ECE 111 - Homework #2

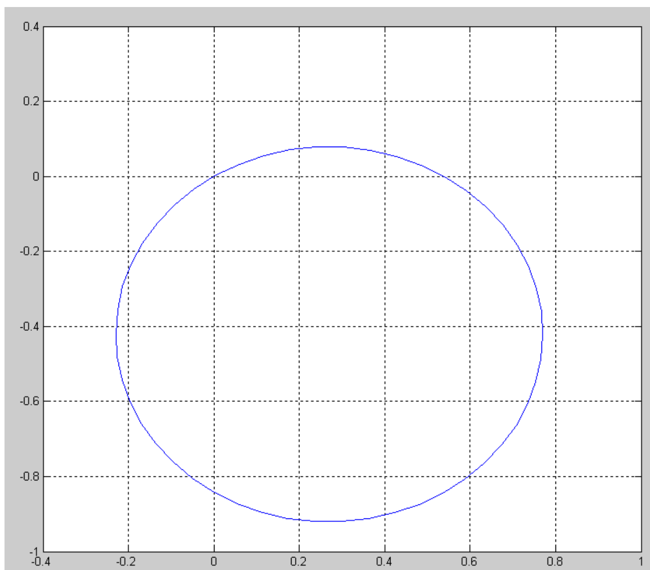
Week #2: Matlab and Trigonometry. Due Tuesday, January 24th

Plot the following functions in Matlab

1) $r = \cos(\theta + 1)$

```
q = [0:0.01:1]' * 2*pi;
r = cos(q + 1);
x = r .* cos(q);
y = r .* sin(q);
plot(x,y)
```

