ECE 111 - Homework #1

Week #1: Algebra. Due Tuesday, January 17th

functions *poly* and *roots*:

1) Use MATLAB, find the roots the the following polynomials:

a) $x^3 - 9x^2 - 49x + 441 = 0$

```
>> P = [1,-9,-49,441]

P =     1    -9   -49   441

>> roots(P)

   -7.0000
    9.0000
    7.0000
```

b) $x^4 - 85x^2 - 60x + 864 = 0$

```
>> P = [1,0,-85,-60,864]

P =

     1     0   -85   -60   864

>> roots(P)

    9.0000
   -8.0000
   -4.0000
    3.0000
```

c) $x^5 - 25x^4 + 144x^3 + 680x^2 - 6800x + 6000 = 0$

```
>> P = [1,-25,144,680,-6800,6000]

P =           1         -25         144         680       -6800        6000

>> roots(P)

  -6.0000
  10.0001
  10.0000 + 0.0001i
  10.0000 - 0.0001i
   1.0000
```

2) Use Matlab to multiply out the following polynomials.

a) $y = (x)(x-10)(x+7)(x-7)$

```
>> R = [0, 10, -7, 7]

R =     0    10    -7     7

>> P = poly(R)

P =     1   -10   -49   490     0
```

meaning

$$y = x^4 - 10x^3 - 49x^2 + 490x$$

b) $y = (x+9)(x+1)(x-4)(x-6)(x-9)(x-10)$

```
>> R = [-9,-1,4,6,9,10]

R =    -9    -1     4     6     9    10

>> P = poly(R)

P =     1         -19          23        1423       -8664        9396       19440
```

meaning

$$y = x^6 - 19x^5 + 23x^4 + 1423x^3 - 8644x^2 + 9396x + 19440$$

Graphing in Matlab

3) Plot the two functions in Matlab and determine all solutions in the range of -4 < x < +4

$$y = \left(\frac{\sin(x)}{x^2+1}\right)$$

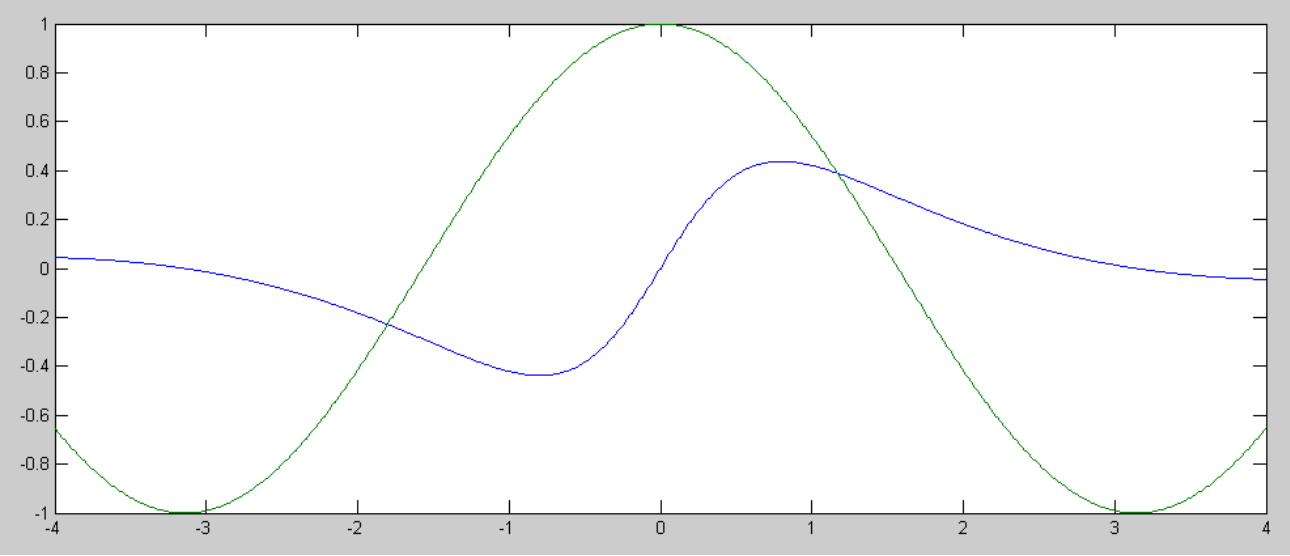
$$y = \cos(x)$$

```
>> x = [-4:0.01:4]';
>> y1 = sin(x) ./ ( x.^2 + 1);
>> y2 = cos(x);
>> plot(x,y1,x,y2)
```

