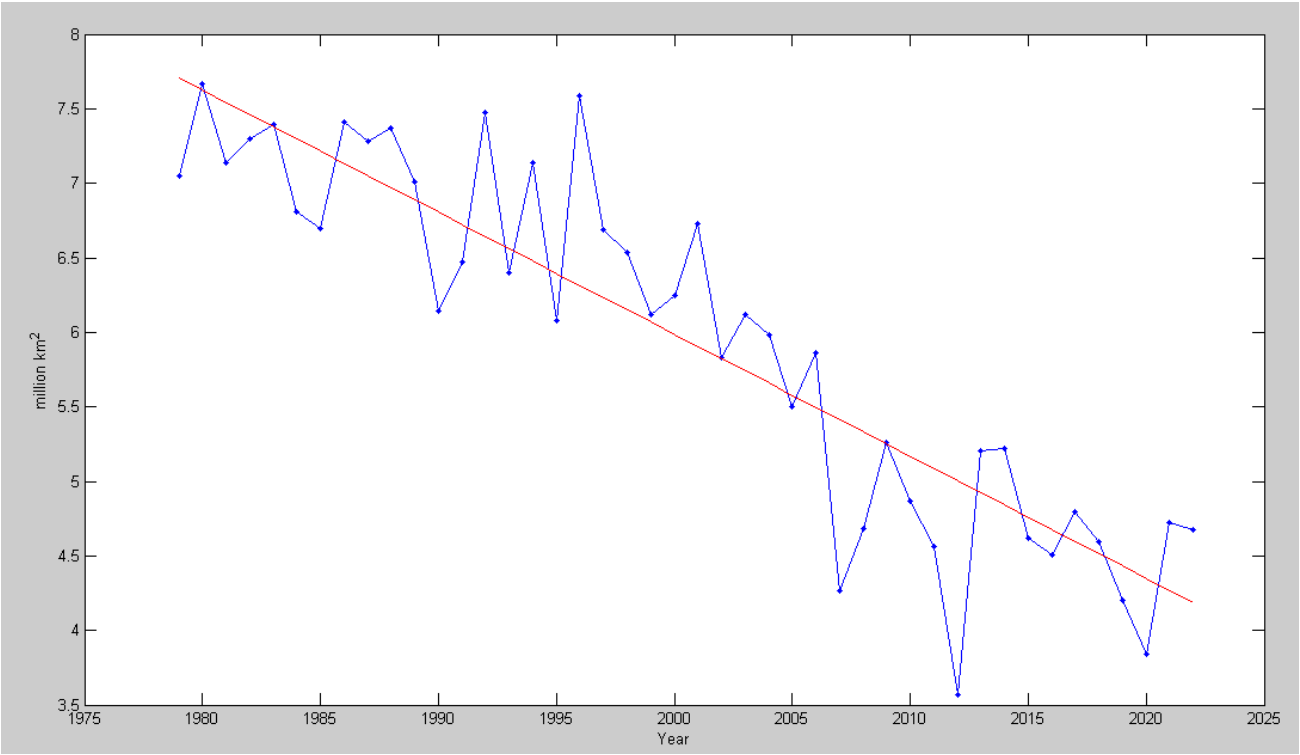
It has warmed up by +3.61F in Fargo in May since 1942

5c) What will the average temperature in Fargo be in May in the year 2050?

```
>> a = A(1);
>> b = A(2);
>> y = a*2050 + b
y =  59.1292
```

From the trend, I'd predict that the average temperature in May 2050 will be +59.12F

Problem 6-7) Sea Ice: The area covered by sea ice is recored by the National Snow and Ice Data Center:



6) Approximate this data from the years 1979 - 2022 with a line

$$Area \approx ay + b$$

From this curve fit, when do you expect the Arctic to be ice free? (First time in 5 million years)

```
>> year = DATA(:,1);
>> ICE = DATA(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*ICE

a   -0.0818
b  169.6574

>> plot(year,ICE,'b.-',year,B*A,'r');
>> xlabel('Year');
>> ylabel('million km^2')

>> roots(A)

  2073.2
```

Assuming a linear curve fit, the Arctic should be ice free in the year 2073.2

7) Approximate this data with a parabolic curve fit:

$$Area \approx ay^2 + by + c$$

From this curve fit, when do you expect the Arctic to be ice free?

