

ECE 111 - Homework #4

Week #4: Math 129 Linear Algebra

N equations & N unknowns

1) Solve for $\{x, y\}$

$$13x + 4y = 13$$

$$15x + 11y = 6$$

Solution:

Step 1: Express in matrix form

$$\begin{bmatrix} 13 & 4 \\ 15 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 6 \end{bmatrix}$$

Solve using matrix operations

$$AX = B$$

$$X = A^{-1}B$$

In Matlab:

```
>> A = [13, 4 ; 15, 11]
```

```
    13     4  
    15    11
```

```
>> B = [13 ; 6]
```

```
    13  
     6
```

```
>> xy = inv(A)*B
```

```
x    1.4337
```

```
y   -1.4096
```

Note: Dimensions are important with matrices. If you input B incorrectly, you get an error

- You can't multiply a 2x2 matrix with a 1x2 matrix.

```
>> A = [13, 4 ; 15, 11]
```

```
    13     4  
    15    11
```

```
>> B = [13 , 6]
```

```
    13     6
```

```
>> xy = inv(A)*B
```

```
??? Error using ==> mtimes  
Inner matrix dimensions must agree.
```

2) Solve for {x, y, z}

$$4x + 7y + 8z = 1$$

$$7x + 2y + 8z = -2$$

$$2x - 2y - 6z = 4$$

Step 1: Place in matrix form

$$\begin{bmatrix} 4 & 7 & 8 \\ 7 & 2 & 8 \\ 2 & -2 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

Solve using Matlab

```
>> A = [ 4, 7, 8 ; 7, 2, 8 ; 2, -2, -6]
```

```
      4      7      8
      7      2      8
      2     -2     -6
```

```
>> B = [1 ; -2 ; 4]
```

```
      1
     -2
      4
```

```
>> inv(A)*B
```

```
x    0.4029
y    0.8417
z   -0.8129
```

```
>>
```

3) Solve for {a, b, c, d}

$$8a + 8b + 5c + d = 3$$

$$9b - 5c + d = 5$$

$$-3a - 3b - 8d = -9$$

$$2a + 2b + 5c + 5d = -3$$

Place in matrix form

$$\begin{bmatrix} 8 & 8 & 5 & 1 \\ 0 & 9 & -5 & 1 \\ -3 & -3 & 0 & -8 \\ 2 & 2 & 5 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ -9 \\ -3 \end{bmatrix}$$

comment: You can't leave an entry blank in a matrix. If a variable isn't present, its coefficient is zero

Solve using Matlab

```
>> a1 = [8, 8, 5, 1];  
>> a2 = [0, 9, -5, 1];  
>> a3 = [-3, -3, 0, -8];  
>> a4 = [2, 2, 5, 5];  
>> A = [a1 ; a2 ; a3 ; a4]
```

```
      8      8      5      1  
      0      9     -5      1  
     -3     -3      0     -8  
      2      2      5      5
```

```
>> B = [3 ; 5 ; -9 ; -3]
```

```
      3  
      5  
     -9  
     -3
```

```
>> inv(A)*B
```

```
a    1.8889  
b   -0.4889  
c   -1.7600  
d    0.6000
```

Comment:

- Matlab makes solving N equations for N unknowns really easy.
- The trick is to express the equations in matrix form.

This is a common theme throughout ECE: whenever you're trying to solve a problem, if you can express it in the form of N equations with N unknowns, then you can solve using matrix operations and Matlab.

Global CO2 Levels

The CO2 levels measured at Mauna Loa observatory for the past 56 years are:

Problem 4) Determine a parabolic curve fit for this data in the form of

$$CO_2 \approx ay^2 + by + c$$

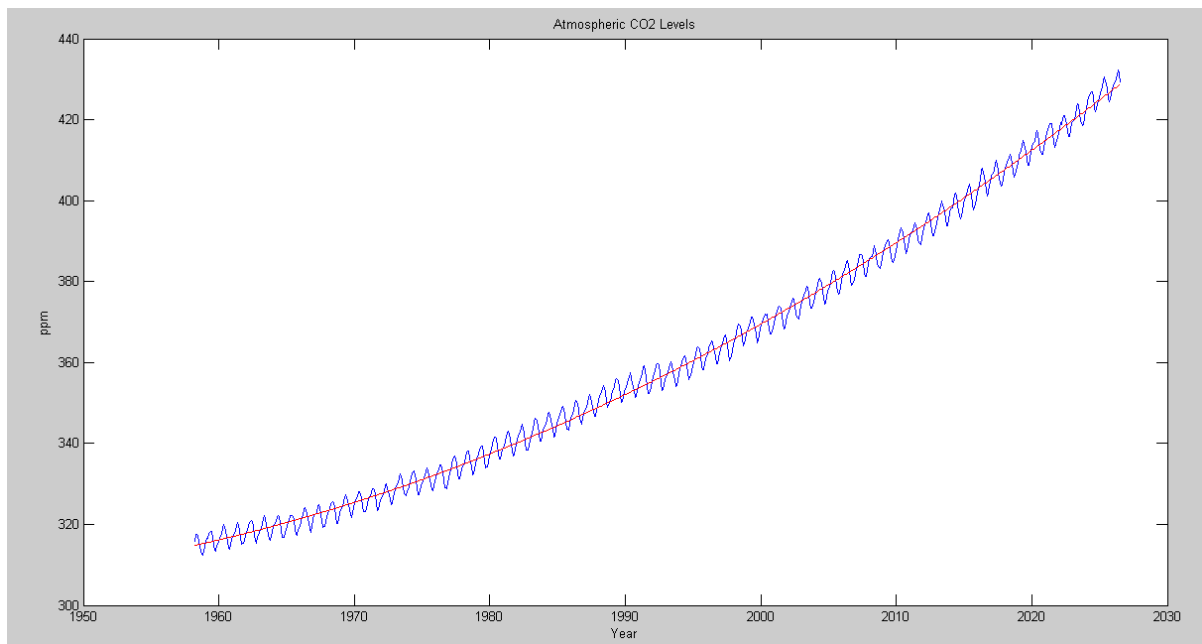
where 'y' is the year. From this data, when do you predict that we will hit

- 350ppm?
- 450ppm?
- 550 ppm?
- Note: Column #3 of the data set is year, #4 is CO2

```
year = DATA(:,3);  
CO2 = DATA(:,4);
```

In Matlab

```
>> year = Data(:,3);  
>> CO2 = Data(:,4);  
>> B = [year.^2, year, year.^0];  
>> A = inv(B'*B)*B'*CO2  
  
a 1.3568e-002  
b -5.2398e+001  
c 5.0891e+004  
  
>> plot(year, CO2, 'b', year, B*A, 'r')  
>> xlabel('Year')  
>> ylabel('ppm')  
>> title('Atmospheric CO2 Levels')
```



When are certain CO2 levels are met?

- Matlab doesn't have a $f(x) = 350$ command.
- It *does* have a $f(x) = 0$ command (roots).

Rewrite the problem

$$ay^2 + by + c = 350$$

as

$$ay^2 + by + (c - 350) = 0$$

350ppm: year = 1988.7

```
>> roots(A - [0;0;350])  
  
1.9887e+003  
1.8731e+003
```

450ppm: year 2034.4

```
>> roots(A - [0;0;450])  
  
2.0344e+003  
1.8274e+003
```

550ppm: year 2065.3

```
>> roots(A - [0;0;550])  
  
2.0653e+003  
1.7964e+003
```

Sea Level

One indication of a warming planet is a rise in ocean levels. About 2/3rds of the change in sea level is due to melting glaciers and ice caps. About 1/3 is due to the thermal expansion of water.

