

ECE 111 - Homework #2

Week #2: Algebra

Newton's Method

Problems 1 & 2) Let x and y be related by:

$$y = \frac{1}{5} \cdot x^2 + 2 \sin(1.4x)$$

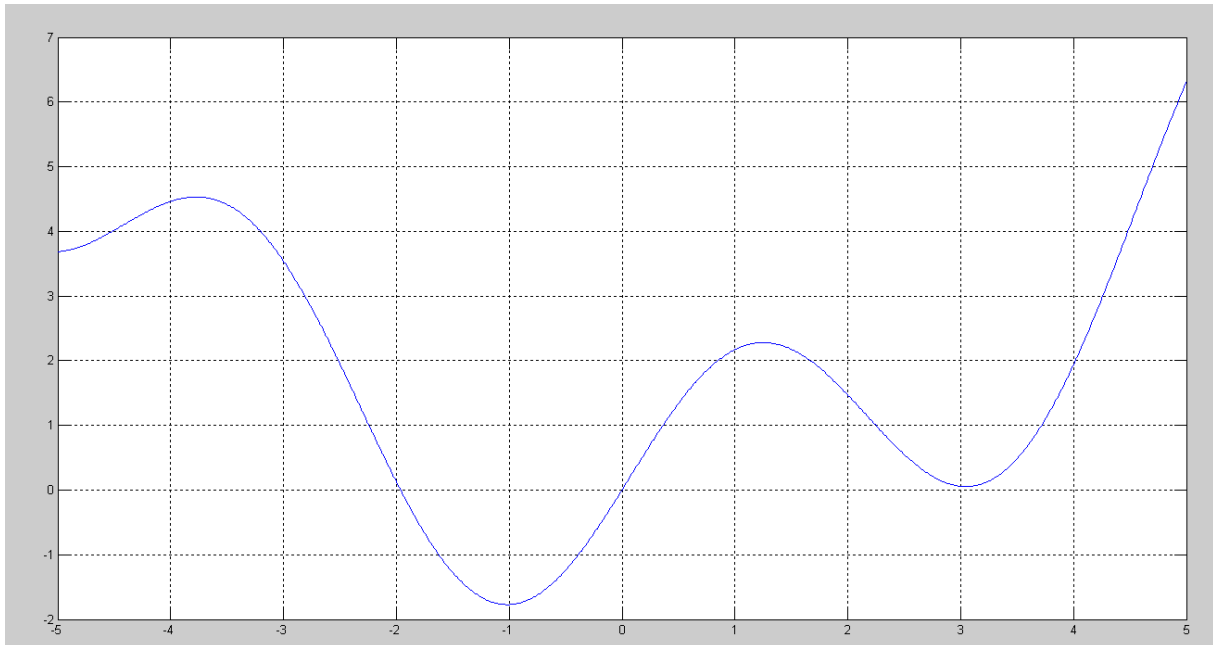
1) Find all solutions in the range of $(-5 < x < 5)$ using graphical methods for $y = 3$. (Plot x vs. y and find all values of x that result in $y = 3$.)

Matlab Code

```
x = [-5:0.01:5]';  
y = 1/5 * x.^2 + 2*sin(1.4*x);  
plot(x, y)  
grid
```

From the graph, there are two solutions:

- $x = -2.8$
- $x = +4.3$



2) Find the solutions to problem #1 using Newton's method

Start with a Matlab function where

- you pass a guess for x , and
- it returns the error in y

```
function [e] = Probl(x)
    y = 1/5 * x.^2 + 2*sin(1.4*x);
    e = y - 3;
end
```

Using Newton's method, iterate to find x so that the error is zero. The answer depends upon your initial guess

```
x3 = -3;
for n=1:10
    x1 = x3;
    y1 = Probl(x1);
    x2 = x1 + 0.1;
    y2 = Probl(x2);
    x3 = x1 - (x2-x1)/(y2-y1) * y1;
    disp([n, x1, y1])
end
```

Result: Initial guess is $x = -3$

1	-2.0000000000000000	-2.869976300311810
2	-2.836832519603086	0.085349876976371
3	-2.808727709354906	-0.000622053939242
4	-2.808928056582905	-0.000002425416752
5	-2.808928837865252	-0.000000009275237
6	-2.808928840853020	-0.000000000035468
7	-2.808928840864446	-0.000000000000135
8	-2.808928840864489	-0.000000000000001
9	-2.808928840864490	0.000000000000000
10	-2.808928840864490	0.000000000000000

Result: Initial guess is $x = +4$

n	x	error
1	4.0000000000000000	-1.062533275744643
2	4.271885052708536	0.053896869261505
3	4.259776556237603	0.000959008280514
4	4.259560319745593	0.000015777113910
5	4.259556762096143	0.000000259160226
6	4.259556703656922	0.000000004256949
7	4.259556702697004	0.000000000069925
8	4.259556702681236	0.000000000001148
9	4.259556702680976	0.000000000000017
10	4.259556702680973	0.000000000000001