There are two solutions over this interval:

(-1.7, -0.21)

(+1.2, +0.4)



4) Plot the two functions in Matlab and determine all solutions in the range of -4 < x < +4

$$y = \frac{1}{4}\exp\left(\frac{x}{2}\right) = \frac{1}{4}e^{x/2}$$

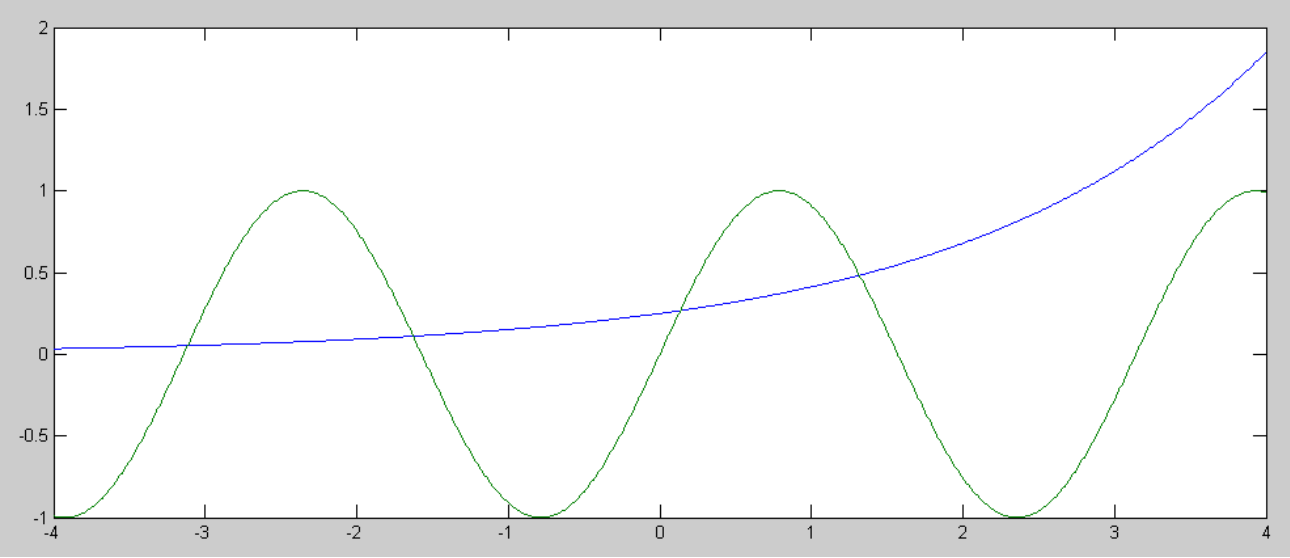
$$y = \sin(2x)$$

In Matlab:

```
>> x = [-4:0.01:4]';
>> y1 = exp(x/2) / 4;
>> y2 = sin(2*x);
>> plot(x,y1,x,y2)
>>
```

There are four solutions from the graph:

- (-3.2, +0.1)
- (-1.6, +0.2)
- (+0.2, +0.25)
- (+1.3, +0.5)