5) Using the change in global sea levels since 1992 to present:

5a) Determine a curve fit of the form of

$$mm = ay + b$$

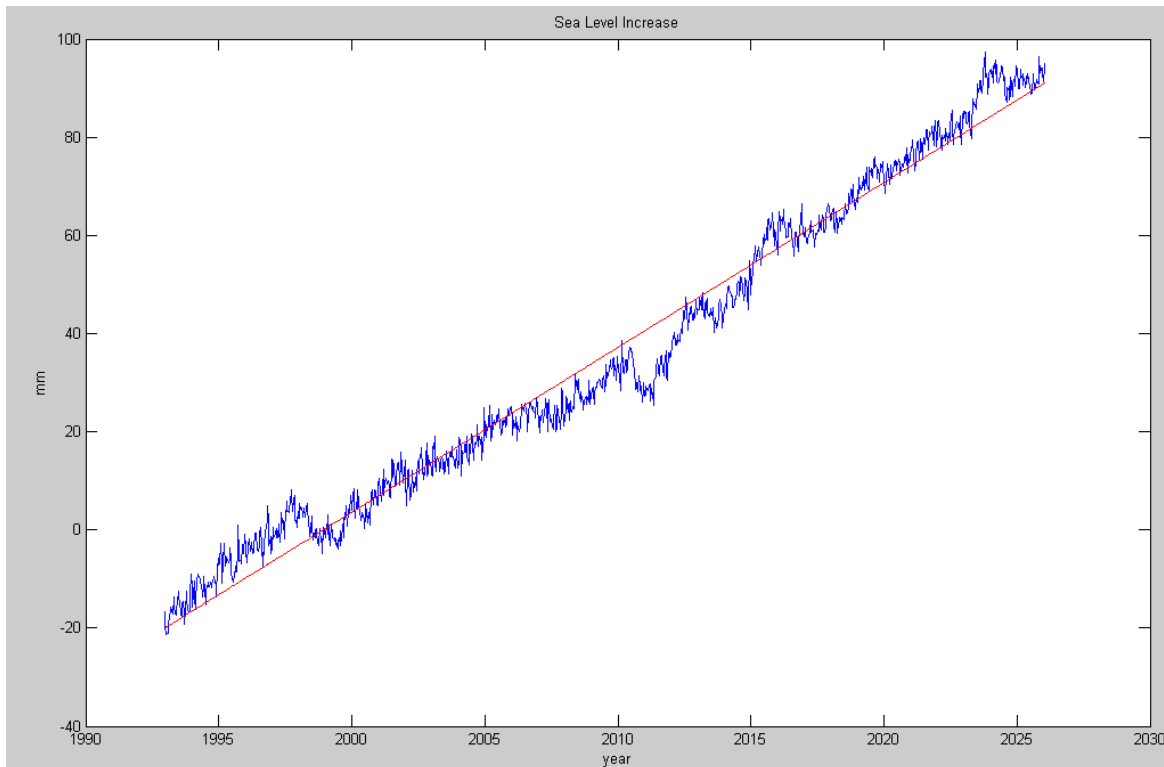
Matlab Code:

```
>> year = Data(:,1);  
>> mm = Data(:,2);  
>> B = [year, year.^0];  
>> A = inv(B'*B)*B'*mm
```

```
a 3.3582e+000  
b -6.7128e+003
```

From the data, sea levels are rising by about 3.3mm per year on average

```
>> plot(year,mm,'b',year,B*A,'r');  
>> xlabel('year');  
>> ylabel('mm')  
>> title('Sea Level Increase')
```



5b) What will global sea levels be

- In the year 2050? (24 years from now)
- In the year 2080? (50 years from now)

```
>> [2050, 1] * A
```

```
ans = 1.7149e+002
```

In the year 2050, sea levels should be up 171mm at this rate

```
>> [2080, 1] * A
```

```
ans = 2.7224e+002
```

```
>>
```

In the year 2080, sea levels should be up 272mm at this rate

5c) The average elevation of Miami, FL is 1.0 meters above sea level at high tide. How long will it take for sea levels to rise 1.0 meter?

```
>> roots(A - [0 ; 1000])
```

```
ans = 2.2967e+003
```

At this rate of increase, sea levels should up 1.0m by the year 2296.

