

ECE 111 - Homework #15

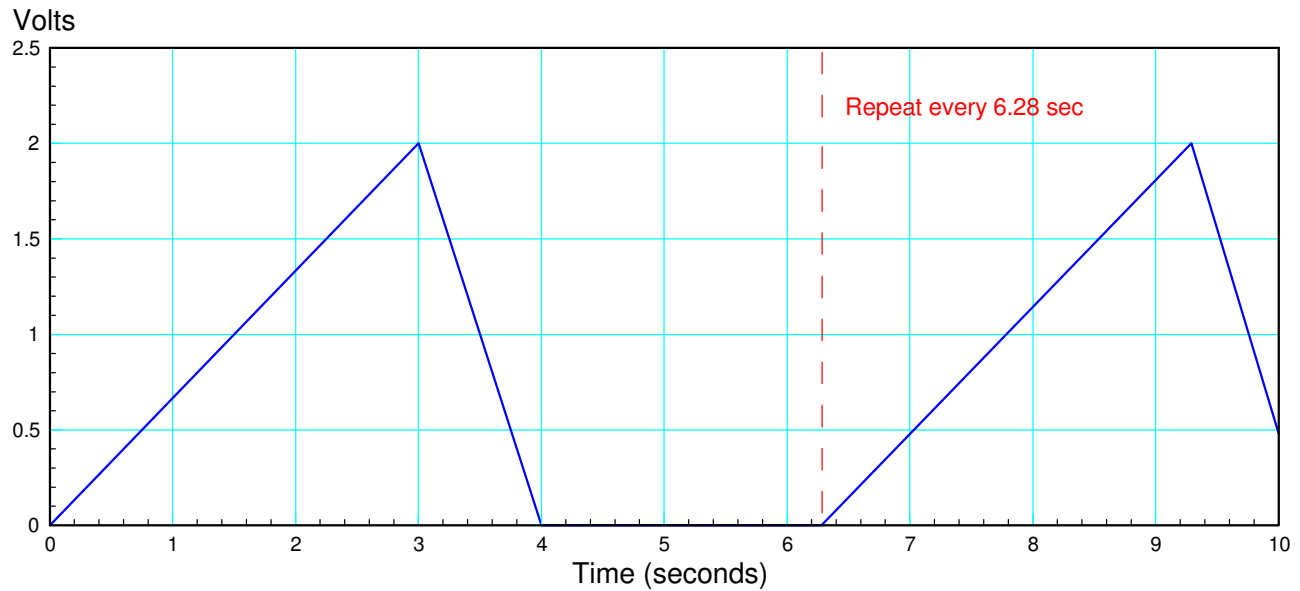
Fourier Series

Problem 1-5) Let $x(t)$ be a function which is periodic in 2π as shown below

$$x(t) = x(t + 2\pi)$$

or in Matlab:

```
t = [1:10000]' / 10000 * 2*pi + 1e-6;  
x = 1.5*t .* (t<3) + (14 - 4*t) .* (t>3) .* (t<4);  
plot(t,x)
```



$x(t)$ Note that $x(t)$ repeats every 2π seconds

Curve Fitting with a power series:

1) Using least squares, approximate $x(t)$ over the interval $(0, 2\pi)$ as

$$x(t) \approx a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

Plot $x(t)$ along with its approximation.

Curve Fitting using a Fourier Series

2) Using least squares, approximate $x(t)$ over the interval $(0, 2\pi)$ as

$$x(t) = a_0 + a_1 \cos(t) + b_1 \sin(t) + a_2 \cos(2t) + b_2 \sin(2t) + a_3 \cos(3t) + b_3 \sin(3t)$$

Plot $x(t)$ along with its approximation.

Curve Fitting using a Fourier Series (take 2)

3a) Determine $x(t)$ in terms of its Fourier Transform out to 3 rad/sec

$$a_0 = \text{mean}(x)$$

$$a_n = 2 \cdot \text{mean}(x \cdot \cos(nt))$$

$$b_n = 2 \cdot \text{mean}(x \cdot \sin(nt))$$

3b) Plot $x(t)$ and its Fourier approximation taken out to 3 rad/sec

Superposition

Assume X and Y are related by

$$Y = \left(\frac{1}{3s^2 + s + 1} \right) X$$

4) Determine the output, $y(t)$, at DC ($\omega = 0$)

5) Determine the output, $y(t)$, at 1 rad/sec

6) Determine the output, $y(t)$, at 2 rad/sec

7) Determine the output, $y(t)$, at 3 rad/sec

8) Determine the total answer, $y(t)$

- Plot $x(t)$ and $y(t)$