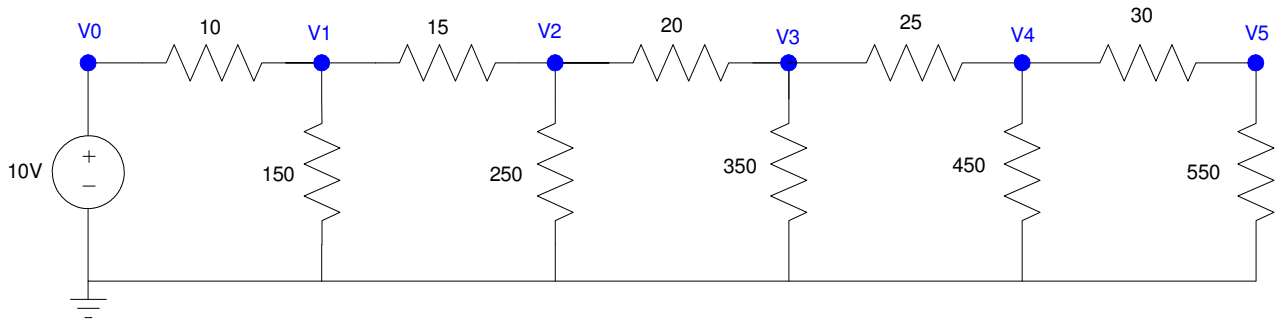
ECE 111 - Homework #9

EE 206 Circuts I
Due Monday, October 23rd

Voltage Nodes

1) Use Voltage Nodes write N equations for N unknowns for the following circuit.



$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{10}\right) + \left(\frac{V_1}{150}\right) + \left(\frac{V_1 - V_2}{15}\right) = 0$$

$$\left(\frac{V_2 - V_1}{15}\right) + \left(\frac{V_2}{250}\right) + \left(\frac{V_2 - V_3}{20}\right) = 0$$

$$\left(\frac{V_3 - V_2}{20}\right) + \left(\frac{V_3}{350}\right) + \left(\frac{V_3 - V_4}{25}\right) = 0$$

$$\left(\frac{V_4 - V_3}{25}\right) + \left(\frac{V_4}{450}\right) + \left(\frac{V_4 - V_5}{30}\right) = 0$$

$$\left(\frac{V_5 - V_4}{30}\right) + \left(\frac{V_5}{550}\right) = 0$$

2) Solve for the node voltages in Matlab.

Group terms

$$V_0 = 10$$

$$-\left(\frac{1}{10}\right)V_0 + \left(\frac{1}{10}+\frac{1}{150}+\frac{1}{15}\right)V_1 - \left(\frac{1}{15}\right)V_2 = 0$$

$$-\left(\frac{1}{15}\right)V_1 + \left(\frac{1}{15}+\frac{1}{250}+\frac{1}{20}\right)V_2 - \left(\frac{1}{20}\right)V_3 = 0$$

$$-\left(\frac{1}{20}\right)V_2 + \left(\frac{1}{20}+\frac{1}{350}+\frac{1}{25}\right)V_3 - \left(\frac{1}{25}\right)V_4 = 0$$

$$-\left(\frac{1}{25}\right)V_3 + \left(\frac{1}{25}+\frac{1}{450}+\frac{1}{30}\right)V_4 - \left(\frac{1}{30}\right)V_5 = 0$$

$$-\left(\frac{1}{30}\right)V_4 + \left(\frac{1}{30}+\frac{1}{550}\right)V_5 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \left(\frac{-1}{10}\right) & \left(\frac{1}{10}+\frac{1}{150}+\frac{1}{15}\right) & \left(\frac{-1}{15}\right) & 0 & 0 & 0 \\ 0 & \left(\frac{-1}{15}\right) & \left(\frac{1}{15}+\frac{1}{250}+\frac{1}{20}\right) & \left(\frac{-1}{20}\right) & 0 & 0 \\ 0 & 0 & \left(\frac{-1}{20}\right) & \left(\frac{1}{20}+\frac{1}{350}+\frac{1}{25}\right) & \left(\frac{-1}{25}\right) & 0 \\ 0 & 0 & 0 & \left(\frac{-1}{25}\right) & \left(\frac{1}{25}+\frac{1}{450}+\frac{1}{30}\right) & \left(\frac{-1}{30}\right) \\ 0 & 0 & 0 & 0 & \left(\frac{-1}{30}\right) & \left(\frac{1}{30}+\frac{1}{550}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