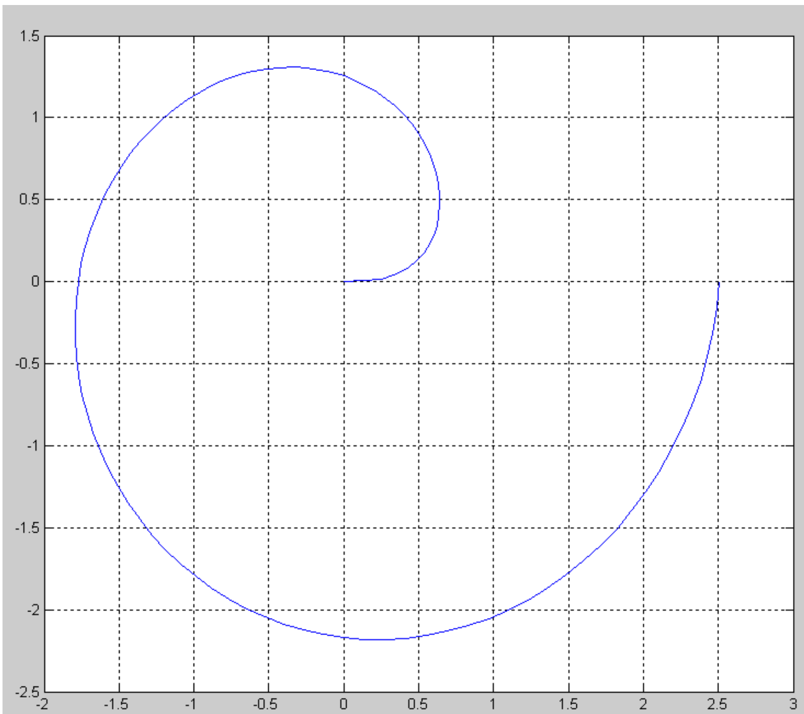
This produces a circle



2) $r = \sqrt{\theta}$

This produces a spiral

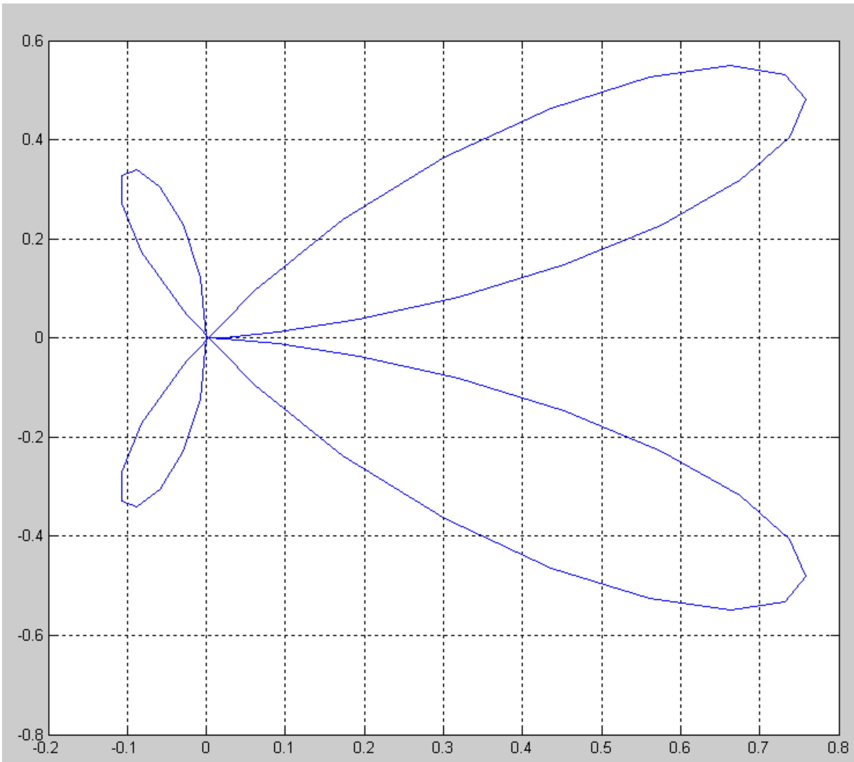
```
q = [0:0.01:1]' * 2*pi;
r = sqrt(q);
x = r .* cos(q);
y = r .* sin(q);
plot(x,y)
grid
```



3) $r = \sin(2\theta) \cdot \cos(3\theta)$

This produces a butterfly

```
q = [0:0.001:1]' * 2*pi;
r = sin(2*q) .* sin(3*q);
x = r .* cos(q);
y = r .* sin(q);
plot(x,y)
grid
```