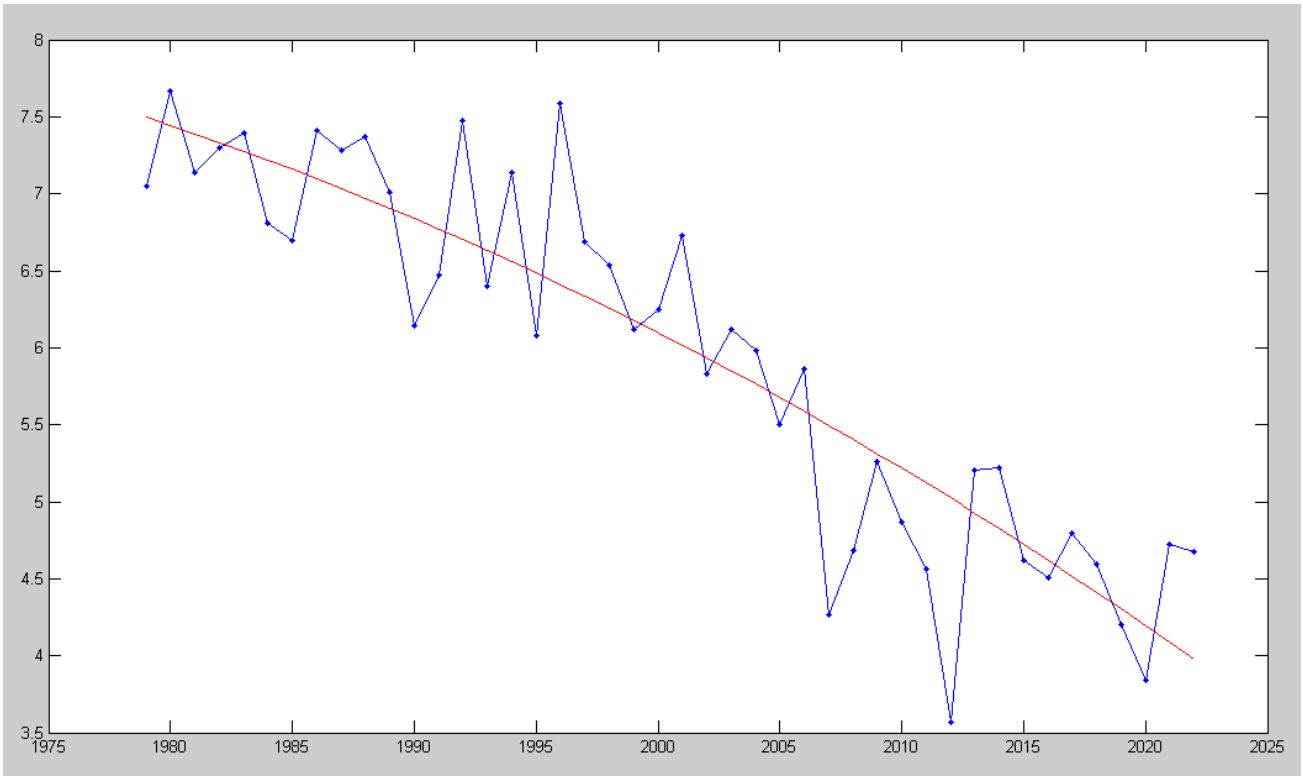
```
>> B = [year.^2, year, year.^0];
>> format short e
>> A = inv(B'*B)*B'*ICE
```

```
a -6.9420e-004
b  2.6957e+000
c -2.6084e+003
```

```
>> roots(A)
```

```
  2052.0        The Arctic should be ice free by 2052 (29 years from now)
  1831.1        stray solution
```

```
>> plot(year,ICE,'b.-',year,B*A,'r');
>>
```



Problem 8-9: World Temperatures. NASA Goddard has been keep records since 1880 (139 years of data).

8) Determine a least-squares curve fit for this data from the year 1880 - 1930 in the form of

$$\delta T = ay + b$$

```
>> DATA = [ <paste data from 1880 - 1930>
   ];
>> year = DATA(:,1);
>> dT = DATA(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*dT

a -9.6729e-004
b  1.6024e+000

>> plot(year,dT,'b.-',year,B*A,'r');
>> xlabel('year');
>> ylabel('degrees C')
```

Based upon this data, what *should* the temperature deviation be in the year 2022?