Problem 6-7) Sea Ice: The area covered by sea ice is recored by the National Snow and Ice Data Center:

6) Approximate this data from the years 1979 - present with a line

$$Area \approx ay + b$$

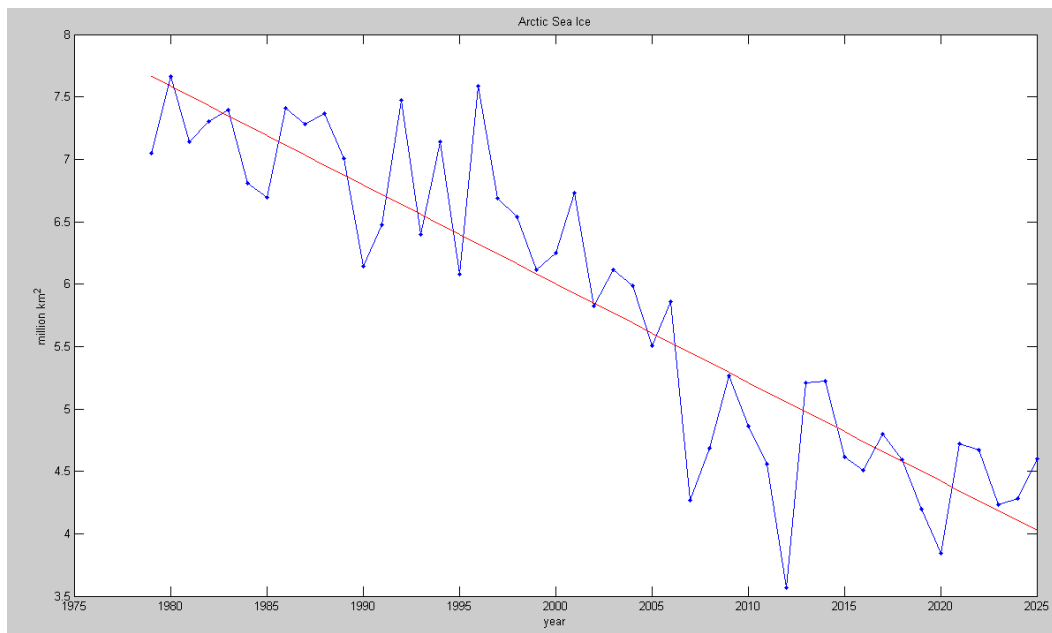
Matlab Code

```
>> year = Data(:,1);  
>> ICE = Data(:,2);  
>> B = [year, year.^0];  
>> format short e  
>> A = inv(B'*B)*B'*ICE
```

A =

```
-7.9116e-002  
1.6424e+002
```

```
>> plot(year,ICE,'b.-',year,B*A, 'r');  
>> xlabel('year')  
>> ylabel('million km^2')  
>> title('Arctic Sea Ice')  
>>
```



From this curve fit, when do you expect the Arctic to be ice free? (First time in 5 million years)

```
>> roots(A)  
  
ans = 2.0759e+003
```

Assuming a constant rate of ice melt, the Arctic should be ice free in the year 2075