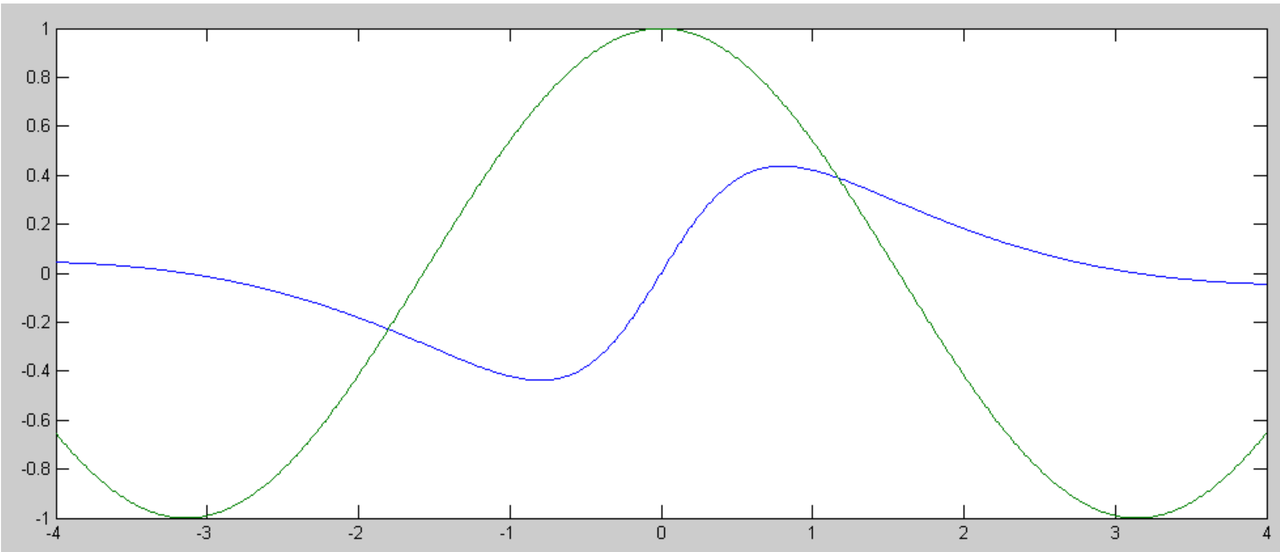
f(x) = 0: Newton's Method

4) Use Newton's method to find the solutions to problem #3 for homework set #1

$$y = \left(\frac{\sin(x)}{x^2+1}\right) \qquad y = \cos(x)$$

or

$$f(x) = \left(\frac{\sin(x)}{x^2+1}\right) - \cos(x) = 0$$



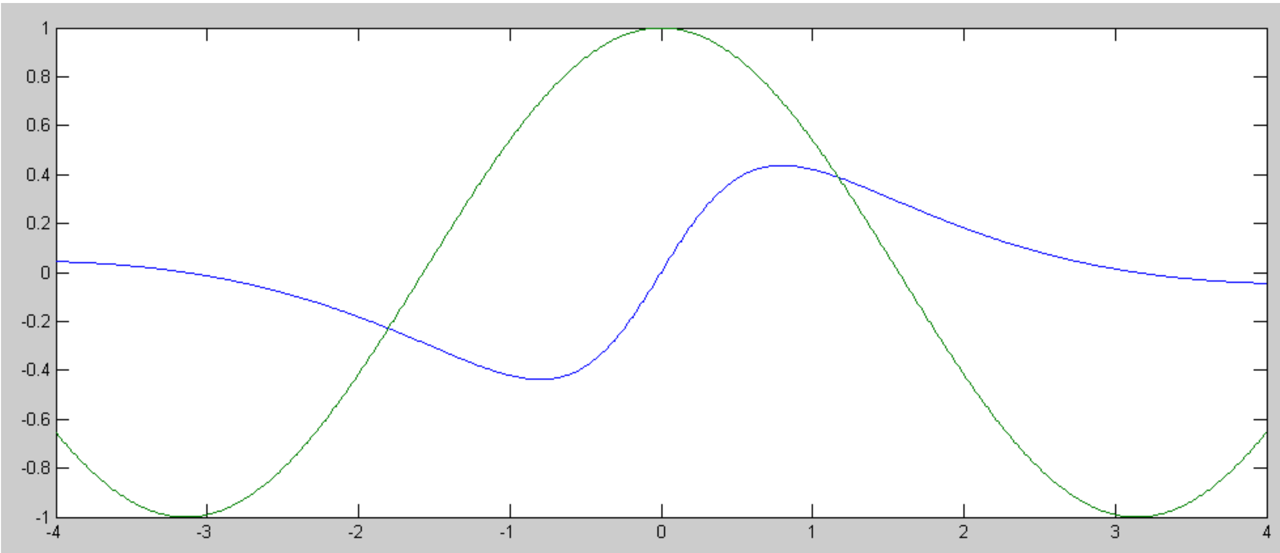
Matlab Code:

```
x1 = -2.5;
dx = 0.01;

for n=1:10
    y1a = sin(x1) / (x1^2 + 1);
    y1b = cos(x1);
    e1 = y1a - y1b;

    x2 = x1 + dx;
    y2a = sin(x2) / (x2^2 + 1);
    y2b = cos(x2);
    e2 = y2a - y2b;

    x3 = x2 - (x2-x1)/(e2-e1) * e2;
    disp([n, x1, y1a, e1])
    x1 = x3;
end
```



Starting at -2.5

	x1	y1	e1
1	-2.500000000000000	-0.082547881945373	0.718595733601560
2	-1.567398622620712	-0.289288369522754	-0.292686067159536
3	-1.799430711630337	-0.229822582371489	-0.003174924595416
4	-1.802024273411177	-0.229177988115020	-0.000005026403429
5	-1.802028382092756	-0.229176967408609	-0.000000006366988
6	-1.802028387297263	-0.229176966115671	-0.000000000008062
7	-1.802028387303853	-0.229176966114033	-0.000000000000010
8	-1.802028387303862	-0.229176966114031	0
9	-1.802028387303862	-0.229176966114031	0
10	**-1.802028387303862**	**-0.229176966114031**	**0**