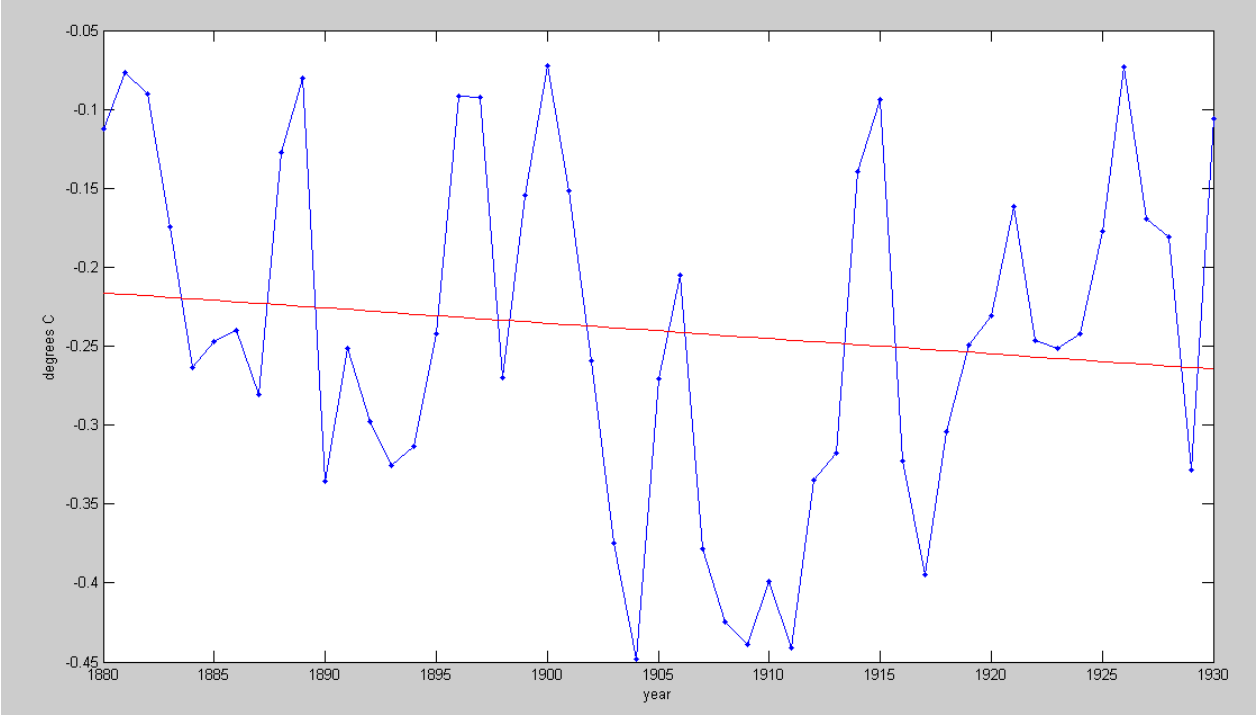
```
>> [2022, 1] * A

 -0.3535
```

Based upon the data from 1880 - 1930, the global average temperature in 2022 should be -0.35C

sidelight: Back in 1900, there were worries that we were entering a new ice age

The actual teperature deviation in 2022 was +0.8591C, or +1.2126C higher

Global warming might actually be more than we're measuring...



9) Determine a least-squares curve fit for this data from the year 1980 - 2022 in the form of

$$\delta T \approx ay^2 + by + c$$

Based upon this data, predict when we will see a 10 degree temperature increase if nothing changes?