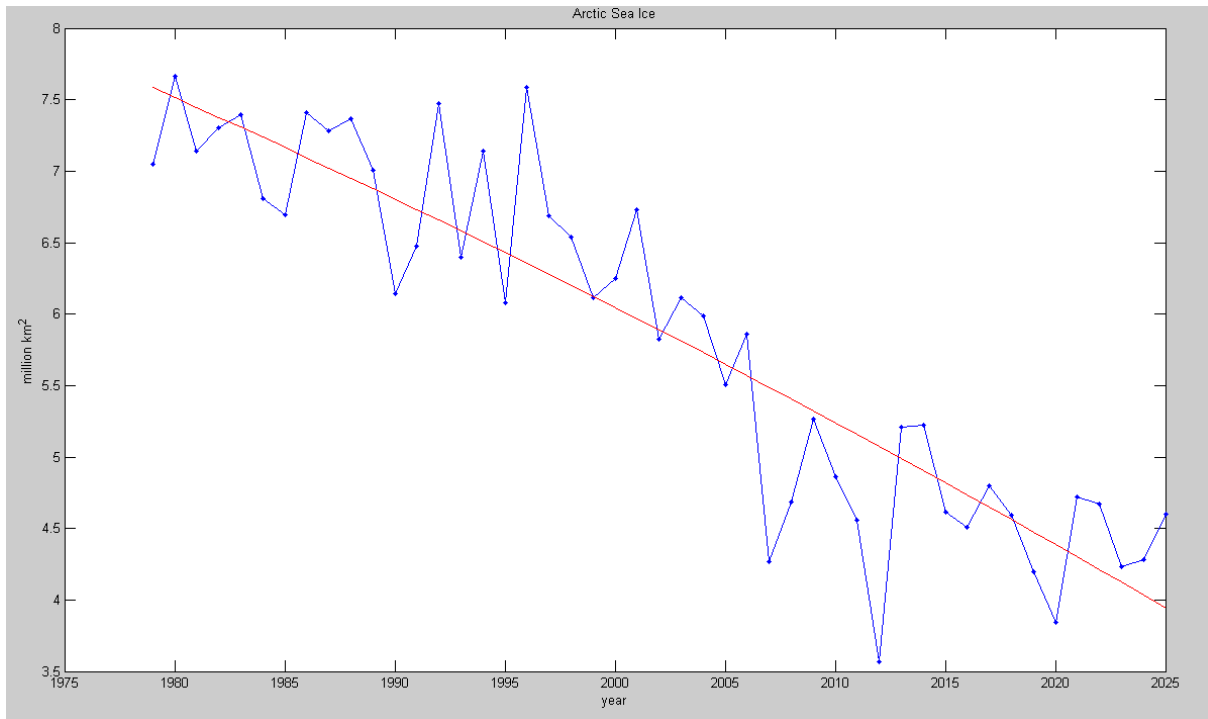
7) Approximate this data with a parabolic curve fit:

$$Area \approx ay^2 + by + c$$

From this curve fit, when do you expect the Arctic to be ice free?

In Matlab

```
>> B = [year.^2, year, year.^0];  
>> A = inv(B'*B)*B'*ICE  
  
a -2.3336e-004  
b 8.5526e-001  
c -7.7103e+002  
  
>> plot(year,ICE,'b.-',year,B*A, 'r');  
>> xlabel('year')  
>> ylabel('million km^2')  
>> title('Arctic Sea Ice')  
>>
```



Find the year where the Arctic will be ice free

```
>> roots(A)  
  
2.0648e+003  
1.6002e+003
```

Assuming the rate of ice melt increases each year at a constant rate, the Arctic will be ice free in the year 2064

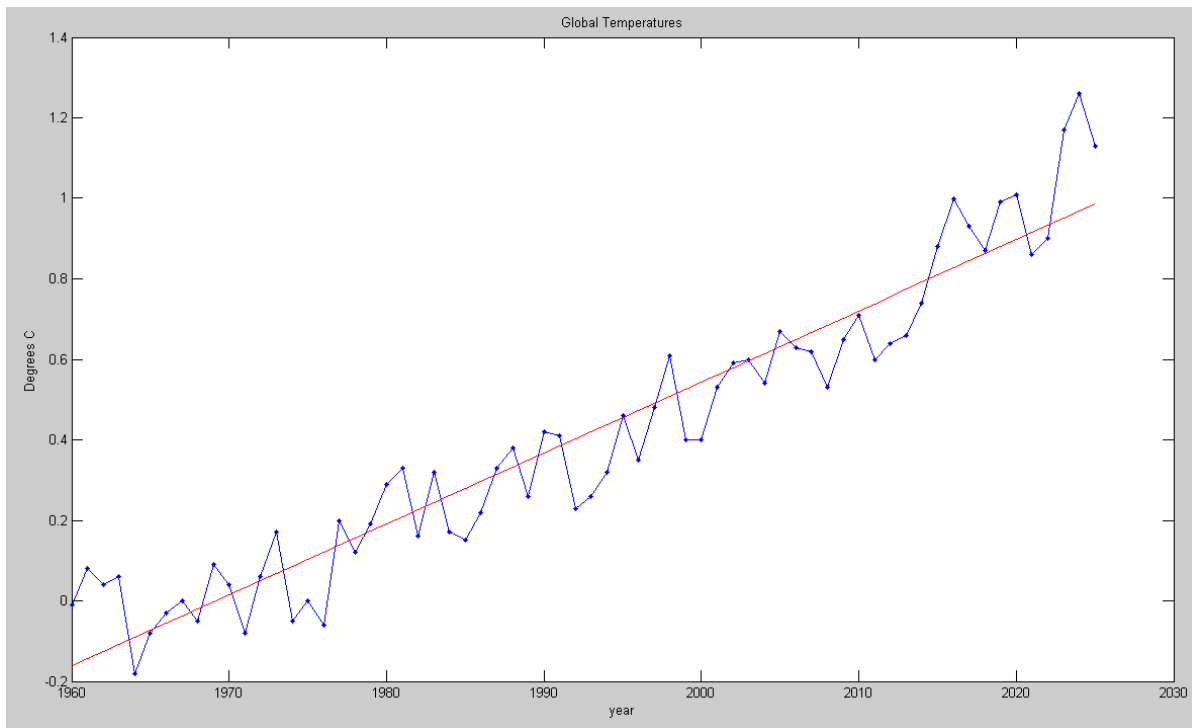
Problem 8-9: World Temperatures. NASA Goddard has been keep records since 1880 (144 years of data).