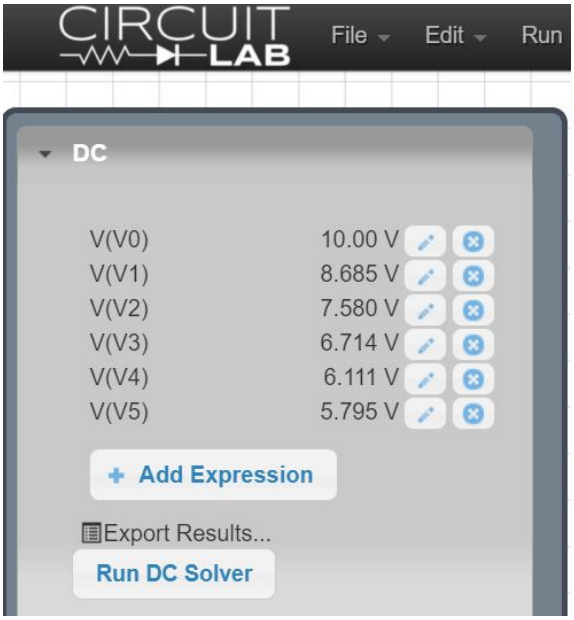
Solve in Matlab

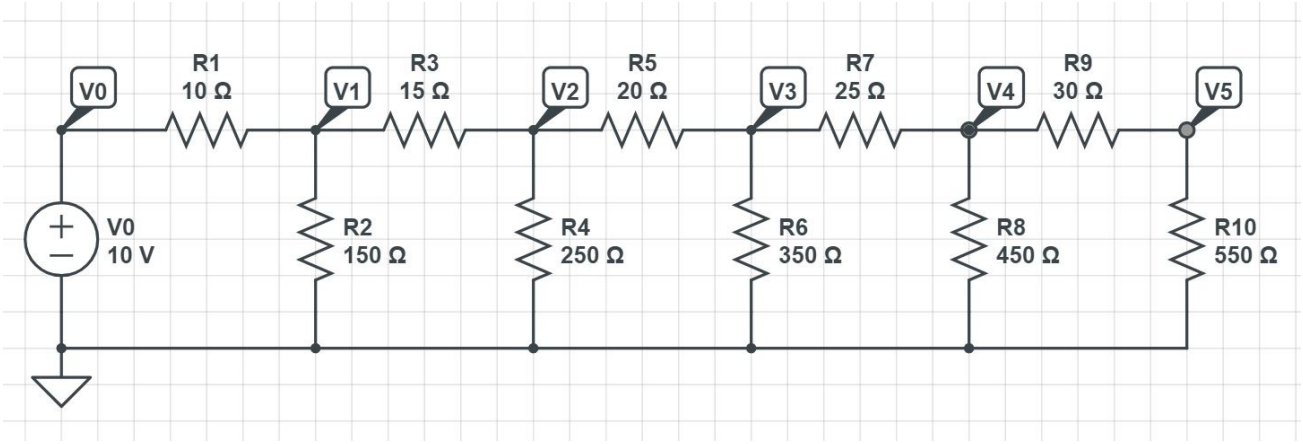
```
>> b1 = [1,0,0,0,0,0];
>> b2 = [-1/10,1/10+1/150+1/15,-1/15,0,0,0];
>> b3 = [0,-1/15,1/15+1/250+1/20,-1/20,0,0];
>> b4 = [0,0,-1/20,1/20+1/350+1/25,-1/25,0];
>> b5 = [0,0,0,-1/25,1/25+1/450+1/30,-1/30];
>> b6 = [0,0,0,0,-1/30,1/30+1/550];
>> B = [b1;b2;b3;b4;b5;b6]
>> B = [b1;b2;b3;b4;b5;b6]

    1.0000         0         0         0         0         0
   -0.1000    0.1733   -0.0667         0         0         0
         0   -0.0667    0.1207   -0.0500         0         0
         0         0   -0.0500    0.0929   -0.0400         0
         0         0         0   -0.0400    0.0756   -0.0333
         0         0         0         0   -0.0333    0.0352

>> A = [10;0;0;0;0;0]

    10
     0
     0
     0
     0
     0
```

```
>> V = inv(B)*A

V0   10.0000
V1    8.6848
V2    7.5804
V3    6.7144
V4    6.1114
V5    5.7953

>>
```

