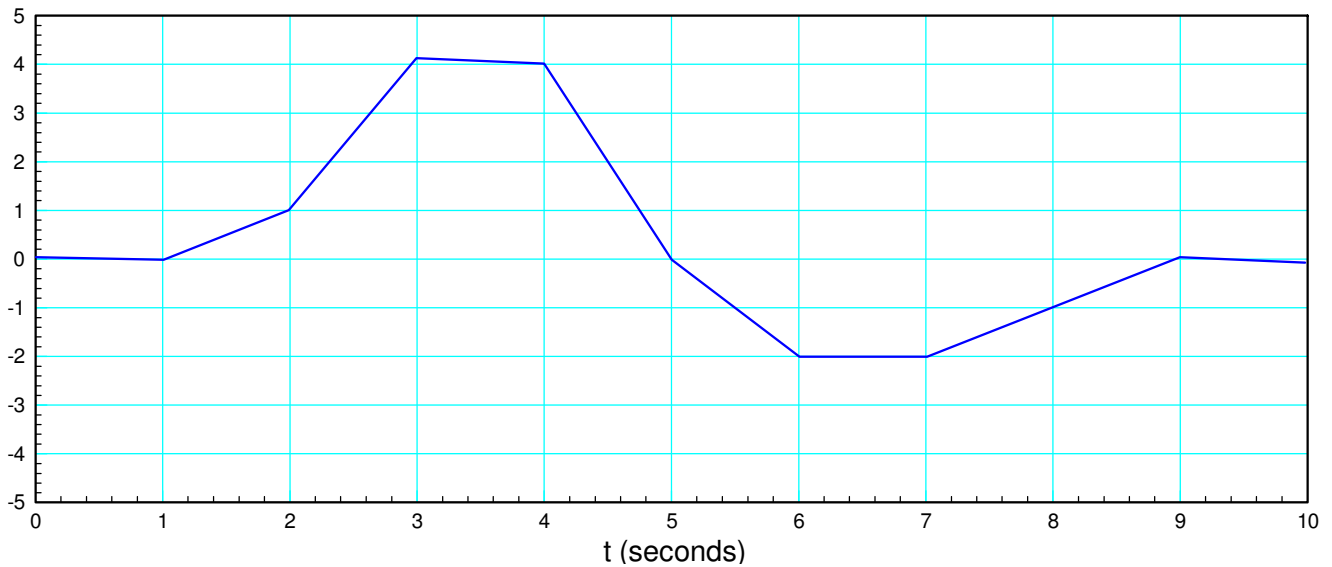


ECE 111 - Homework #6:

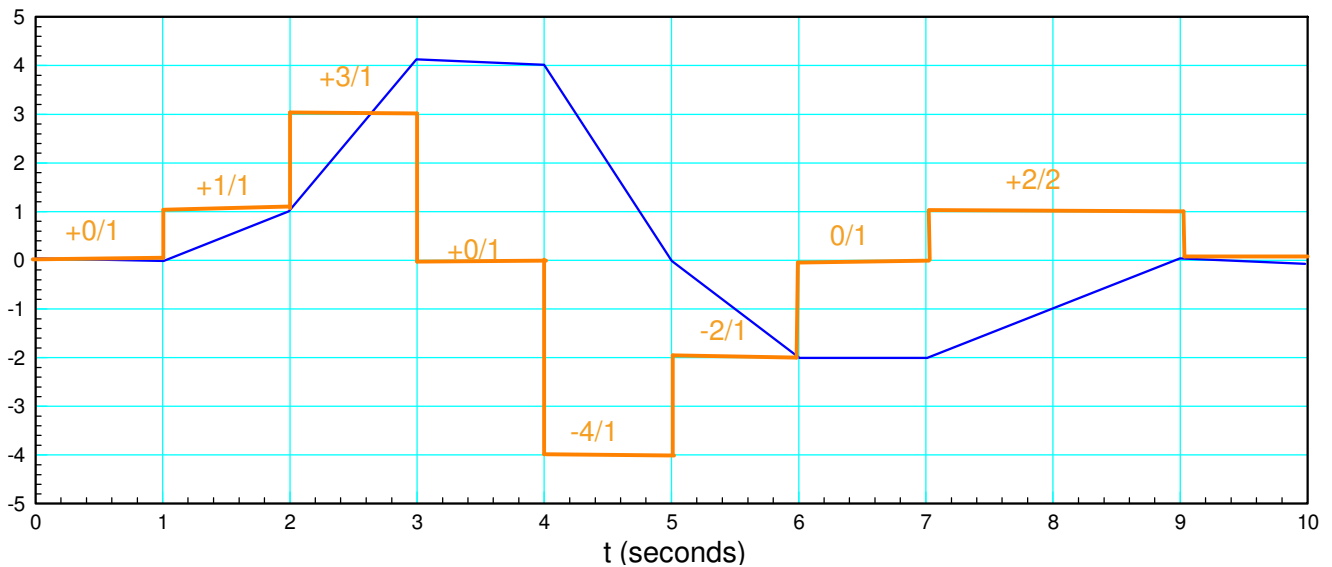
Math 165: Differentiation

1) Sketch the derivative of the following function

If this is the balance of your checking account, how much money are you adding (positive) or withdrawing (negative) for the balance to be as shown?



Draw the derivative (the slope) on top of the graph



Numerical Differentiation:

2) Use numerical methods to determine the derivative of y:

$$y = \exp\left(\frac{-x^2}{10} + \sin(2x)\right)$$

$$z = \frac{d}{dx}(y)$$

for $-10 < x < 10$. (a plot is sufficient).

Write a Matlab function to find the derivative

$$\frac{dy}{dx}(i) \approx \left(\frac{y(i+1) - y(i-1)}{x(i+1) - x(i-1)} \right)$$

```
function [ dy ] = derivative( x, y )

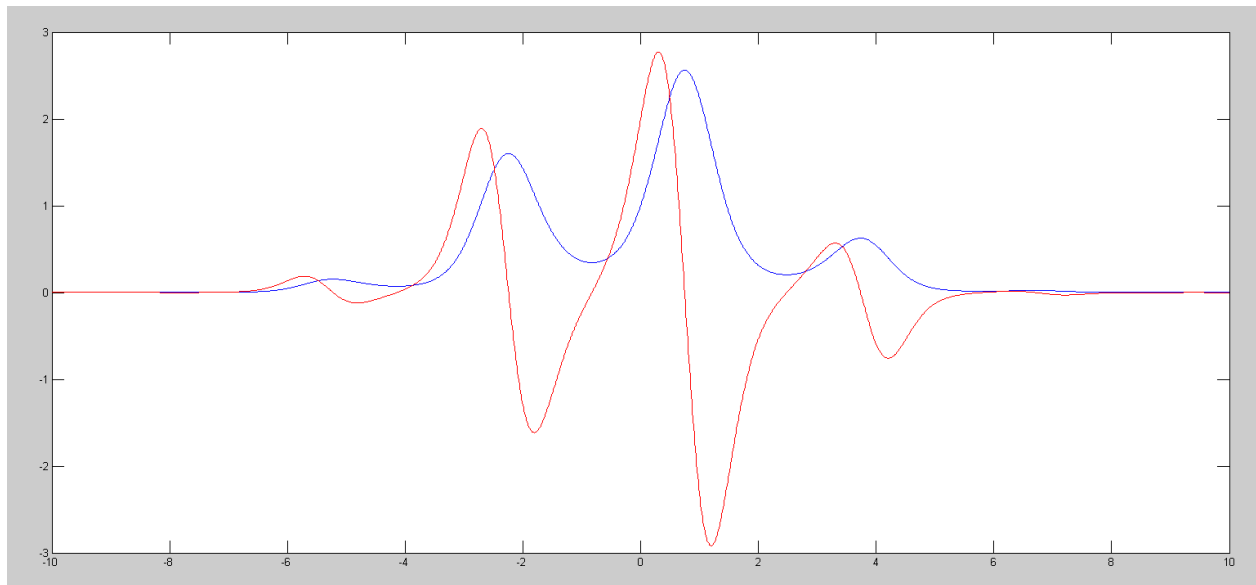
dy = 0*y;
n = length(y);
for i=2:n-1
    dy(i) = ( y(i+1) - y(i-1) ) / ( x(i+1) - x(i-1) );
end

dy(1) = 2*dy(2) - dy(3);
dy(n) = 2*dy(n-1) - dy(n-2);

end
```

Using this in Matlab

```
>> x = [-10:0.01:10]';
>> y = exp( -(x.^2)/10 + sin(2*x) );
>> z = derivative(x,y);
>> plot(x,y,'b',x,z,'r');
>>
```



y(x) (blue) and dy/dx (red)

note:

- With Matlab and numerical techniques, you can find the derivative of a function even if you haven't had calculus
- Matlab doesn't give you the expression for the derivative - it just tells you what it looks like