8) Determine a least-squares curve fit for this data from the year 1960 - resent in the form of

$$\delta T = ay + b$$

In Matlab

```
>> year = Data(:,1);  
>> dT = Data(:,2);  
>> B = [year, year.^0];  
>> A = inv(B'*B)*B'*dT  
  
a 1.7656e-002  
b -3.4767e+001  
  
>> plot(year,dT,'b.-',year,B*A, 'r');  
>> xlabel('year')  
>> ylabel('Degrees C')  
>> title('Global Temperatures')  
>>
```



Based upon this data, what will the temperature be in the year
2050? (24 years from now)

```
>> y = 2050;  
>> [y, 1] * A  
  
ans = 1.4269e+000
```

Assuming constant increase each year (linear curve fit), in 2050 the planet should be at +1.4269C.

2080? (50 years from now)

```
>> y = 2080;  
>> [y, 1] * A  
  
ans = 1.9565e+000
```

In 2080, the planet will be +1.95C

When will we reach +6 degrees C?

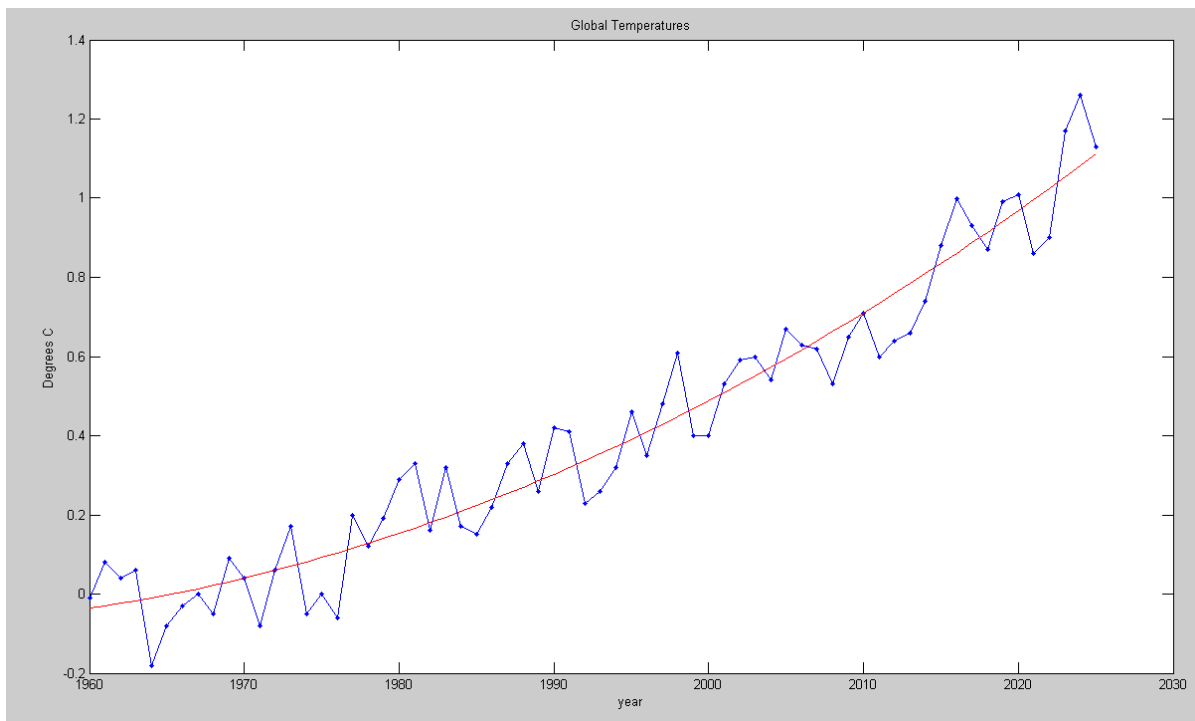
```
>> roots(A - [0; 6])  
  
ans = 2.3090e+003
```