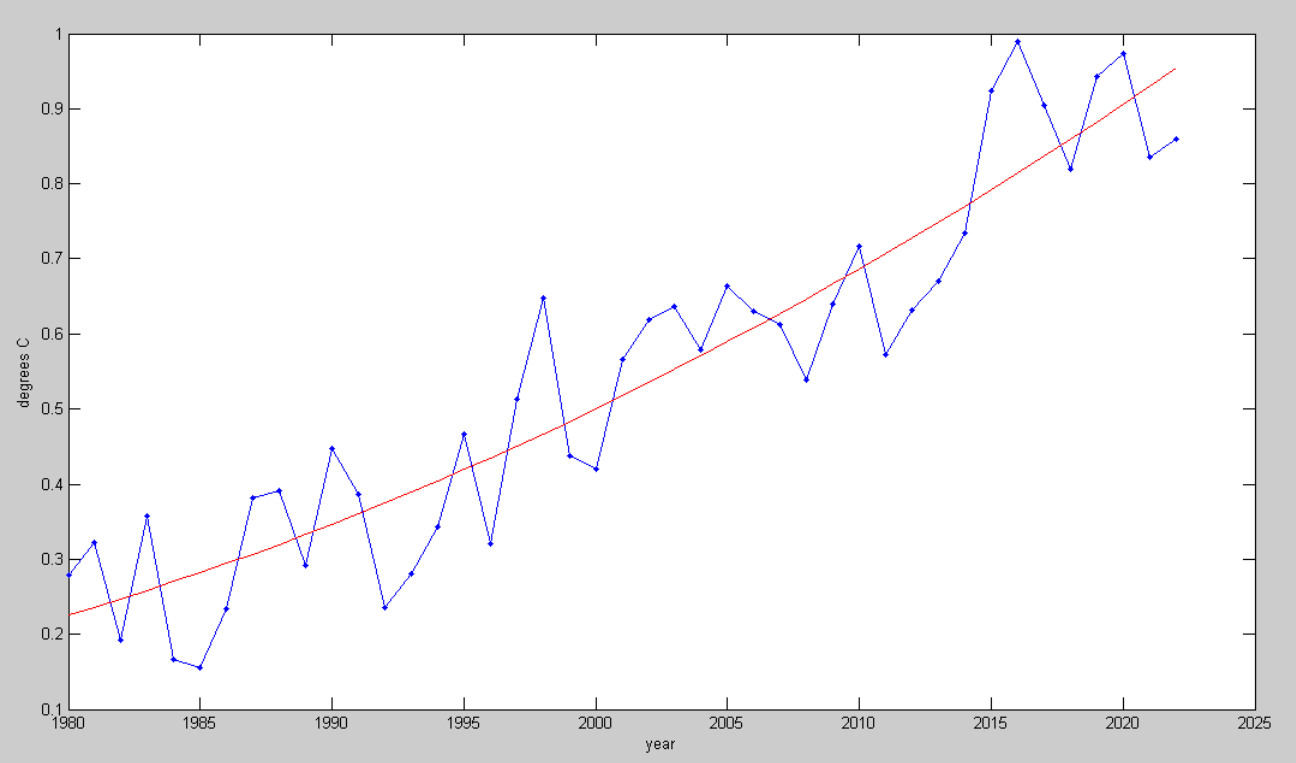
```
>> DATA = [ < paste data from 1880 - 2022>
>> year = DATA(:,1);
>> dT = DATA(:,2);
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*dT
```

```
a  1.6382e-004
b -6.3828e-001
c  6.2177e+002
```

```
>> roots(A - [0;0;10])
```

```
  2194.4        +10C in the year 2194 (171 years from now)
  1701.8        stray solution
```

```
>> plot(year,dT,'b.-',year,B*A,'r');
>> xlabel('year');
>> ylabel('degrees C')
```



10) What does a temperature rise of 10 degrees mean for the planet?

not graded - too political

The Permian Extinction Event suggests that it's not good: no animals larger than a mouse survived the Permian Extinction - which was triggered by CO2 levels at 2000ppm and a +10 degree C temerature rise.

One Degree: 2023 Summers like 2003 where a heat wave in France caused 10,000 deaths become the norm. Flows of the Po and Rhine river decrease. Crop production drops.

```
-->roots(A - [0;0;1])

  2023.9
  1872.3
```

Two Degrees: 2057. Oceans absorb less CO2 (too hot) and soils start to release CO2. Vacations to the Mediterranean in the summer are just too hot. Crop failures in Africa and Central America cause mass migration. Coastal cities flood. 1/3rd of species face extinction.

```
-->roots(A - [0;0;2])

  2057.0
  1839.3
```

Three Degrees: 2082. Crop failures in China cause the migration of more than 1 billion people. Collapse of equatorial governments.

```
-->roots(A - [0;0;3])

  2082.1
  1814.1
```

Four Degrees: 2103. Spain becomes a desert. Mass migration to Northern latitudes. Rain forests burn up.

```
-->roots(A - [0;0;4])

  2103.2
  1793.0
```

Six Degrees: 2138. Ice caps are gone. Methane hydrates become unstable raising temperatures in a positive-feedback loop. Ocean circulation stops. Hydrogen sulfide producing bacteria flourish poisoning the air. The Ozone layer dissipates leaving the land sterilized with UV radiation. End-Permian-like conditions make life nearly impossible.

```
-->roots(A - [0;0;0;6])

  2138.5
  1757.7
```

Scary? Yes. That's why the rest of the world sees the Paris Climate Accord as being important. That's why the United Nations sees Global Warming as the #1 threat - far greater than terrorism. Far greater than COVID.