3) Use numerical methods to determine the derivative of y:

$$y = \sin(x) + 0.1 \cos(7x)$$

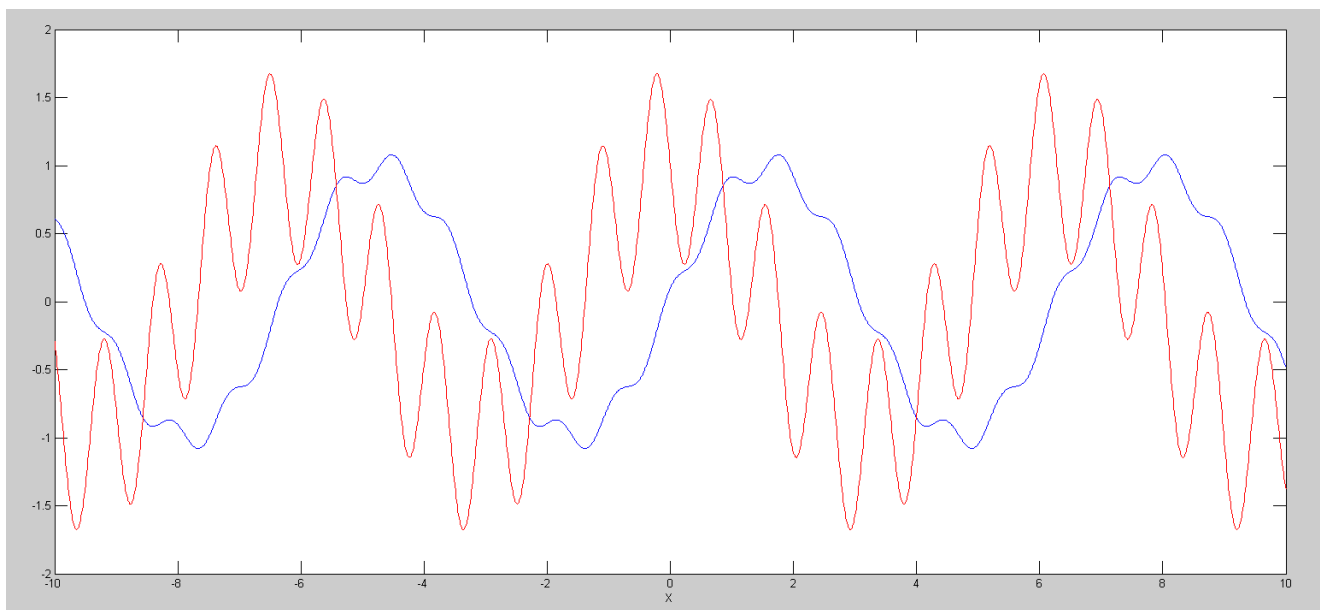
$$z = \frac{d}{dx}(y)$$

for $-10 < x < 10$. (a plot is sufficient).

Matlab Code

```
>> x = [-10:0.01:10]';  
>> y = sin(x) + 0.1*cos(7*x);  
>> z = derivative(x,y);  
>> plot(x,y,'b',x,z,'r');  
>> xlabel('X')  
>>
```

Result



y(x) (blue) and dy/dx (red)

Differentiation with Real Data

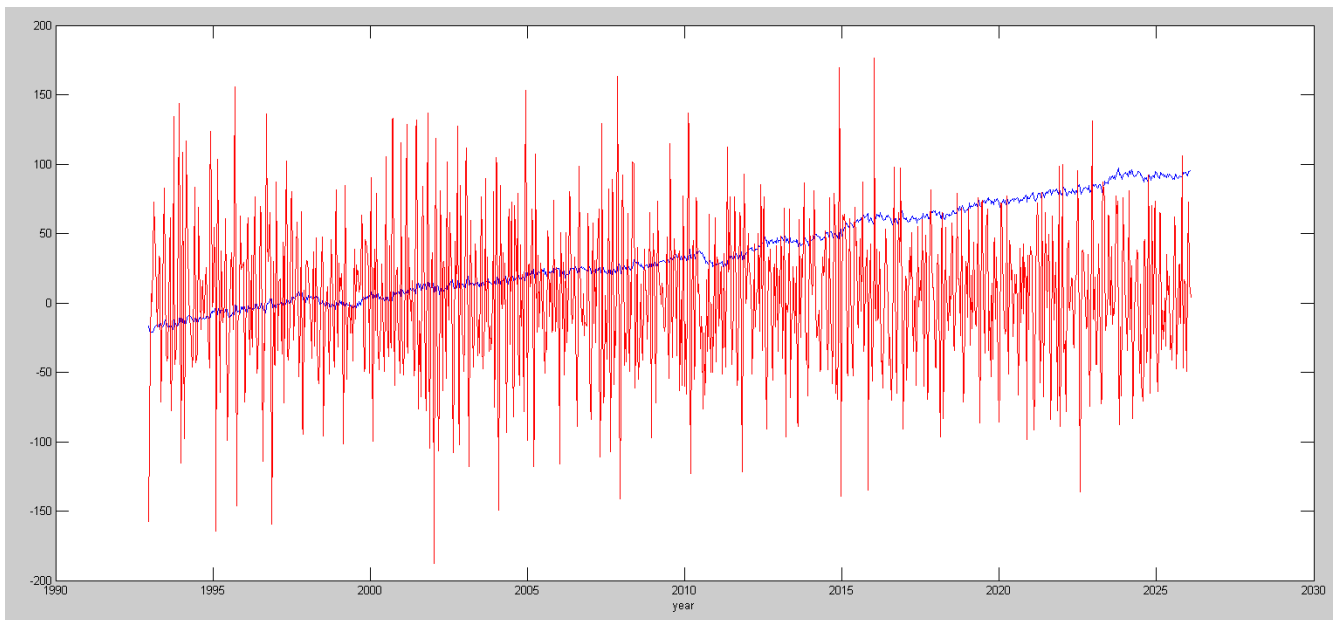
4) Global sea levels are measured every ten days since 1992:

Compute and plot the derivative to see the change in sea level in units of mm / year

- note: it will probably be a really noisy graph. Differentiation amplifies noise.

In Matlab

```
>> Data = [ <paste> ];  
>> x = Data(:,1);  
>> y = Data(:,2);  
>> z = derivative(x,y);  
>> plot(x,y,'b',x,z,'r');  
>> xlabel('year')
```



blue: Sea Level (mm) - red: Sea Level Rise (mm/y)

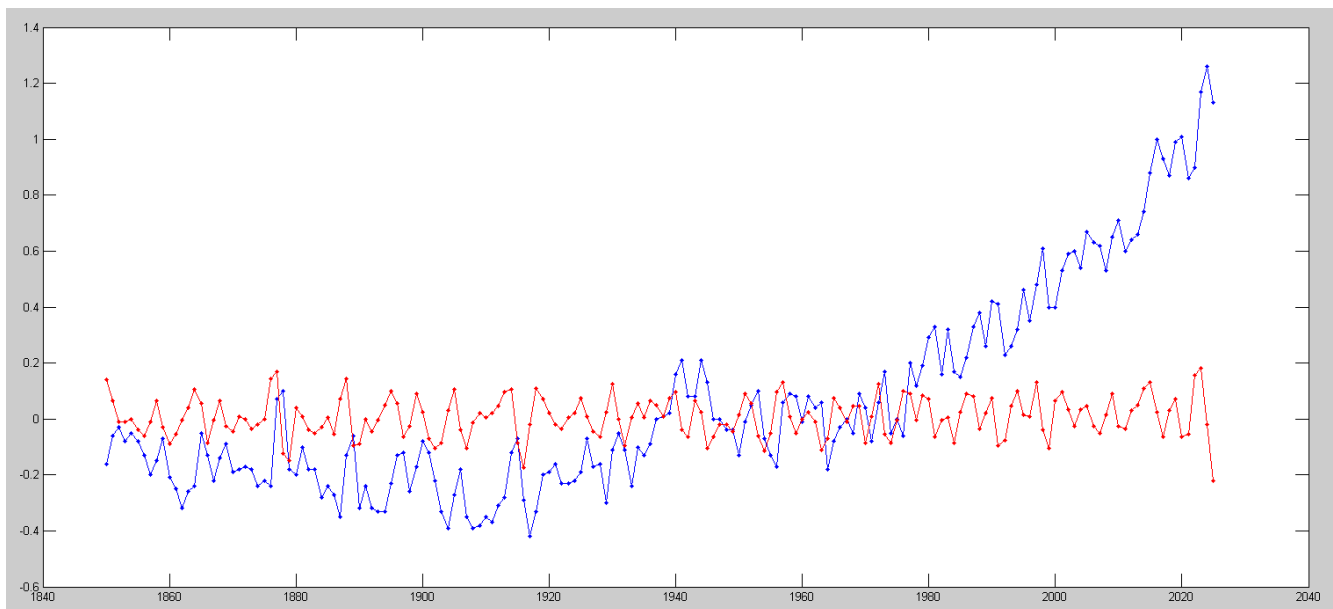
5) Global temperatures have been measured since 1850

Compute and plot the derivative to see the change in temperature on a year-by-year basis

- note: it will probably be a really noisy graph. Differentiation amplifies noise.