Starting at +2.0

1	2.000000000000000	0.181859485365136	0.598006321912279
2	1.119339293100155	0.399398206051919	-0.036878847659213
3	1.172248029706980	0.388189182714031	0.000108357201021
4	1.172093737969097	0.388223104756238	0.000000084687257
5	1.172093617378534	0.388223131266241	0.000000000065166
6	1.172093617285740	0.388223131286640	0.000000000000050
7	1.172093617285669	0.388223131286656	0.000000000000000
8	1.172093617285669	0.388223131286656	0.000000000000000
9	1.172093617285669	0.388223131286656	0.000000000000000
10	**1.172093617285669**	**0.388223131286656**	**0.000000000000000**

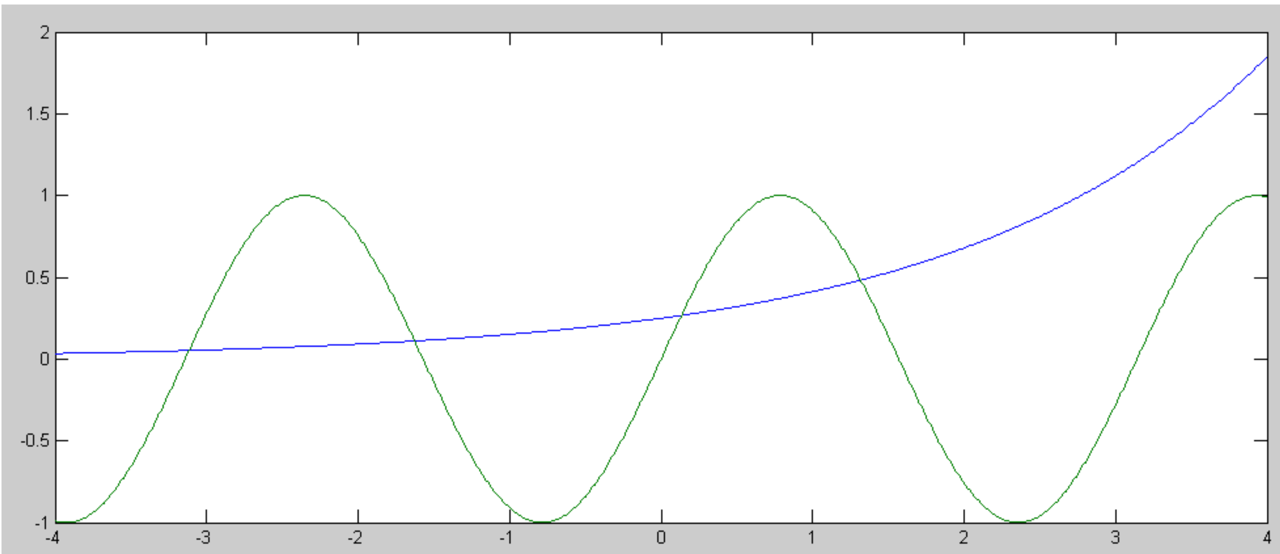
5) Use Newton's method to find the solutions to problem #4 for homework set #1

$$y = \tfrac{1}{4}\exp\left(\tfrac{x}{2}\right) = \tfrac{1}{4}e^{x/2}$$

$$y = \sin(2x)$$

or

$$f(x) = \tfrac{1}{4}\exp\left(\tfrac{x}{2}\right) - \sin(2x) = 0$$



Solution near x = -3:

```
x3 = -3;
dx = 0.01;

for n=1:10
    x1 = x3;
    y1a = exp(x1/2) / 4;
    y1b = sin(2*x1);
    e1 = y1a - y1b;

    disp([n, x1, y1a, e1])

    x2 = x1 + dx;
    y2a = exp(x2/2) / 4;
    y2b = sin(2*x2);
    e2 = y2a - y2b;

    x3 = x2 - (x2-x1)/(e2-e1) * e2;
end
```