3) Check your answers in CircuitLab

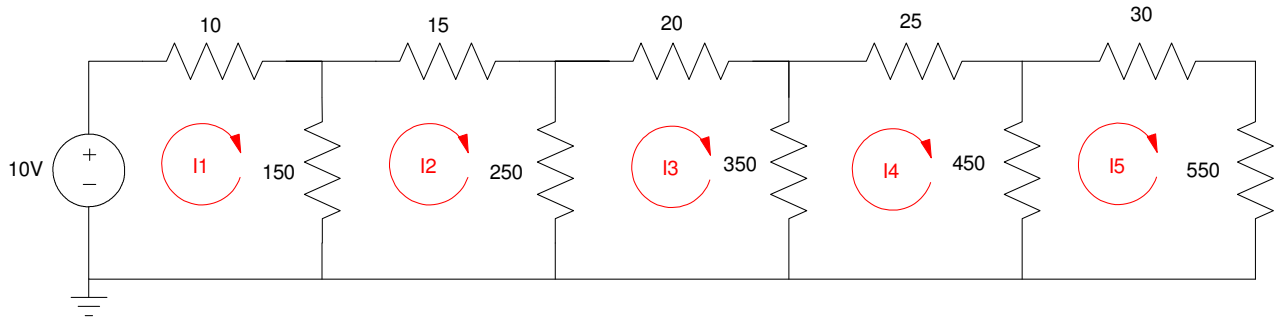
- Same results





Current Loops

4) Use Current Loops to write N equations for N unknowns for the following circuit.



Use concervation of voltage: the sum of the voltages around any closed loop is zero

$$-10 + 10(I_1) + 150(I_1 - I_2) = 0$$

$$150(I_2 - I_1) + 15(I_2) + 250(I_2 - I_3) = 0$$

$$250(I_3 - I_2) + 20(I_3) + 350(I_3 - I_4) = 0$$

$$350(I_4 - I_3) + 25(I_4) + 450(I_4 - I_5) = 0$$

$$450(I_5 - I_4) + 30(I_5) + 550(I_5) = 0$$

5) Solve for the currents in Matlab

Group terms

$$160I_1 - 150I_2 = 10$$

$$-150I_1 + 415I_2 - 250I_3 = 0$$

$$-250I_2 + 620I_3 - 350I_4 = 0$$

$$-350I_3 + 825I_4 - 450I_5 = 0$$

$$-450I_4 + 1030I_5 = 0$$

Place in matrix form

$$\begin{bmatrix} 160 & -150 & 0 & 0 & 0 \\ -150 & 415 & -250 & 0 & 0 \\ 0 & -250 & 620 & -350 & 0 \\ 0 & 0 & -350 & 825 & -450 \\ 0 & 0 & 0 & -450 & 1030 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> B = [160,-150,0,0,0 ; -150,415,-250,0,0 ; 0,-250,620,-350,0];
>> B = [B ; 0,0,-350,825,-450 ; 0,0,0,-450,1030]

          160        -150           0           0           0
         -150         415        -250           0           0
            0        -250         620        -350           0
            0           0        -350         825        -450
            0           0           0        -450        1030

>> A = [10;0;0;0;0];
>> I = inv(B)*A

    0.1315
    0.0736
    0.0433
    0.0241
    0.0105

>> mA = I * 1000
```

I1 131.5221 mA
I2   73.6236 mA
I3   43.3019 mA
I4   24.1179 mA
I5   10.5370 mA

6) Check your answers in CircuitLab.

Same results as Matlab