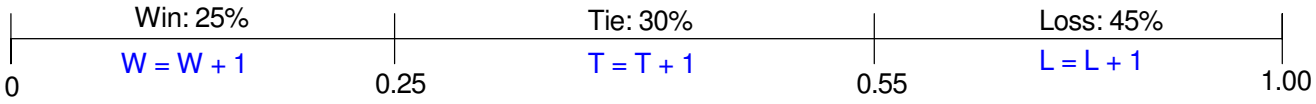
Monte-Carlo Simulations:

Two teams, A and B, are playing a game. Team A has a

- 25% chance of winning any given game (+1 point)
- 30% chance of a tie (+1/2 point), and
- 45% chance of a loss (+0 points)

5) For Loops: Suppose the two teams play a 5-game match. Determine the probability that

- Team A wins the match (A has more than 2.5 points),
- There is a tie (A has 2.5 points), and
- Team A loses (A has less than 2.5 points)



This is a for-loop (fixed number of games)

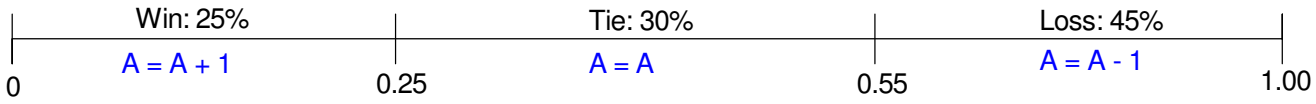
```
W = 0;
T = 0;
L = 0;
for n=1:1e5
    A = 0;
    for i=1:5
        x = rand;
        if(x < 0.2) A = A + 1;
        elseif(x < 0.55) A = A + 0.5;
        else A = A;
        end
    end
    if(A > 2.5) W = W + 1;
    elseif(A == 2.5) T = T + 1;
    else L = L + 1;
    end
end

disp([W,T,L]);
```

```
        Wins        Ties        Losses
       15504       16816       67680
```

Result: A has a

- 15.5% chance of winning
- 16.8% chance of a tie
- 67.68% chance of losing

6) While Loops: Suppose the two teams play until one team is up by 2 points. Determine the probability that team A will win the match.



hint: use a while-loop and keep looping until one team is up by 2 games.

```
W = 0;
L = 0;
for n=1:1e5
    A = 0;
    while(abs(A) < 2)
        x = rand;
        if(x < 0.2) A = A + 1;
        elseif(x < 0.55) A = A;
        else A = A - 1;
        end
    end
    if(A > 0) W = W + 1;
    else L = L + 1;
    end
end

disp([W,L]);

        Wins        Losses
       16416       83584
```

With this format, A has a

- 16.416% chance of winning, and
- 83.584% chance of losing

7) Gauss' Dilema: Play the following game 1000 times. (i.e. use Matlab and a for loop along with a while loop)

- It costs $20 to play. The pot starts at $1.
- Flip a coin. If you get a heads, the pot doubles. If you get a tails, the game is over and you collect the money in the pot.
- Keep flipping until you get a tails.

How much money do you expect to win (or lose) each time you play this game?

Matlab Code:

```
Winnings = 0;
for n=1:1e3
    Pot = 1;
    while(rand < 0.5)
       Pot = 2*Pot;
    end
    Winnings = Winnings + Pot - 20;
end

Winnings
```

Results

```
Winnings =      -15674
Winnings =      -14909
Winnings =      -12069
Winnings =        1273
Winnings =      -14976
```

Most of the time I'm down $12k to $15k after 1000 games

Dice:

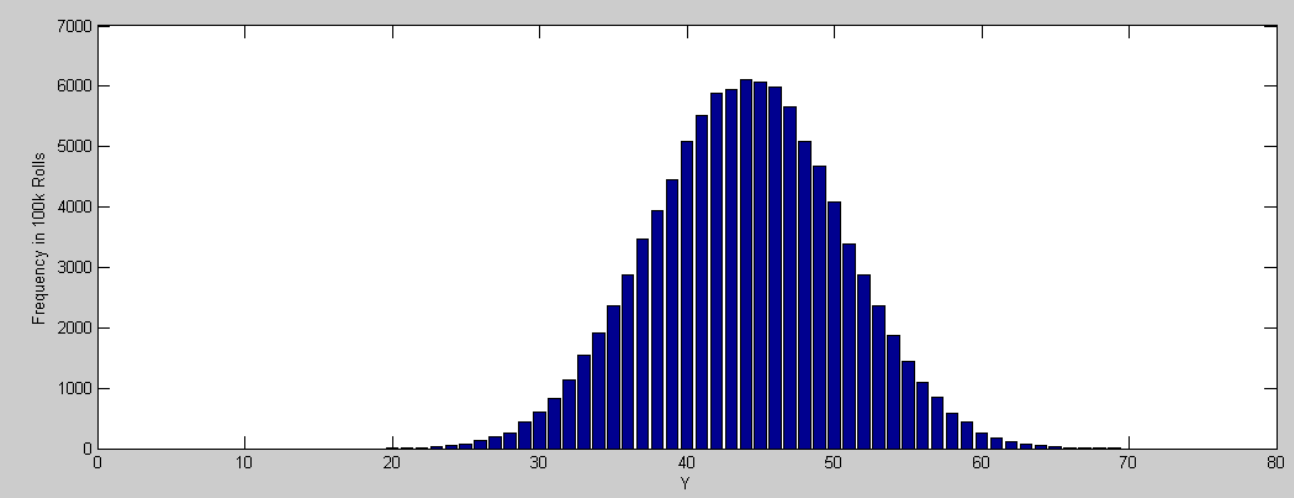
8a) Determine the probability distribution for the following:

- Roll three 4-sided dice, four 6-sided dice, and five 8-sided dice.
- The total is the sum of all of the dice.

  Y = 3d4 + 4d6 + 5d8

Matlab Code:

```
Result = zeros(76,1);
for n=1:1e5
    d4 = ceil(4*rand(1,3));
    d6 = ceil(6*rand(1,4));
    d8 = ceil(8*rand(1,5));
    Y = sum(d4) + sum(d6) + sum(d8);
    Result(Y) = Result(Y) + 1;
end
bar(Result)
xlabel('Y');
ylabel('Frequency in 100k Rolls')
```



8b) What is the probability of the total being 50?

```
>> Result(50) / 1e5
ans =    0.0408
```

There is a 4.08% chance of rolling 50

8c) What is the probability of the total being 50 or more?

```
>> sum(Result(50:76)) / 1e5
ans =    0.1971
```

There is a 19.71% chance of rolling 50 or more

9) Two people are playing a dice game:

- Player A rolls three 4-sided dice, four 6-sided dice, and five 8-sided dice
- Player B rolls two 100-sided dice.
- Whoever has the highest total wins.

Determine the probability that

- A wins
- There is a tie, and
- B wins

Matlab Code

```
W = 0;
T = 0;
L = 0;
for n=1:1e5
    d4 = ceil(4*rand(1,3));
    d6 = ceil(6*rand(1,4));
    d8 = ceil(8*rand(1,5));
    d100 = ceil(100*rand(1,2));
    A = sum(d4) + sum(d6) + sum(d8);
    B = sum(d100);
    if(A > B) W = W + 1; end
    if(A == B) T = T + 1; end
    if(A < B) L = L + 1; end
end
disp([W, T, L]) / 1e5
```

```
          W           T          L
    0.0922    0.0041    0.9037
    0.0910    0.0043    0.9047
    0.0936    0.0039    0.9024
    0.0911    0.0047    0.9042
    0.0925    0.0041    0.9035
    0.0931    0.0042    0.9027
    0.0927    0.0045    0.9028
```

A has a

- 9.2% chance of winning
- 0.4% chance of a tie, and
- 90.4% chance of losing