Solution near x = -3:

n	x	y	error
1	-3.000000000000000	0.055782540037107	-0.223632958161818
2	-3.118533580630941	0.052572550375312	0.006470750705550
3	-3.115249712727348	0.052658941935915	-0.000002568907846
4	-3.115251016977774	0.052658907595702	0.000000001633469
5	-3.115251016148452	0.052658907617538	-0.000000000001039
6	-3.115251016148979	0.052658907617524	0.000000000000000
7	-3.115251016148979	0.052658907617524	0.000000000000000
8	-3.115251016148979	0.052658907617524	0.000000000000000
9	-3.115251016148979	0.052658907617524	0.000000000000000
10	**-3.115251016148979**	**0.052658907617524**	**0.000000000000000**

Solution near x = -2

n	x	y	error
1	-2.000000000000000	0.091969860292861	-0.664832635015068
2	-1.514165974354776	0.117258151897151	0.230276860898823
3	-1.626849656264651	0.110834277449173	-0.001037704739818
4	-1.626342247494458	0.110862400158657	-0.000001077445236
5	-1.626341720710345	0.110862429358937	-0.000000001171237
6	-1.626341720137704	0.110862429390679	-0.000000000001273
7	-1.626341720137081	0.110862429390713	-0.000000000000001
8	-1.626341720137081	0.110862429390713	-0.000000000000000
9	-1.626341720137081	0.110862429390713	-0.000000000000000
10	**-1.626341720137081**	**0.110862429390713**	**-0.000000000000000**

Solution near x = +0

n	x	y	error
1	0	0.250000000000000	0.250000000000000
2	0.133365081488579	0.267239018576066	0.003660375952924
3	0.135410073717302	0.267512409180762	-0.000009371940302
4	0.135404830772754	0.267511707905318	0.000000030496412
5	0.135404847833303	0.267511710187266	-0.000000000099181
6	0.135404847777818	0.267511710179845	0.000000000000323
7	0.135404847777999	0.267511710179869	-0.000000000000001
8	0.135404847777998	0.267511710179869	0
9	0.135404847777998	0.267511710179869	0
10	**0.135404847777998**	**0.267511710179869**	**0**

Solution near x = +2

n	x	y	error
1	2.000000000000000	0.679570457114761	1.436372952422690
2	1.120247212361906	0.437722226903082	-0.346286880209863
3	1.354778722031198	0.492182842660308	0.073462969109731
4	1.319309782949071	0.483531184068988	0.001498638275048
5	1.318562034512684	0.483350438015823	0.000008147208684
6	1.318557966386326	0.483349454851494	0.000000041328813
7	1.318557945749624	0.483349449864125	0.000000000209569
8	1.318557945644979	0.483349449838835	0.000000000001063
9	1.318557945644449	0.483349449838707	0.000000000000006
10	**1.318557945644446**	**0.483349449838706**	**-0.000000000000000**

f(x) = 0: Shoot Game:

Pick a random number from 50 to 100 for your target.

Pick a random number from 30 to 70 for your firing angle

6) Use trial and error to find the initial velocity (X) to fire a tennis ball to hit the target (result is zero)

```
>> Target = 50*rand + 50
Target =   90.7362

>> Angle = 50*rand + 20
Angle =   65.2896

>> Shoot(30, Angle, Target)
ans =   30.6515

>> Shoot(50, Angle, Target)
ans =  -28.2262

>> Shoot(40, Angle, Target)
ans =   -0.9533
```

7) Repeat using California method to find the initial velocity (X) to fire the tenis ball to hit the target