In Matlab

```
>> x = Data(:,1);  
>> y = Data(:,2);  
>> z = derivative(x,y);  
>> plot(x,y,'b',x,z,'r');  
>> xlabel('year')
```



Global Temperature Deviations:
blue: Degrees C - red: Degrees C / year

Path Planning

6) Assume a motor's angle is as follows:

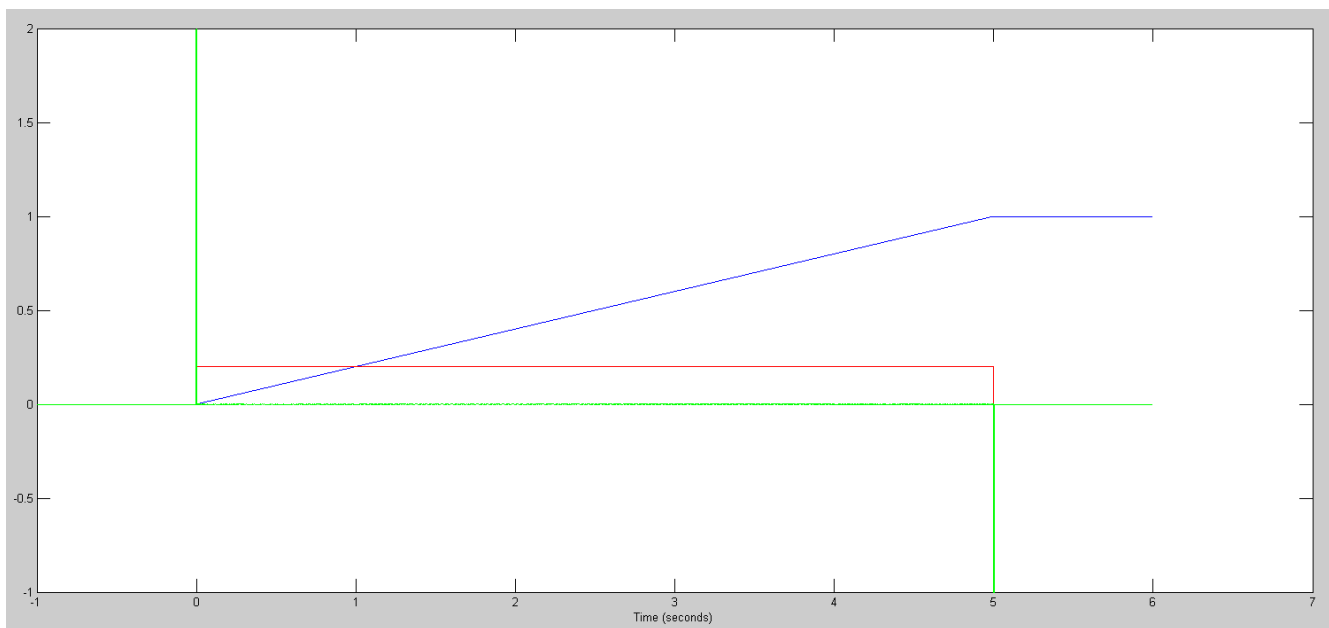
$$\theta = \begin{cases} 0 & t < 0 \\ t/5 & 0 < t < 5 \\ 1 & t > 5 \end{cases}$$

Calculate and plot using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

In Matlab

```
>> t = [-1:0.001:6]' + 1e-6;  
>> q = (t/5) .* (t>0) .* (t<5) + 1*(t>5);  
>> dq = derivative(t,q);  
>> d2q = derivative(t,dq);  
>> plot(t,q,'b',t,dq,'r',t,d2q,'g')  
>> xlabel('Time (seconds)')  
>> ylim([-1,2])
```



Angle vs time (blue)
Velocity vs time (red)
Acceleration vs time (green)

7) Assume a motor's angle is as follows:

$$\theta = \begin{cases} 0 & t < 0 \\ \frac{1}{2} - \frac{1}{2} \cos\left(\frac{1}{5}\pi t\right) & 0 < t < 5 \\ 1 & t > 5 \end{cases}$$

Calculate and plot using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

In Matlab

```
>> t = [-1:0.001:6]' + 1e-6;  
>> q1 = ( 1/2 - 1/2 * cos(pi*t/5) ) .* (t>0) .* (t<5);  
>> q2 = 1 * (t>5);  
>> q = q1 + q2;  
  
>> dq = derivative(t,q);  
  
>> d2q = derivative(t,dq);  
  
>> plot(t,q,'b',t,dq,'r',t,d2q,'g')  
>> xlabel('Time (seconds)')  
>>
```