Assuming constant temperature increase each year, we'll hit +6C in the year 2309

9) Determine a least-squares curve fit for this data in the form of

$$\delta T \approx ay^2 + by + c$$

```
>> B = [year.^2, year, year.^0];  
>> A = inv(B'*B)*B'*dT  
Warning: Matrix is close to singular or badly scaled.  
Results may be inaccurate. RCOND = 4.230324e-022.  
  
a 1.8222e-004  
b -7.0848e-001  
c 6.8858e+002  
  
>> plot(year,dT,'b.-',year,B*A, 'r');  
>> xlabel('year')  
>> ylabel('Degrees C')  
>> title('Global Temperatures')  
>>
```



Based upon this data, what will the temperature be in the year

2050? (24 years from now)

```
>> y = 2050;  
>> [y^2, y, 1] * A  
  
ans = 1.9632e+000
```

Assuming global temperatures are increasing each year at an increasing rate, we should be at + 1.96C in the year 2050

```
>> y = 2080;  
>> [y^2, y, 1] * A  
  
ans = 3.2855e+000
```

Assuming global temperatures are increasing each year at an increasing rate, we should be at + 3.28C in the year 2080

```
>> roots(A - [0;0;6])  
  
2.1268e+003  
1.7614e+003
```

Assuming global temperatures are increasing each year at an increasing rate, we should be at + 6C in the year 2126 (100 years from now)

10) What does a temperature rise of 10 degrees mean for the planet?

not graded - too political

The Permian Extinction Event suggests that it's not good: no animals larger than a mouse survived the Permian Extinction - which was triggered by CO2 levels at 2000ppm and a +10 degree C temperature rise.

One Degree: 2021 Summers like 2003 where a heat wave in France caused 10,000 deaths become the norm. Flows of the Po and Rhine river decrease. Crop production drops.

```
-->roots(A - [0;0;1])
```

```
2.0211e+003
```

```
1.8670e+003
```

Two Degrees: 2050. Oceans absorb less CO2 (too hot) and soils start to release CO2. Vacations to the Mediterranean in the summer are just too hot. Crop failures in Africa and Central America cause mass migration. Coastal cities flood. 1/3rd of species face extinction.

```
-->roots(A - [0;0;2])
```

```
2.0509e+003
```

```
1.8372e+003
```

Three Degrees: 2074. Crop failures in China cause the migration of more than 1 billion people. Collapse of equatorial governments.

```
-->roots(A - [0;0;3])
```

```
2.0741e+003
```

```
1.8140e+003
```

Four Degrees: 2093. Spain becomes a desert. Mass migration to Northern latitudes. Rain forests burn up.

```
-->roots(A - [0;0;4])
```

```
2.0937e+003
```

```
1.7944e+003
```

Six Degrees: 2126. Ice caps are gone. Methane hydrates become unstable raising temperatures in a positive-feedback loop. Ocean circulation stops. Hydrogen sulfide producing bacteria flourish poisoning the air. The Ozone layer dissipates leaving the land sterilized with UV radiation. End-Permian-like conditions make life nearly impossible.

```
-->roots(A - [0;0;0;6])
```

```
2.1268e+003
```

```
1.7614e+003
```

Scary? Yes. That's why the rest of the world sees the Paris Climate Accord as being important. That's why the United Nations sees Global Warming as the #1 threat - far greater than terrorism. Far greater than COVID.