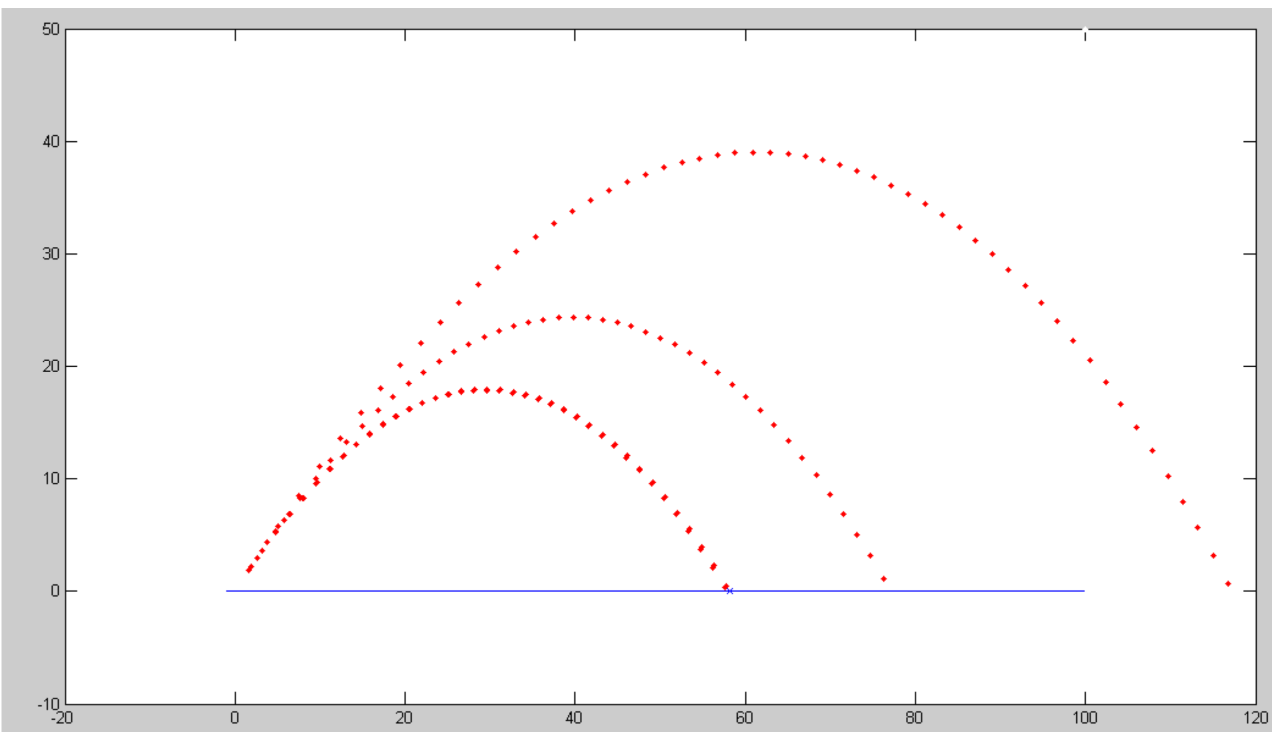
```
Target = 50*rand + 50
X1 = 30;
Y1 = Shoot(X1, 50, Target);
disp([0, X1, Y1]);

X2 = 40;
Y2 = Shoot(X2, 50, Target);
disp([0, X2, Y2]);

for n=1:5
  X3 = X2 - (X2 - X1)/(Y2 - Y1) * Y2;
  Y3 = Shoot(X3, 50, Target);

  X1 = X2;
  Y1 = Y2;
  X2 = X3;
  Y2 = Y3;
  disp([n, X3, Y3]);
end
```

n	x	y
0	30.000000000000000	19.015707849940156
0	40.000000000000000	59.054707224434154
1	25.250703527307998	-0.278143452572778
2	25.319846003981951	-0.003291963405090
3	25.320674140309816	0.000004550766121
4	25.320672997085627	-0.000000000095000
5	**25.320672997109490**	**-0.000000000000028**



7) Repeat using Newton's method to find the initial velocity (X) to fire the tenis ball to hit the target

```
% Newton's Method

Target = 50*rand + 50
X3 = 30;
dx = 0.01;

for n=1:5
   X1 = X3;
   Y1 = Shoot(X1, 50, Target);
   X2 = X1 + dx;
   Y2 = Shoot(X2, 50, Target);

   X3 = X2 - (X2 - X1)/(Y2 - Y1) * Y2;
   disp([n, X1, Y1]);
end

1  30.000000000000000  20.650643733969879
2  24.978175939159318   0.276013824983387
3  24.908380891072920  -0.000108755970906
4  24.908408418108760  -0.000000028308811
5  24.908408425273944  -0.000000000007418
```