Problems 3 & 4) Let x and y be related by

$$y = \sin(1.7x)$$

$$y = \frac{1}{5} \cdot (x^2 - 1)$$

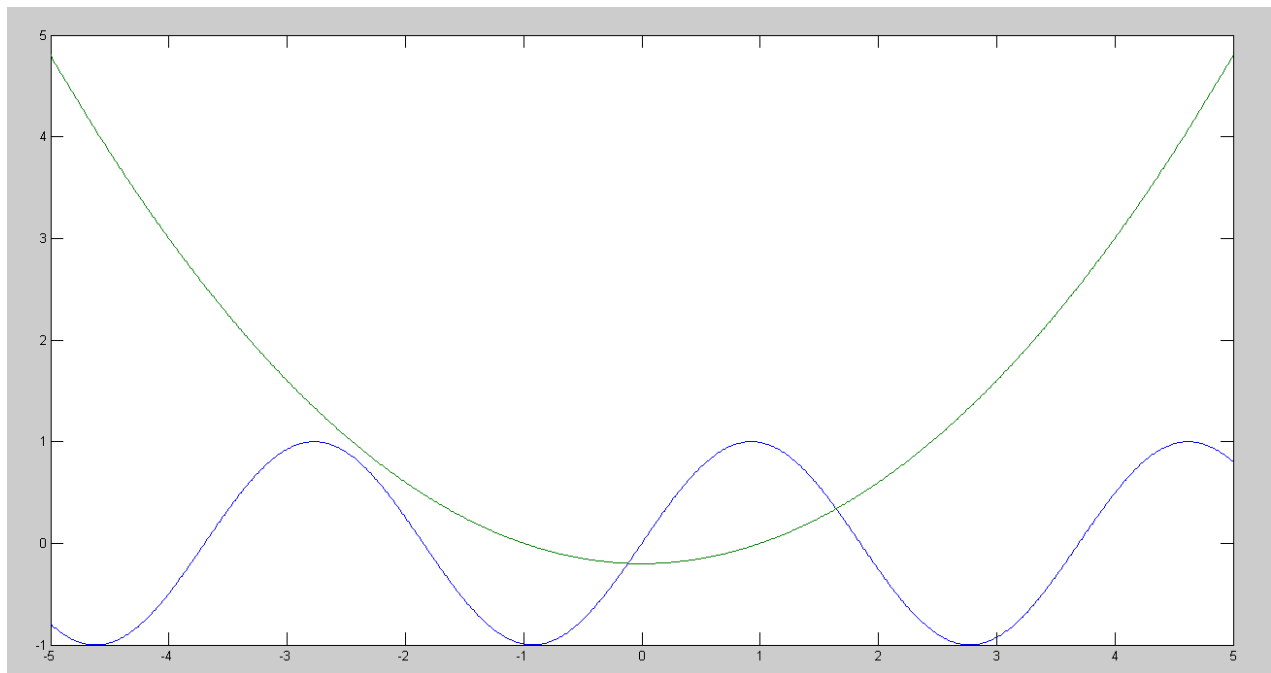
3) Find all solutions in the range of $(-5 < x < 5)$ using graphical methods. (Plot both functions on the same graph. The solution is when the two functions intersect.)

In Matlab

```
x = [-5:0.01:5]';  
y1 = sin(1.7*x);  
y2 = 1/5 * ( x.^2 - 1);  
plot(x,y1,x,y2)
```

From the graph, there are two solutions (where the graphs intersect)

- $x = -0.1$
- $x = +1.6$



4) Find the solutions to problem #3 using Newton's method.

Start with a Matlab function where

- You pass x, and
- It returns in error in y

```
function [e] = Prob3(x)
    y1 = sin(1.7*x);
    y2 = 1/5 * (x^2 - 1);
    e = y1 - y2;
end
```

Find value of x that results in zero error. Note that you converge to different answers based upon your initial guess

```
x3 = 0;
for i=1:10
    x1 = x3;
    y1 = Prob3(x1);

    x2 = x1 + 0.01;
    y2 = Prob3(x2);

    x3 = x1 - (x2-x1)/(y2-y1) * y1;
    disp([i, x1, y1]);
    pause(0.1);
end
```

Results for an initial guess of $x = 0$

n	x	error
1	0	0.2000000000000000
2	-0.117791310368527	-0.001684622779364
3	-0.116808415967771	-0.000000696651727
4	-0.116808009540391	-0.000000000313031
5	-0.116808009357768	-0.000000000000141
6	-0.116808009357686	-0.000000000000000
7	-0.116808009357686	0.000000000000000
8	-0.116808009357686	0
9	-0.116808009357686	0
10	-0.116808009357686	0

Result for an initial guess of $x = +2.00$

n	x	error
1	2.000000000000000	-0.855541102026831
2	1.649624761162746	-0.013377456784388
3	1.643733685889617	-0.000063183976538
4	1.643705763800772	-0.000000191536201
5	1.643705679156201	-0.000000000579058
6	1.643705678900300	-0.000000000001751
7	1.643705678899527	-0.000000000000005
8	1.643705678899525	-0.000000000000000
9	1.643705678899525	-0.000000000000000
10	1.643705678899525	-0.000000000000000

Newton's Method with a Temperature Sensor

Assume the temperature - resistance relationship of a sensor is:

$$R = 3000 \cdot \exp\left(\frac{3500}{T+273} - \frac{3500}{298}\right) \Omega$$

$$V = \left(\frac{R}{R+10,000}\right) 10V$$

```
T = [-20:0.01:20]';  
R = 3000 * exp(3500 ./ (T + 273) - 3500/298);  
V = R ./ (R + 10000) * 10;  
plot(T,R)  
plot(T,V)
```

5) Use Newton's method to find the temperature when

- R = 15,000 Ohms
- R = 5,000 Ohms

Start with a Matlab function that returns the error in T

```
function [e] = Prob5(T)  
R = 3000 * exp(3500/(T+273) - 3500/298);  
e = R - 15000;  
end
```

Solve using Newton's method

```
x3 = 2;  
for i=1:10  
    x1 = x3;  
    y1 = Prob5(x1);  
  
    x2 = x1 + 0.01;  
    y2 = Prob5(x2);  
  
    x3 = x1 - (x2-x1)/(y2-y1) * y1;  
    disp([i, x1, y1]);  
    pause(0.1);  
end
```

Result for R = 15,000 Ohms

n	T	error
1	2.0000000000000000	-6988.175018752198
2	-16.851501446160	5442.064538102073
3	-11.8593552301319	742.8329795551890
4	-10.939716488811312	19.531601634838808
5	-10.914192821914677	0.008878916967660
6	-10.914181201686722	-0.000002598038918
7	-10.914181205086891	0.000000000734872
8	-10.914181205085930	-0.000000000010914
9	-10.914181205085944	-0.000000000010914
10	-10.914181205085958	0.000000000041837

Result for R = 5000 Ohms

1	2.0000000000000000	3011.824981247802
2	10.1247854893754	560.4893497492476
3	12.433935954668193	31.296214097968004
4	12.578767638057881	0.105141241541787
5	12.579257739681408	-0.000024957885216
6	12.579257623340768	0.000000006225491
7	12.579257623369788	-0.000000000000909
8	12.579257623369784	-0.000000000000909
9	12.579257623369781	0.000000000007276
10	12.579257623369815	-0.000000000000909

6) Use Newton's method to determine the temperature when

- $V = 6.00V$
- $V = 4.00V$

Assume temperature and voltage are related with:

$$R = 3000 \cdot \exp\left(\frac{3500}{T+273} - \frac{3500}{298}\right) \Omega$$

$$V = \left(\frac{R}{R+10,000}\right) 10V$$

Solution: Start with a Matlab function

- Which is passed T and
- Returns the error in the voltage

```
function [e] = Prob7(T)
    R = 3000 * exp(3500/(T+273) - 3500/298);
    V = R / (R + 10000) * 10;
    e = V - 6;
end
```

Find the solution to $e = 0$ using Newton's method

```
clc
x3 = 2;
for i=1:10
    x1 = x3;
    y1 = Prob7(x1);

    x2 = x1 + 0.01;
    y2 = Prob7(x2);

    x3 = x1 - (x2-x1)/(y2-y1) * y1;
    disp([i, x1, y1]);
    pause(0.1);
end
```

Results for $V = 6$:

n	T	error
1	2.000000000000000	-1.551908266048026
2	-11.579202517281226	0.081248749883476
3	-10.913498739233475	-0.000083459071117
4	-10.914181195772940	-0.000000001138893
5	-10.914181205085841	-0.000000000000007
6	-10.914181205085900	-0.000000000000002
7	-10.914181205085914	-0.000000000000002
8	-10.914181205085928	-0.000000000000002
9	-10.914181205085942	-0.000000000000002
10	-10.914181205085956	0.000000000000006

Results for $V = 4.00V$

n	T	error
1	2.0000000000000000	0.448091733951974
2	5.920804170434935	0.011743296878477
3	6.029468036855720	0.000009320422354
4	6.029554432372176	-0.000000000746661
5	6.029554425451004	0.000000000000057
6	6.029554425451531	0.000000000000002
7	6.029554425451548	0.000000000000002
8	6.029554425451565	-0.000000000000007
9	6.029554425451499	0.000000000000002
10	6.029554425451516	0.000000000000002

Comment:

- Numerical methods give you a lot more decimal places than graphical methods
- Newton's method is pretty much the same for any function
- The solution you converge to depends upon your initial guess. It helps to have a good guess of the answer before you zero in using numerical methods.